

Embedding the Universal See-Saw Mechanism in Pati-Salam Model

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CETUP* 2024

Lead, South Dakota

July 17, 2024

- ▶ The model is based on the gauge group defined as

$$SU(4)_C \times SU(2)_L \times SU(2)_R$$

(J.C.Pati, A.Salam,1973)

- ▶ The electric charge generator is given by

$$Q = T_{3L} + T_{3R} + \frac{B - L}{2}$$

- ▶ This gauge group extends the Standard Model (SM) by identifying the $SU(3)$ color group as a subgroup of $SU(4)$ and treating the lepton number as the fourth color.

$$4 \rightarrow 3_{\frac{1}{3}} \oplus 1_{-1}$$



- ▶ Quark-lepton unification through $SU(4)_c$.
- ▶ Quantization of electric charge leading to $Q_p + Q_e = 0$.
- ▶ The electroweak sector extends to be left-right (LR) symmetric.
- ▶ Existence of right-handed neutrinos.
- ▶ It treats $B - L$ as local gauge symmetry and gauge interaction itself conserves $B - L$ and fermion number.
- ▶ The absence of proton decay mediating processes opens the window to realize the symmetry at a low scale.
- ▶ A stepping stone to higher unification such as $SO(10)$, E_6 , $SU(16)$ etc.

- ▶ SM fermion spectrum (including right-handed neutrino)

$$Q_L^{i=1} = \begin{pmatrix} u_r & u_g & u_b & \nu_e \\ d_r & d_g & d_b & e \end{pmatrix}_L \sim (4, 2, 1);$$

$$Q_R^{i=1} = \begin{pmatrix} u_r & u_g & u_b & \nu_e \\ d_r & d_g & d_b & e \end{pmatrix}_R \sim (4, 1, 2)$$

- ▶ The simple Higgs sector comprises only left- and right-handed doublets of $SU(2)$.

$$\chi_L = \begin{pmatrix} \chi_u^r & \chi_u^b & \chi_u^g & \chi_\nu \\ \chi_d^r & \chi_d^b & \chi_d^g & \chi_e \end{pmatrix}_L \sim (4, 2, 1);$$

$$\chi_R = \begin{pmatrix} \chi_u^r & \chi_u^b & \chi_u^g & \chi_\nu \\ \chi_d^r & \chi_d^b & \chi_d^g & \chi_e \end{pmatrix}_R \sim (4, 1, 2);$$

- ▶ The general Higgs potential is given by:

$$\begin{aligned}
 V = & -\mu_L^2 \text{Tr}(\chi_L^\dagger \chi_L) - \mu_R^2 \text{Tr}(\chi_R^\dagger \chi_R) + \lambda_0 [(\text{Tr}(\chi_L^\dagger \chi_L))^2 + (\text{Tr}(\chi_R^\dagger \chi_R))^2] + \\
 & \lambda_1 [\text{Tr}(\chi_L^\dagger \chi_L \chi_L^\dagger \chi_L) + \text{Tr}(\chi_R^\dagger \chi_R \chi_R^\dagger \chi_R)] + \lambda_2 \text{Tr}(\chi_L^\dagger \chi_L) \text{Tr}(\chi_R^\dagger \chi_R) + \\
 & \lambda_3 \text{Tr}(\chi_L^\dagger \chi_L \chi_R^\dagger \chi_R) + \lambda_4 (\text{Tr}(\chi_L^T \tilde{\chi}_L^* \chi_R^\dagger \tilde{\chi}_R) + \text{h.c.}).
 \end{aligned}$$

- ▶ V invariant with respect to parity operation, except for the scalar mass terms for $\chi_{L/R}$ which breaks the symmetry softly.
- ▶ No CP-violating term is arising from the Higgs sector.
- ▶ $\epsilon_{\alpha\beta\gamma\delta} \epsilon_{ab}\epsilon_{cd} \chi_L^{\alpha a} \chi_L^{\beta b} \chi_R^{\gamma c} \chi_R^{\delta d}$ term will be there. This type of pfaffian structure leads to neutron oscillation ($\Delta B = 2$) but doesn't play a role in the mass spectrum of physical Higgs.

