E_6: THE MINIMAL GRAND UNIFIED THEORY OF NEUTRINO MASS

Borut Bajc

J. Stefan Institute, Ljubljana, Slovenia

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Introduction

In the SM there is no correlation between the charged fermion sector and neutral one. Actually, strictly speaking, in the SM neutrinos are massless.

$$\mathcal{L}_Y = \underbrace{Y_u^{ij} H Q_i u_j^c + Y_d^{ij} H^* Q_i d_j^c + Y_e^{ij} H^* L_i e_j^c}_{\mathcal{L}_Y^{SM}} + \ldots$$

Simplest additions (for example ν^c) which incorporate the nonzero neutrino mass

$$\ldots = Y_\nu^{ij} H L_i \nu_j^c$$

do not connect the two sectors: the Yukawa from the neutrino sector (Y_ν) has nothing in common with the Yukawas from the charged sector (Y_u, Y_d, Y_e)

This however not that surprising: the SM is anyway not a theory of flavour, not even in the charged fermion sector

(no relations among Y_u, Y_d and Y_e)

Let's upgrade the SM embedding it into a GUT: this means that

- instead of three gauge couplings from

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

  we have now a simple gauge group

  (SU(5) or SO(10) or E_6)

- instead of five irreps + those needed for neutrino mass

$$\underbrace{Q}_{6} + \underbrace{L}_{2} + \underbrace{u^c}_{3} + \underbrace{d^c}_{3} + \underbrace{e^c}_{1} + \underbrace{?}_{?}$$

  we will have one or at most two irreps per generation

  ($\bar{5} + 10$ or 16 or 27)

What can we say in GUTs about fermion masses and mixings?

SU(5)

The SM fermions get unified, instead of 5 irreps of the SM (Q, L, u^c, d^c, e^c) one gets only 2 irreps of SU(5):

$$\bar{5} = (d^c, L) \quad , \quad 10 = (Q, u^c, e^c)$$

The Yukawa sector is more economical than in the SM

$$\mathcal{L}_Y = {Y_{10}}^{ij} 5_H 10_i 10_j + {Y_5}^{ij} 5^*_H 10_i \bar{5}_j + \ldots$$

GOOD: it has only two Yukawa matrices

BAD:

- wrong relation $M_D = M_E$
- no neutrino mass either, or if we add an SU(5) singlet ν^c

$$\ldots = {Y_\nu}^{ij} 5_H \bar{5}_i \nu^c_j$$

the same problem as in the SM: no relation between neutral sector Y_ν and charged sector Y_{10}, Y_5

SO(10)

The situation here more promising than in SU(5):

$$16 = (\underbrace{Q, L, u^c, d^c, e^c}_{SU(5)}, \nu^c)$$

ν^c automatically included, so neutrino masses nonzero and somehow related to other fermion masses

The minimal model

$$\mathcal{L}_Y = Y_{10}^{ij} 10_H 16_i 16_j + Y_{126}^{ij} 126_H 16_i 16_j$$

however does not work because in SO(10) 10_H is a real representation and so it has only one Higgs doublet in it:

the fit turns out not to work

Possible solutions are for example

1. add another (real) $10'_H$ with extra Yukawa

$$\delta\mathcal{L}_Y = Y^{ij}_{10'} 10'_H 16_i 16_j$$

   But now 3 Yukawa matrices (Y^{ij}_{10}, Y^{ij}_{126}, $Y^{ij}_{10'}$), not predictive

2. add an extra U(1) symmetry (for example a Peccei-Quinn global) so that 10_H is now automatically complex

   But now the symmetry is not SO(10) but instead SO(10)×U(1), i.e. it is not minimal

3. Another possibility is to supersymmetrise: 10_H is then automatically complex; but again non minimal, $SO(10)\times$ supersymmetry

Fits of fermion masses and mixings work well for $10 + \overline{126}$

Example susy SO(10) with $10 + 126$ Yukawas

$$\chi^2 \sim 4$$

mainly in m_d (pull ~ 2)