- ▶ **Two Neutral Higgs:** The mixing matrix between $\sigma_L \equiv \text{Re}(\chi_{\nu L})$ and $\sigma_R \equiv \text{Re}(\chi_{\nu R})$:

$$M_{\text{neutral}}^2 = \begin{pmatrix} 2(\lambda_0 + \lambda_1)\kappa_L^2 & (\lambda_2 + \lambda_3)\kappa_L\kappa_R \\ (\lambda_2 + \lambda_3)\kappa_L\kappa_R & 2(\lambda_0 + \lambda_1)\kappa_R^2 \end{pmatrix}.$$

- ▶ **Six up-type Higgs:** The mixing matrix between χ_{uL}^i and χ_{uR}^i :

$$M_{\text{up}}^2 = \frac{1}{2} \begin{pmatrix} -\lambda_3\kappa_R^2 & \lambda_3\kappa_L\kappa_R \\ \lambda_3\kappa_L\kappa_R & -\lambda_3\kappa_L^2 \end{pmatrix},$$

with $M_{Hu1}^2 = -\frac{1}{2}\lambda_3(\kappa_L^2 + \kappa_R^2)$ and $M_{Hu2}^2 = 0$.

- ▶ **Twelve down-type Higgs:** The mixing matrix between χ_{dL}^i and χ_{dR}^i :

$$M_{\text{down}}^2 = \begin{pmatrix} -\lambda_1\kappa_L^2 - \frac{1}{2}\lambda_3\kappa_R^2 & -\lambda_4\kappa_L\kappa_R \\ -\lambda_4\kappa_L\kappa_R & -\lambda_1\kappa_R^2 - \frac{1}{2}\lambda_3\kappa_L^2 \end{pmatrix}.$$

- ▶ The gauged Higgs sector Lagrangian, without the scalar potential, is simply:

$$\mathcal{L} = (D_\mu \chi_L)^\dagger (D_\mu \chi_L) + (D_\mu \chi_R)^\dagger (D_\mu \chi_R)$$

- ▶ We get 12 massive gauge bosons which is equal to the number of broken generators

$$SU(4)_C \times SU(2)_L \times SU(2)_R \rightarrow SU(3) \times U(1)_{EM}$$

- ▶ The charged gauge bosons don't mix at tree level and their masses are:

$$\begin{aligned} m_{W_L^{\mu\pm}}^2 &= \frac{1}{4} g_L^2 \kappa_L^2, \\ m_{W_R^{\mu\pm}}^2 &= \frac{1}{4} g_R^2 \kappa_R^2, \\ m_{Y_{i=1,2,3}^{\mu\pm}}^2 &= \frac{1}{4} g_4^2 (\kappa_L^2 + \kappa_R^2). \end{aligned}$$

- ▶ The mass part of the Lagrangian is:

$$\mathcal{L}_{mass}^{Neutral} = \frac{1}{8} \begin{pmatrix} W_{\mu L}^0 & W_{\mu R}^0 & G_{\mu 15} \end{pmatrix} M_0^2 \begin{pmatrix} W_L^{\mu 0} \\ W_R^{\mu 0} \\ G^{\mu 15} \end{pmatrix}.$$

where the mass matrix square is given by

$$\begin{pmatrix} g_L^2 \kappa_L^2 & 0 & -\sqrt{\frac{3}{2}} g_4 g_L \kappa_L^2 \\ 0 & g_R^2 \kappa_R^2 & -\sqrt{\frac{3}{2}} g_4 g_R \kappa_R^2 \\ -\sqrt{\frac{3}{2}} g_4 g_L \kappa_L^2 & -\sqrt{\frac{3}{2}} g_4 g_R \kappa_R^2 & \frac{3}{2} g_4^2 (\kappa_L^2 + \kappa_R^2) \end{pmatrix}.$$

- ▶ Matching condition:

$$\frac{1}{e^2} = \frac{1}{g_L^2} + \frac{1}{g_R^2} + \frac{2}{3g_4^2}.$$

- ▶ After decoupling the photon, the massive gauge boson mixing matrix looks like:

$$M_{Z_L-Z_R}^2 = \frac{1}{4} \begin{pmatrix} (g_Y^2 + g_L^2) \kappa_L^2 & g_Y^2 \sqrt{\frac{g_Y^2 + g_L^2}{g_R^2 - g_Y^2}} \kappa_L^2 \\ g_Y^2 \sqrt{\frac{g_Y^2 + g_L^2}{g_R^2 - g_Y^2}} \kappa_L^2 & \frac{g_Y^4}{g_R^2 - g_Y^2} \kappa_L^2 + \frac{g_R^4}{g_R^2 - g_Y^2} \kappa_R^2 \end{pmatrix}.$$

- ▶ The mixing angle β given approximately by

$$\beta \simeq \frac{g_Y^2}{g_R^4} \sqrt{(g_L^2 + g_Y^2)(g_R^2 - g_Y^2)} \frac{\kappa_L^2}{\kappa_R^2}.$$

- ▶ We see, at this point, no Yukawa couplings are allowed.
- ▶ That motivates us to introduce $SU(4)_C$ charged vector-like fermions to give consistent mass to fermions and help realize the Universal Seesaw mechanism in this model.

$$\Psi_{10}^i(10, 1, 1) \rightarrow \Omega(6_{1/3}) \oplus D(3_{-\frac{1}{3}}) \oplus E(1_{-1}).$$

Both handedness $((10, 1, 1) \oplus (\bar{10}, 1, 1))$ are required for mass generation which helps in anomaly cancellation as well.

- ▶ And we have the adjoint representation as well:

$$\Psi_{15}^i(15, 1, 1) \rightarrow \Sigma(8_0) \oplus U(3_{\frac{2}{3}}) \oplus \bar{U}(\bar{3}_{-\frac{2}{3}}) \oplus N(1_0).$$

where i runs on three generations of fermions.

- ▶ The Yukawa Lagrangian gives 6×6 mass matrices for up-type quarks (u, U), down-type quarks (d, D) and charged leptons (e, E). With parity symmetry:

$$M_u = \begin{pmatrix} 0 & y_{15}\kappa_L \\ y_{15}^\dagger\kappa_R & M_{15} \end{pmatrix}$$

$$M_d = \begin{pmatrix} 0 & y_{10}\kappa_L \\ y_{10}^\dagger\kappa_R & M_{10} \end{pmatrix}$$

$$M_e = \begin{pmatrix} 0 & \sqrt{2}y_{10}\kappa_L \\ \sqrt{2}y_{10}^\dagger\kappa_R & M_{10} \end{pmatrix}$$

- ▶ The overall non-degeneracy factor in the down and electron sector may not have the necessary flexibility to generate masses accurately for all three generations.

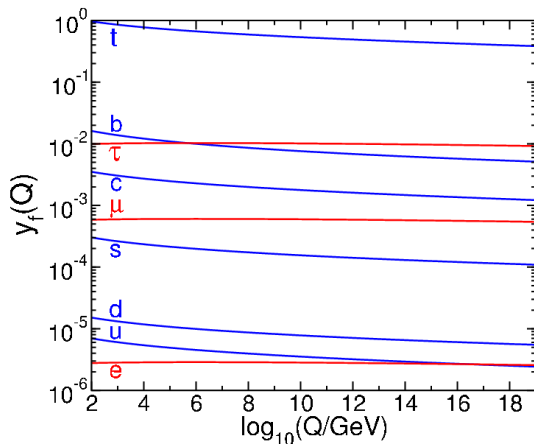


Figure: Standard model parameters in the tadpole-free pure \overline{MS} scheme.

(S.P.Martin, D.G.Robertson, 2019)

- ▶ The Seesaw mass matrix:

$$M_f = \begin{pmatrix} 0 & y_f \kappa_L \\ y_f^\dagger \kappa_R & M_F \end{pmatrix},$$

- ▶ One-loop contribution can be decomposed as

$$(\delta M)_{1\text{-loop}} = \begin{pmatrix} \delta M_{LL} & \delta M_{LH} \\ \delta M_{HL} & \delta M_{HH} \end{pmatrix}$$

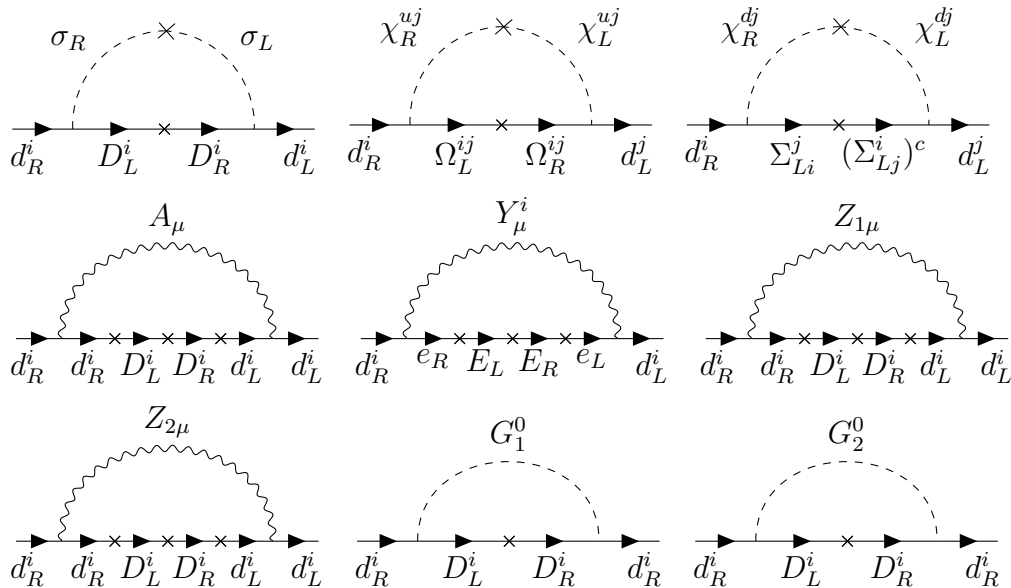
- ▶ Effective light fermion mass matrix, up to one-loop order:

$$m_f^{\text{light}} \simeq -y_f M_F^{-1} y_f^\dagger \kappa_L \kappa_R + \delta M_{LL} - \delta M_{LH} M_F^{-1} y_f^\dagger \kappa_R - y_f \kappa_L M_F^{-1} \delta M_{HL} \\ + y_f \kappa_L M_F^{-1} \delta M_{HH} M_F^{-1} y_f^\dagger \kappa_R$$

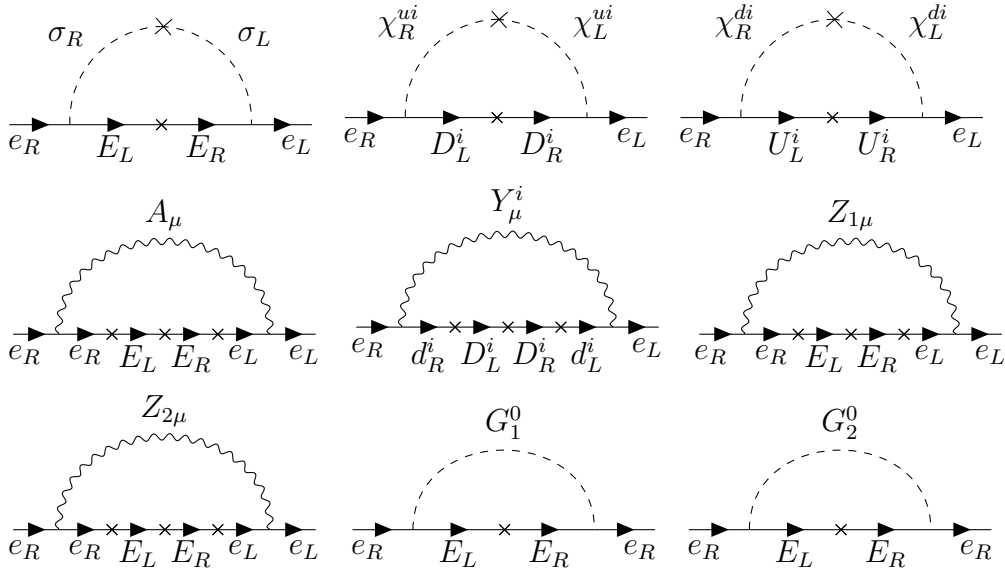
- ▶ In a renormalizable theory, δM_{LL} has to be finite and gauge independent through radiative corrections since no counter-term is associated.

(Gerard 't Hooft, 1971.)

One-Loop Corrections To Light Down-Quark Mass



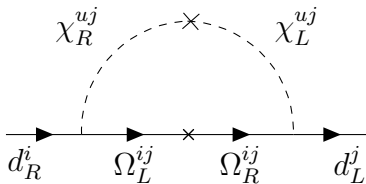
One-Loop Corrections To Light Electron Mass



Example With One Diagram



- ▶ Unlike charged lepton, the down sector can get one-loop radiative mass corrections through heavy sextet and up-type Higgs.



$$m_d^i \sim \frac{2y_{10}^2 M_{10}^i \sin\theta \cos\theta}{(4\pi)^2} \left[\frac{M_{Hu1}^2}{(M_{10}^i)^2 - M_{Hu1}^2} \ln \left(\frac{(M_{10}^i)^2}{M_{Hu1}^2} \right) - \frac{M_Y^2}{(M_{10}^i)^2 - M_Y^2} \ln \left(\frac{(M_{10}^i)^2}{M_Y^2} \right) \right]$$

- ▶ Hierarchy in bare mass terms can give rise to hierarchical radiative corrections throughout the three generations.