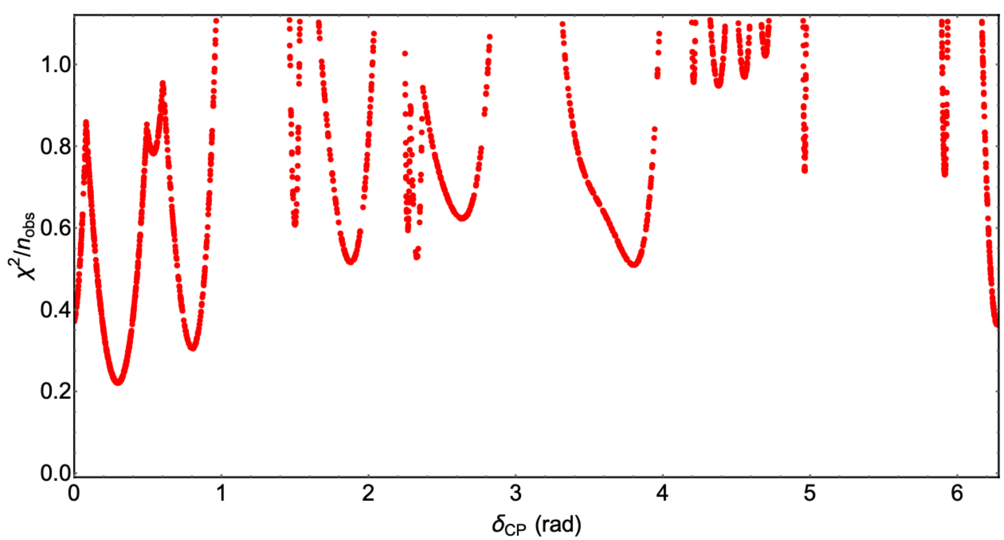
sfermion scale preferred high $\sim 10^3$ TeV

Babu, BB, Saad, '18

Predictions for neutrino

Quantity	Predicted Value
$\{m_1, m_2, m_3\}$ (in eV)	$\{3.32 \times 10^{-3}, 9.89 \times 10^{-3}, 5.42 \times 10^{-2}\}$
$\{\delta^{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$	$\{17.0°, 344.13°, 337.45°\}$
$\{m_{cos}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{6.74 \times 10^{-2}, 6.47 \times 10^{-3}, 6.11 \times 10^{-3}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{1.29 \times 10^{10}, 6.25 \times 10^{11}, 4.13 \times 10^{12}\}$

$$m_{cos} = \sum_i m_i \quad , \quad m_\beta = \sum_i |U_{ei}|^2 m_i \quad , \quad m_{\beta\beta} = \left|\sum_i U_{ei}^2 m_i\right|$$



E_6

The fundamental respresentation is the complex 27

In the decomposition $E_6 \to SO(10) \times U(1)$ we have

$$27 = 16_1 + 10_{-2} + 1_4$$

So 10 in SO(10) coming from 27 of E_6 is automatically complex

E_6 automatically contains the extra U(1) that was needed (but missing) in the minimal SO(10)

Few facts about E_6

- it is a rank 6 Lie group
- the algebra has 78 generators (78 is the adjoint representation)
- the fundamental representation is 27
- each irreducible representation can be denoted in tensor notation as

$$\phi^{\alpha_1\alpha_2\ldots}_{\beta_1\beta_2\beta_3\ldots} \quad , \quad \alpha_i, \beta_i = 1, \ldots, 27$$

- invariant tensors

  $d^{\alpha\beta\gamma}$, $d_{\alpha\beta\gamma}$ ... completely symmetric made out of 0, ±1, and $= 0$ if any two indices the same

- Irreducible representations we will need:

  27^α

  $351'^{\alpha\beta}$ ... two-index symmetric with $d_{\alpha\beta\gamma}351'^{\beta\gamma} = 0$

  $650^\alpha{}_\beta$ ... two indices with $650^\alpha{}_\beta \left(T^A\right)^\beta{}_\alpha = 0 \quad , \quad A = 1, \ldots, 78$

- invariants are made out of products of irreducible representations $\phi^{\alpha_1\alpha_2\ldots}_{\beta_1\beta_2\beta_3\ldots}$ and $d^{\alpha\beta\gamma}$, $d_{\alpha\beta\gamma}$ with each index up is paired with an index down (and implicitly summed over)

  Example:

$$d_{\alpha\beta\gamma}27^\alpha 27^\beta 27^\gamma \quad , \quad 27^\alpha 27^\beta 351'^*_{\alpha\beta} \quad , \quad \ldots$$

The Yukawa sector

What are the possible Yukawas in E_6?

$$27 \times 27 = \overline{27} + 351 + 351'$$

The minimal Yukawa thus seems to be

$$\mathcal{L}_Y = Y_{27}^{ij}\, 27_i\, 27_H\, 27_j + Y_{351'}^{ij}\, 27_i\, 351'^*_H\, 27_j$$