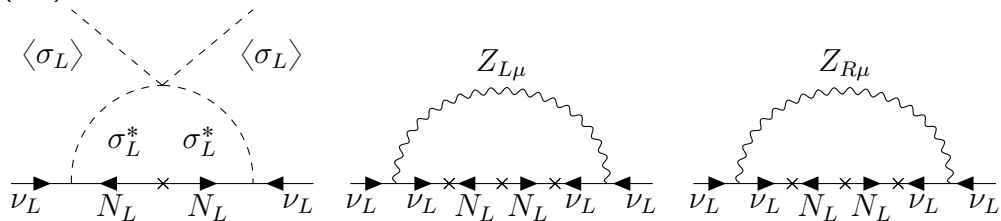
- ▶ The Lagrangian gives 9×9 mass matrix for neutrinos.

$$\mathcal{L}_{\nu N} \supset \frac{1}{2} \overline{F_L^\nu} \begin{pmatrix} 0 & 0 & -\frac{3}{2\sqrt{6}} y_{15} \kappa_L \\ 0 & 0 & -\frac{3}{2\sqrt{6}} y_{15} \kappa_R \\ -\frac{3}{2\sqrt{6}} y_{15}^T \kappa_L & -\frac{3}{2\sqrt{6}} y_{15}^T \kappa_R & M_{15} \end{pmatrix} F_R^\nu$$

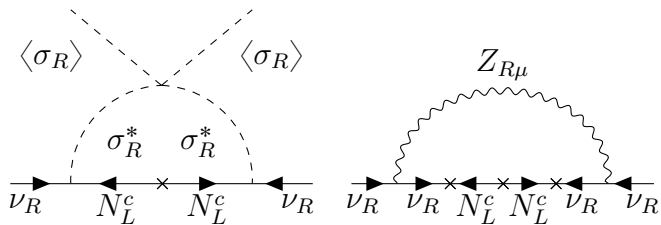
- ▶ The basis is written in $F_L^\nu \equiv (\nu_L, \nu_R^c, N_L)$, $F_R^\nu \equiv (\nu_L^c, \nu_R, N_L^c)$
- ▶ This mass matrix leads to the lightest neutrino being massless.
- ▶ In our case, neutrinos will get Dirac and Majorana mass contributions at one loop.

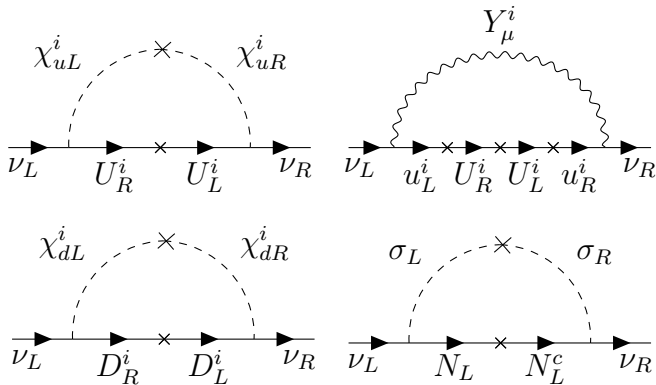
Radiative Majorana Mass Through One-Loop Correction

- ▶ (1,1) block:



- ▶ (2,2) block:





$$\begin{aligned}
 (m_\nu^D)^i_{1+2} = & \frac{3y_{15}^2 M_{15}^i \sin\theta \cos\theta}{16\pi^2} \left[\frac{M_{Hu1}^2}{(M_{15}^i)^2 - M_{Hu1}^2} \ln \left(\frac{(M_{15}^i)^2}{M_{Hu1}^2} \right) - \frac{M_Y^2}{(M_{15}^i)^2 - M_Y^2} \right. \\
 & \left. \ln \left(\frac{(M_{15}^i)^2}{M_Y^2} \right) \right] + \frac{3g_4^2 y_{15}^2 \kappa_L \kappa_R M_{15}^i}{16\pi^2} \left[\frac{(M_{15}^i)^2}{(M_{15}^i)^2 - M_Y^2} \ln \left(\frac{(M_{15}^i)^2}{M_Y^2} \right) \right]
 \end{aligned}$$

Now the mass matrix looks like this:

$$\mathcal{L}_{\nu N} \supset \frac{1}{2} \overline{F_L^\nu} \begin{pmatrix} m_L & m_\nu^D & -\frac{3}{2\sqrt{6}} y_{15} \kappa_L \\ (m_\nu^D)^T & m_R & -\frac{3}{2\sqrt{6}} y_{15} \kappa_R \\ -\frac{3}{2\sqrt{6}} y_{15}^T \kappa_L & -\frac{3}{2\sqrt{6}} y_{15}^T \kappa_R & M_{15} \end{pmatrix} F_R^\nu$$

The lightest neutrino mass:

$$\begin{aligned} M_\nu^\ell &\approx -m_L + \begin{pmatrix} m_\nu^D & -\frac{3}{2\sqrt{6}} y_{15} \kappa_L \end{pmatrix} \begin{pmatrix} m_R & -\frac{3}{2\sqrt{6}} y_{15} \kappa_R \\ -\frac{3}{2\sqrt{6}} y_{15}^T \kappa_R & M_{15} \end{pmatrix}^{-1} \begin{pmatrix} (m_\nu^D)^T \\ -\frac{3}{2\sqrt{6}} y_{15}^T \kappa_L \end{pmatrix} \\ &\approx \frac{\kappa_L}{\kappa_R} (m_\nu^D + (m_\nu^D)^T) + \frac{\kappa_L^2}{\kappa_R^2} m_R - m_L \end{aligned}$$

- ▶ The Strong CP Problem is related to the presence of this term in Lagrangian (a consequence of non-perturbative effects):

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

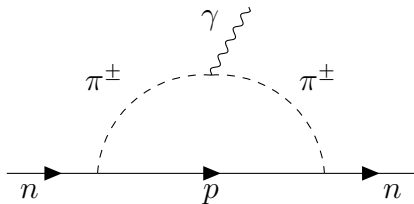
- ▶ Violates P and T but conserves C, so it violates CP.

(J.Schwinger, 1951)

- ▶ θ carries the information of CP violation in strong interactions.
- ▶ θ is shifted by the chiral transformation needed to diagonalize the quark mass matrix M.
- ▶ The invariant physical parameter $\bar{\theta}$ is

$$\bar{\theta} = \theta + \text{ArgDet}(M_Q)$$

- ▶ The Feynman diagram giving the leading-order contribution to the neutron EDM



$$d_n \simeq \frac{e \bar{\theta} g_A c_+ \mu}{8\pi^2 f_\pi^2} \log\left(\frac{\Lambda^2}{m_\pi^2}\right) \simeq 3 \times 10^{-16} \bar{\theta} e \text{ cm}$$

$$\mu = \frac{m_u m_d}{m_u + m_d}, g_A \simeq 1.27, c_+ \simeq 1.6, \Lambda = 4\pi f_\pi$$

(A.Hook,2018)

- ▶ Experimental bound, $d_n < 10^{-26} e \text{ cm} \implies \bar{\theta} < 10^{-10}$

(C.Abel et al.2020)

- ▶ Extreme smallness in this dimensionless parameter $\bar{\theta}$, is the strong CP problem.

$$\bar{\theta} = \theta + \text{ArgDet}(M_u M_d) + 5 \text{ArgDet}(M_{\text{sextet}}) + 6 \text{ArgDet}(M_{\text{octet}})$$

$$M_u = \begin{pmatrix} 0 & y_{15} \kappa_L \\ y_{15}^\dagger \kappa_R & M_{15} \end{pmatrix}; \quad M_d = \begin{pmatrix} 0 & y_{10} \kappa_L \\ y_{10}^\dagger \kappa_R & M_{10} \end{pmatrix}$$

- ▶ Anomaly coefficients are different for different representations of $SU(3)_C$
- ▶ It is clear that $\text{Det}(M_u)$ and $\text{Det}(M_d)$ are separately real.
- ▶ $M_{\text{sextet}} \equiv M_{10}$ and $M_{\text{octet}} \equiv M_{15}$ are the hermitian bare mass matrices realized in left-right symmetric Lagrangian. Parity is only softly broken in dimension 2 terms.
- ▶ Therefore, we have $\bar{\theta}=0$ at the tree level.

- ▶ We choose to work on the weak basis rather than the mass eigenstate.
- ▶ Radiatively corrected quark ($q = u, d$) mass matrix:

$$M^q = M_0^q(1 + C).$$

- ▶ So $\bar{\theta}$ can be expressed as

$$\bar{\theta} = \text{ArgDet}(1 + C) = \text{ImTr} \ln(1 + C) = \text{ImTr} C_1 + \text{ImTr}(C_2 - \frac{1}{2}C_1^2) + \dots$$

Using the loop expansion: $C = C_1 + C_2 + C_3 + \dots$

- ▶ The correction matrix for up- and down-type quark can be denoted by

$$\delta M^q = \begin{pmatrix} \delta M_{LL}^q & \delta M_{LH}^q \\ \delta M_{HL}^q & \delta M_{HH}^q \end{pmatrix}$$

- ▶ Elements of correction matrix enter into $\bar{\theta}$ in the following manner