Y_{27}^{ij}, $Y_{351'}^{ij}$ ... 3×3 symmetric Yukawa matrices

351 seems less promising since the Yukawa matrix is antisymmetric

E_6 compared to SO(10): $27 \leftrightarrow 10 \quad , \quad 351' \leftrightarrow 126 \quad , \quad 351 \leftrightarrow 120$

On top of the usual SM model particle we have and extra $5+\bar{5}$ plus two SM singlets:

$$27 = \underbrace{16}_{10+\bar{5}+1} + \underbrace{10}_{5+\bar{5}} + 1$$

$$\begin{aligned}
10 &= \begin{pmatrix} u & u^c & d & e^c \end{pmatrix} \\
\bar{5} &= \begin{pmatrix} d^c & e & \nu \end{pmatrix} \\
1 &= \begin{pmatrix} \nu^c \end{pmatrix} \\
5 &= \begin{pmatrix} d' & e'^c & \nu'^c \end{pmatrix} \\
\bar{5} &= \begin{pmatrix} d'^c & e' & \nu' \end{pmatrix} \\
1 &= \begin{pmatrix} n \end{pmatrix}
\end{aligned}$$

This means that in general the matrices of charged fermions are $(i, j = 1, \ldots, N_g = 3)$

u-quark. . . 3 × 3 : $\begin{pmatrix} u_i \end{pmatrix} (M_U)_{ij} \begin{pmatrix} u_j^c \end{pmatrix}$

d-quark. . . 6 × 6 : $\begin{pmatrix} d_i^c & d_i'^c \end{pmatrix} (M_D)_{ij} \begin{pmatrix} d_j \\ d_j' \end{pmatrix}$

charged lepton. . . 6 × 6 : $\begin{pmatrix} e_i & e_i' \end{pmatrix} (M_E)_{ij} \begin{pmatrix} e_j^c \\ e_j'^c \end{pmatrix}$

for neutrinos

Dirac neutrino. . . 6×9 : $\begin{pmatrix} \nu_i & \nu'_i \end{pmatrix} (M_N)_{ij} \begin{pmatrix} \nu^c_j \\ n_j \\ \nu'^c_j \end{pmatrix}$

right-handed neutrino. . . 9×9 : $\frac{1}{2} \begin{pmatrix} \nu^c_i & n_i & \nu'^c_i \end{pmatrix} (M_N)_{ij} \begin{pmatrix} \nu^c_j \\ n_j \\ \nu'^c_j \end{pmatrix}$

This complicated structure simplifies considerably if we want to have a dark matter candidate: this is in fact possible when spinorial vevs are zero which conserves an extra Z_2 (equivalent to R-parity in susy SO(10))

Under decomposition $E_6 \to SO(10) \times U(1)$

$$\begin{aligned} 27 &= 1_4 + 10_{-2} + 16_1 \\ 351' &= 1_8 + 10_2 + 16_5 + 54_{-4} + 126_2 + \overline{144}_{-1} \end{aligned}$$

If spinorial vevs are zero, nonzero vevs have only even U(1) charges

$$U(1) \to Z_2$$

The lightest scalar from spinorial Higgses is odd under Z_2 and thus stable

We arrange it to be an inert Higgs doublet $(1, 2, 1/2)$: a fine-tuning in the odd doublet matrix is needed.

An additional fine-tuning in the doublet matrix is employed on top of the usual one to get a light SM Higgs

$Z_2 \to$

- no mixing between the $\bar{5}$ of 16 and the $\bar{5}$ of 10 in 27

- the extra singlets decouple from the usual ν^c from 16

Only the SO(10) degrees of freedom remain:

u-quark. . . 3×3 : $\left(u_i\right) (M_U)_{ij} \left(u^c_j\right)$

d-quark. . . 3×3 : $\left(d^c_i\right) (M_D)_{ij} \left(d_j\right)$

charged lepton. . . 3×3 : $\left(e_i\right) (M_E)_{ij} \left(e^c_j\right)$

Dirac neutrino. . . 3×3 : $\left(\nu_i\right) (M_N)_{ij} \left(\nu^c_j\right)$

right-handed neutrino. . . 3×3 : $\frac{1}{2} \left(\nu^c_i\right) (M_N)_{ij} \left(\nu^c_j\right)$