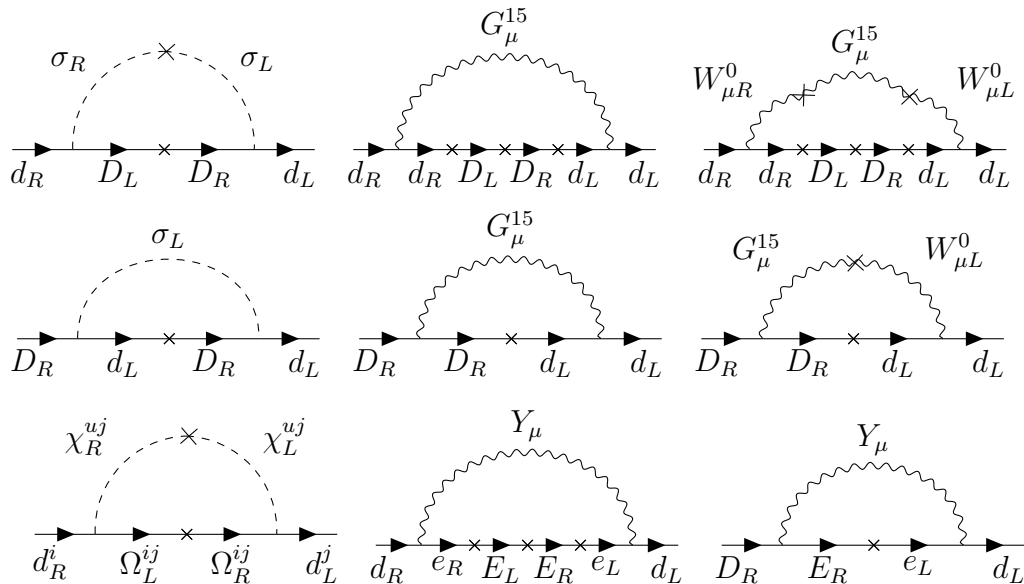
$$\bar{\theta} = \text{ImTr}\left[-\left(\frac{1}{\kappa_L \kappa_R}\right) \delta M_{LL}^q (y_q^\dagger)^{-1} M_P y_q^{-1} + \left(\frac{1}{\kappa_L}\right) \delta M_{LH}^q y_q^{-1} + \left(\frac{1}{\kappa_R}\right) \delta M_{HL}^q (y_q^\dagger)^{-1}\right]$$

where $y_q = y_{15}(y_{10})$ and $M_P = M_{15}(M_{10})$ is for up (down) quark.

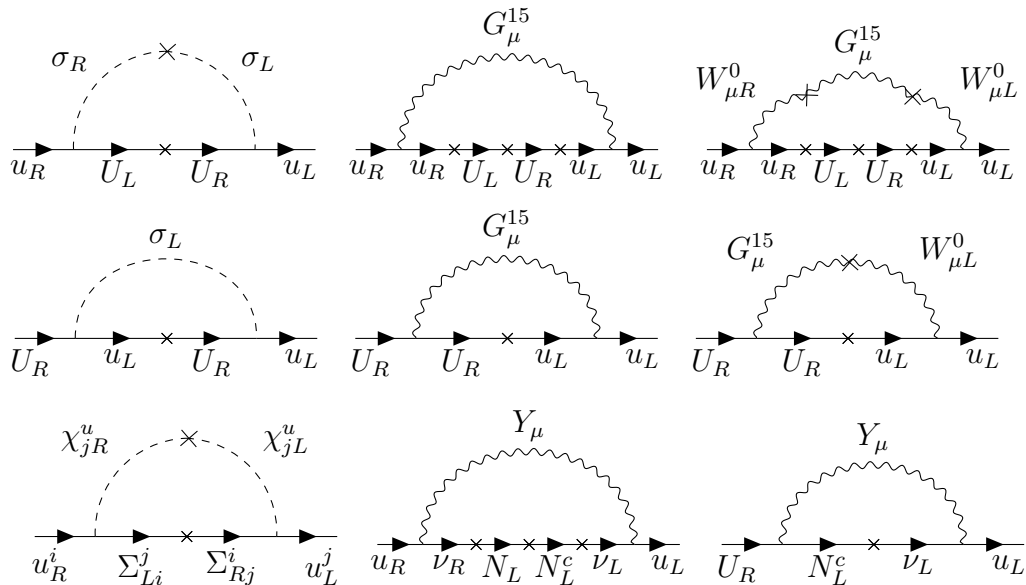
(K.S.Babu, R.N.Mohapatra,1989)

- ▶ δM_{HH}^q doesn't enter into the equation for $\bar{\theta}$. That means we need not compute one-loop correction to vector-like quark mass.

One-Loop Correction For Down Quark Mass

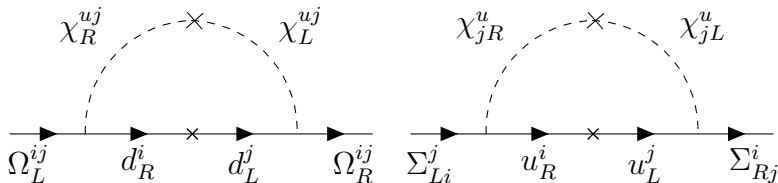


One-Loop Correction For Up Quark Mass



- ▶ The induced $\bar{\theta}$ can be written considering upto one-loop:

$$\bar{\theta} = \text{Im Tr}[M_{10}^{-1}(\delta M_{\Omega})] + \text{Im Tr}[M_{15}^{-1}(\delta M_{\Sigma})]$$

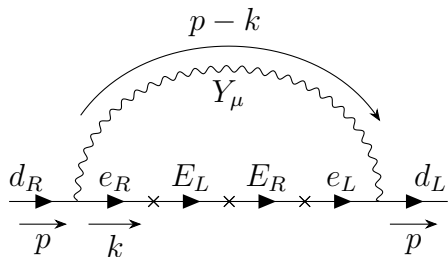


- ▶ Since we are treating mass as a part of the interaction, the cross on the fermion/boson line stands for all possible tree-level diagrams.
- ▶ Propagator with all possible mass insertion can be written as:

$$\overline{F}_R \left((M_F^0)^\dagger \frac{k^2}{k^2 - M_F^0 (M_F^0)^\dagger} \right) F_L$$

- ▶ **Unique features of having Quark-lepton unification:** Corrections coming from the SM lepton, sextet, and octet lines as well.
- ▶ $F_{L/R}^{\text{up}} \equiv (u, U)_{L/R}$, $F_{L/R}^{\text{down}} \equiv (d, D)_{L/R}$, $F_{L/R}^e \equiv (e, E)_{L/R}$, $F_{L/R}^\Omega \equiv \Omega_{L/R}$
- ▶ $F_L^\nu \equiv (\nu_L, \nu_R^c, N_L)$, $F_R^\nu \equiv (\nu_L^c, \nu_R, N_L^c)$, $F_{L/R}^\Sigma \equiv \Sigma_L / \Sigma_L^C$

- Each diagram separately gives zero contribution to $\bar{\theta}$



- The amplitude with leptoquark in the loop is proportional to

$$\delta M_{LL}^d \supset \int \frac{d^4 k}{(2\pi)^4} \left(\frac{g_4}{\sqrt{2}} \right)^2 \left(g_{\mu\nu} - \frac{(p-k)_\mu (p-k)_\nu}{M_Y^2} \right) \frac{\left((M_e^0)^\dagger \frac{k^2}{k^2 - M_e^0 (M_e^0)^\dagger} \right)}{(p-k)^2 - M_Y^2 + i\epsilon}$$

- ▶ It's contribution to $\bar{\theta}$:

$$\text{ImTr} \left[- \left(\frac{1}{\kappa_L \kappa_R} \right) \delta M_{LL}^d (y_{10}^\dagger)^{-1} M_{10} y_{10}^{-1} \right]$$

- ▶ We can evaluate the trace before momentum integration.
- ▶ Expanding $(M_e^0 (M_e^0)^\dagger - k^2)^{-1}$:

$$(M_e^0 (M_e^0)^\dagger - k^2)^{-1} \equiv \begin{pmatrix} A_e(k^2) & B_e(k^2) \\ B_e^\dagger(k^2) & C_e(k^2) \end{pmatrix}$$

with $A_e = A_e^\dagger$, $C_e = C_e^\dagger$ and B_e are 3×3 block matrices

- ▶ The term related to $\bar{e}_L e_R$ from $\left((M_e^0)^\dagger \frac{k^2}{k^2 - M_e^0 (M_e^0)^\dagger} \right)$ expansion:

$$\delta M_{LL}^d \supset k^2 \kappa_R B_e(k^2) y_{15}^\dagger$$

- ▶ So, the contribution to $\bar{\theta}$:

$$\begin{aligned} \bar{\theta} &\sim \text{ImTr} \left[B_e(k^2) y_{10}^\dagger (y_{10}^\dagger)^{-1} M_{10} y_{10}^{-1} \right] \\ &= \text{ImTr} \left[\left(\kappa_L^2 y_{10}^\dagger y_{10} - k^2 \right)^{-1} (M_{10})^\dagger C_e(k^2) M_{10} \right] = 0 \end{aligned}$$

with C_e being hermitian.

- ▶ This setup has the potential to solve the strong CP problem via parity along with some other appropriate symmetry.
- ▶ Fermion masses are generated by mixing usual fermions with vector-like fermions belonging to the "Universal Seesaw" class.

(A Davidson, KC Wali, 1987)

- ▶ All generation fermion mass can be realized through the one-loop radiative corrections.
- ▶ Lightest neutrino acquires Majorana and Dirac mass contribution at one-loop level.

Thank you