But since there are more doublet vevs the relations are a bit less constrained than in SO(10):

$$
\begin{aligned}
M_U &= v_1 Y_{27} + v_2 Y_{351'} \\
M_D &= v_3 Y_{27} + v_4 Y_{351'} \\
M_E &= -v_3 Y_{27} + v_5 Y_{351'} \\
M_N &= -\left(v_1 Y_{27} + v_6 Y_{351'}\right) \left(V Y_{351'}\right)^{-1} \left(v_1 Y_{27} + v_6 Y_{351'}\right)^T
\end{aligned}
$$

This is equivalent to SO(10) when

$$v_5 = 3 v_4 \quad , \quad v_6 = -3 v_2$$

Since in SO(10) there is a solution which fits data, so it is in E_6

The Higgs sector

Only 27_H and $351'_H$ enough?

We were not able so far to find a good solution. We will add another Higgs E_6 multiplet: 650_H

The vev

$$\langle 650_H \rangle \neq 0$$

can bring the theory to interesting intermediate symmetries:

$$E_6 \to SO(10) \times U(1) \quad \text{or} \quad SU^3(3) \quad \text{or} \quad SU(6) \times SU(2)$$

The role of 27_H and $351'_H$ is then to break these intermediate symmetries down to the SM (on top of contributing to Yukawas)

The RGE

Once we found the symmetries of the intermediate scale we want to check which of them are realistic.

We assume

1. a single intermediate scale
2. the extended survival hypothesis: all multiplets which can be heavy are heavy except those which will take part to symmetry breaking
3. an extra Z_2 which will stabilise the dark matter candidate

Define

$$\bar{t} = \log_{10}\left(\frac{\mu}{1 \text{ GeV}}\right)$$

We look for

1. unification of couplings, with not too large threshold corrections (they allow some spreading of the masses around a given scale)
2. couplings at unification still perturbative $\alpha_U^{-1} \gtrsim 10$
3. unification scale allowed by proton decay but smaller than Planck scale

$$16 \lesssim \bar{t}_U \lesssim 18$$

Intermediate symmetries considered:

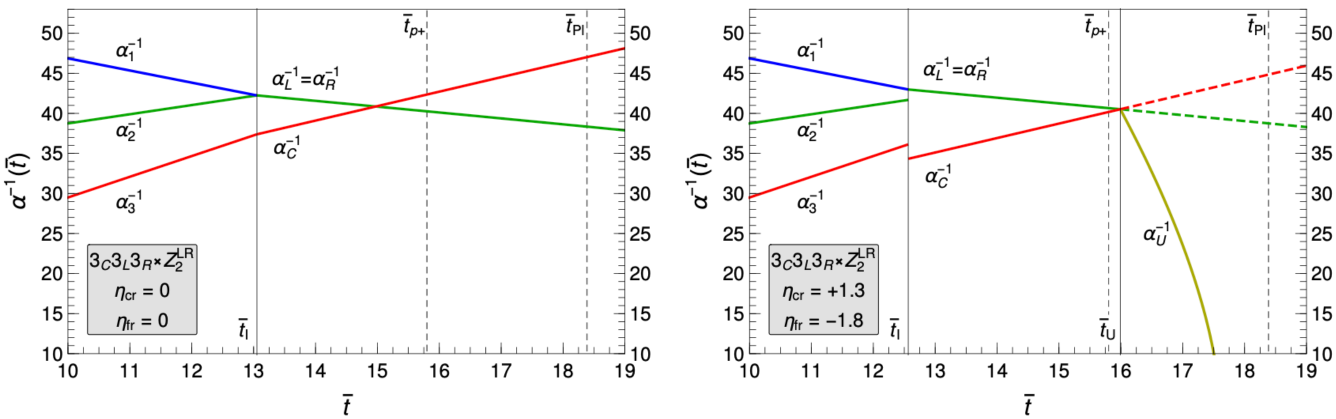
1. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{LR}$
2. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{CL}$
3. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{CR}$
4. $SU(6)_{CL} \times SU(2)_R$
5. $SU(6)_{CR} \times SU(2)_L$
6. $SO(10)' \times U(1)'$

- the extra Z_2 parities above are automatic from 650_H, nothing to do with dark matter Z_2 mentioned before
- the last case is flipped SO(10)

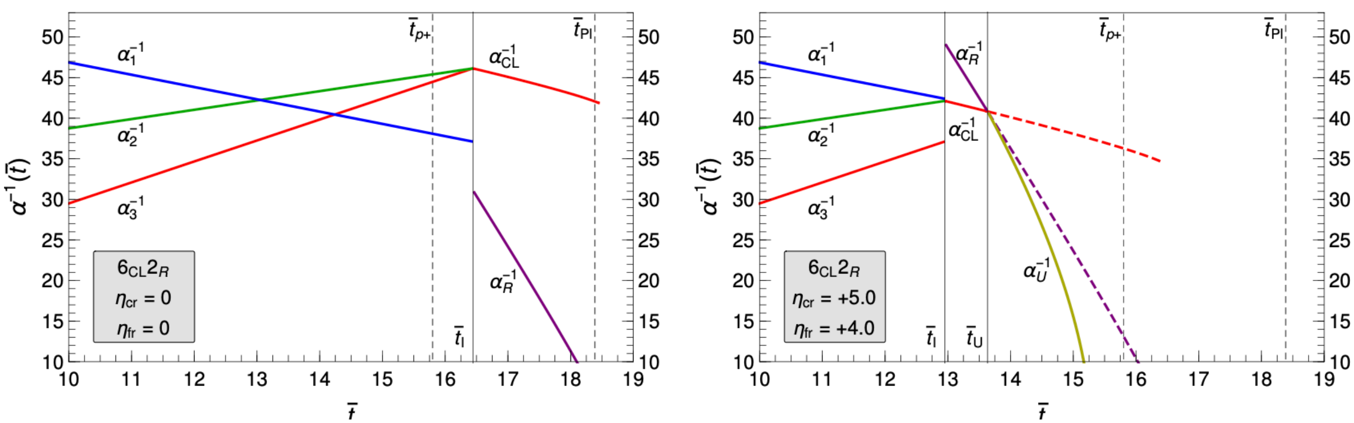
blue. . . successfull

red. . . unsuccessfull

GOOD:

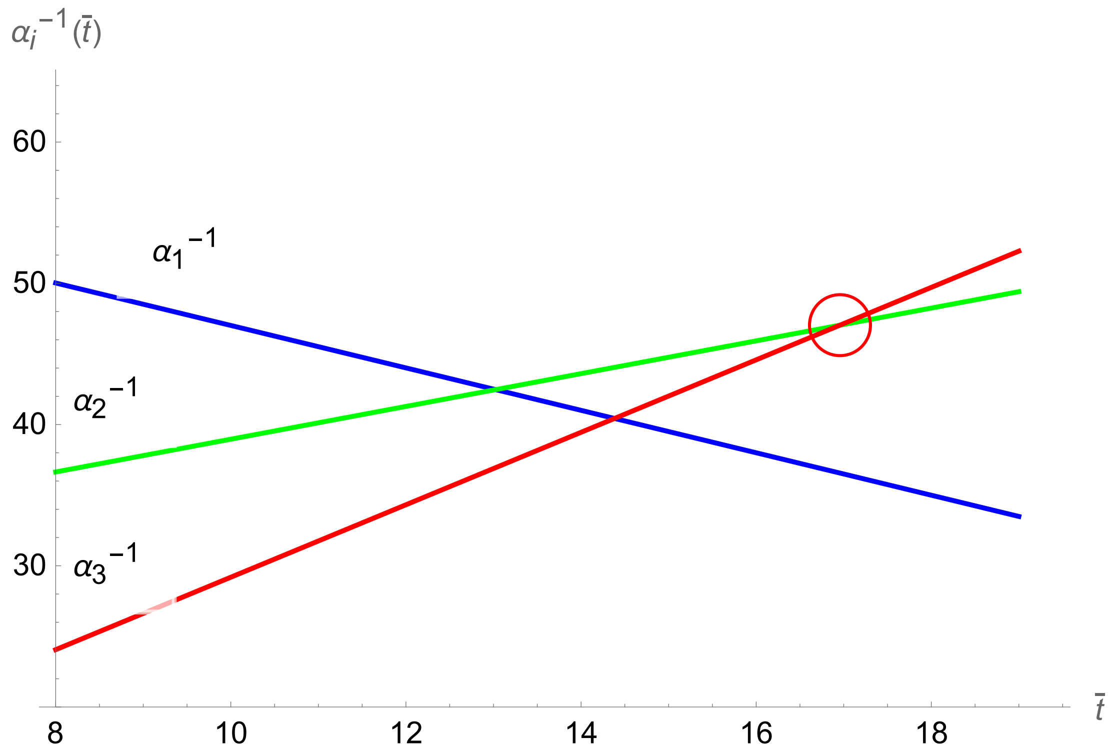

α_1^{-1}
α_2^{-1}
α_3^{-1}
$\alpha_L^{-1}=\alpha_R^{-1}$
α_C^{-1}
α_U^{-1}
$\bar{t}_{p+}$
$\bar{t}_{\rm Pl}$
$\bar{t}_{\rm I}$
$\bar{t}_{\rm U}$
$3_C3_L3_R\times Z_2^{\rm LR}$
$\eta_{\rm cr}=0$
$\eta_{\rm fr}=0$
$\eta_{\rm cr}=+1.3$
$\eta_{\rm fr}=-1.8$
$\alpha^{-1}(\bar{t})$
$\bar{t}$

BAD:


α_1^{-1}
α_2^{-1}
α_3^{-1}
$\alpha_{\rm CL}^{-1}$
α_R^{-1}
α_U^{-1}
$\bar{t}_{p+}$
$\bar{t}_{\rm Pl}$
$\bar{t}_{\rm I}$
$\bar{t}_{\rm U}$
$6_{\rm CL}2_R$
$\eta_{\rm cr}=0$
$\eta_{\rm fr}=0$
$\eta_{\rm cr}=+5.0$
$\eta_{\rm fr}=+4.0$
$\alpha^{-1}(\bar{t})$
$\bar{t}$

The different behaviour is due to different conditions at intermediate scales

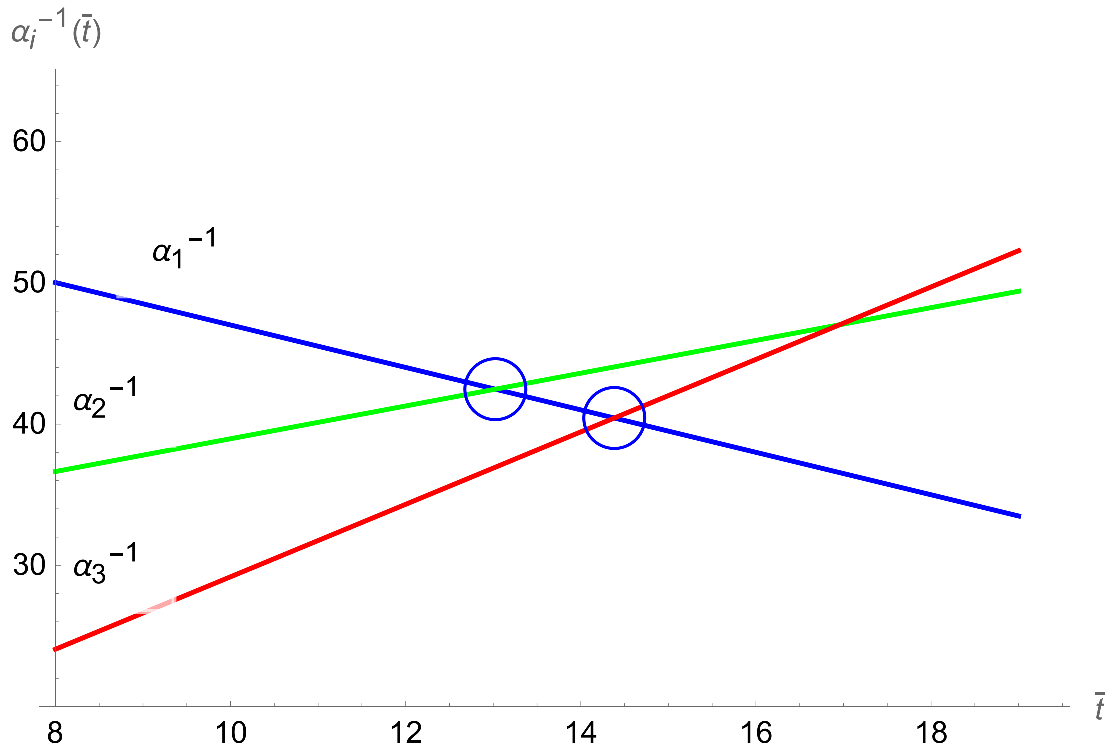
In the RED (unsuccessful) case this condition is $\alpha_2 = \alpha_3$



This happens quite high in energy so that in the meantime α_1^{-1} has decreased too much.

On the contrary the BLUE cases which work need unification

$\alpha_1 = c\,\alpha_2 + (1-c)\,\alpha_3 \quad , \quad 0 \le c \le 1$



This happens much below the scale of α_2 and α_3 unification.

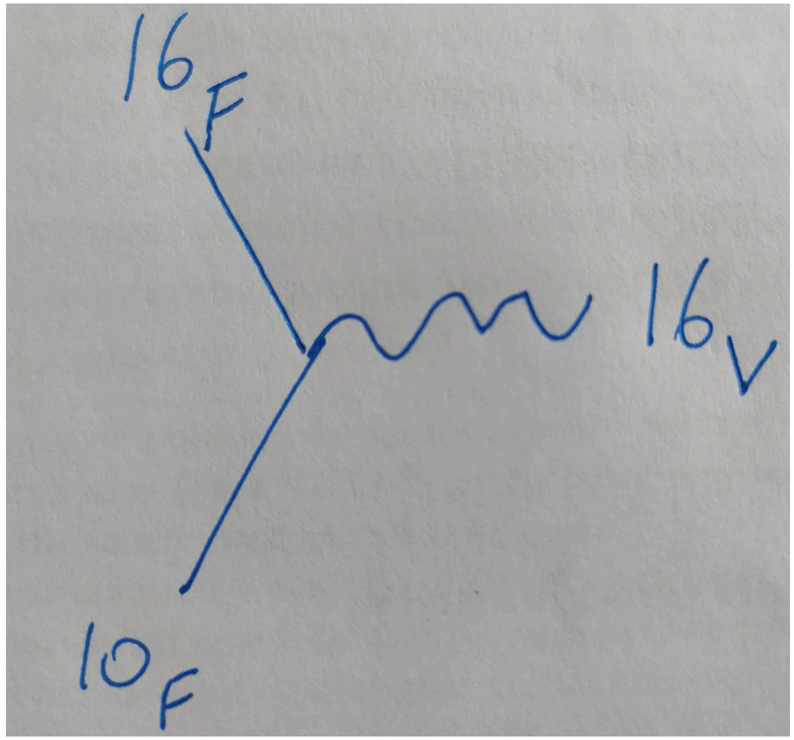
Proton decay

As in all GUTs nucleons decay. The operator responsible for it is $d = 6$, we will consider only the gauge mediated contribution

This is because typical Yukawa couplings are much smaller than the gauge coupling. We neglect the possibility of having small Higgs triplet mass (this would typically make unification harder)

label	$3_C\, 2_L\, 1_Y$	SU(5)	SO(10)	E_6	ψ	comment
X	$\sim (\mathbf{3}, \mathbf{2}, -5/6)$	**24**	**45**	**78**	0	the SU(5) leptoquark
X'	$\sim (\mathbf{3}, \mathbf{2}, +1/6)$	**10**	**45**	**78**	0	the SO(10) leptoquark
X''	$\sim (\mathbf{3}, \mathbf{2}, +1/6)$	**10**	**16**	**78**	−3	the E_6 leptoquark

The first thing to notice is that X'' does not contribute if spinorial Z_2 vacuum is taken



The 10_F is heavy so 16_V does not contribute to pdk

Another consequence of E_6 is that $M_X = M_{X'}$

In SU(5) there is no X' and

$$\mathcal{B}_{SU(5)}(p^+ \to \pi^0 e^+) \approx \frac{5}{2} \mathcal{B}_{SU(5)}(p^+ \to \pi^+ \bar{\nu})$$

so we have an (approximate) relation:

$$\mathcal{B}_{E_6}(p^+ \to \pi^0 e^+) \approx \mathcal{B}_{E_6}(p^+ \to \pi^+ \bar{\nu})$$

This differentiates between the minimal $SU(5)$ and E_6 scenarios

Conclusions

- the minimal grand unified theory of the form $GUT \times$ nothing for neutrino masses is E_6
- assuming extended survival hypothesis and a spinorial parity which gives a dark matter candidate, we analysed 6 possible intermediate symmetries originated by $\langle 650_H \rangle$
- possible realistic intermediate symmetries are
  1. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{LR}$
  2. $SU(3)_C \times SU(3)_L \times SU(3)_R \times Z_2^{CR}$
  3. $SU(6)_{CR} \times SU(2)_L$
- quark and lepton masses and mixings can be properly described on the same footing