CETUP* 2024

Lead/Deadwood Middle School

09 July, 2024

Charged-current (anti)neutrino-nucleon scattering and QED nuclear medium effects

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LA-UR-24-25219

Outline

1) charged-current (anti)neutrino-nucleon scattering

O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion in Phys. Rev. D (2022)

O. T., Rajan Gupta, and Tanmoy Bhattacharya, Phys. Rev. D 108 7 (2023)
Roberto Petti, O. T., and Richard J. Hill, letter in Phys. Rev. D 109 5 (2024)

O. T., Minerba Betancourt, Kaushik Borah, Richard J. Hill, and Thomas Junk
Phys. Lett. B 854 (2024)

O. T., Minerba Betancourt, Kaushik Borah, Richard J. Hill, and Thomas Junk,
2403.04687

2) QED nuclear medium effects

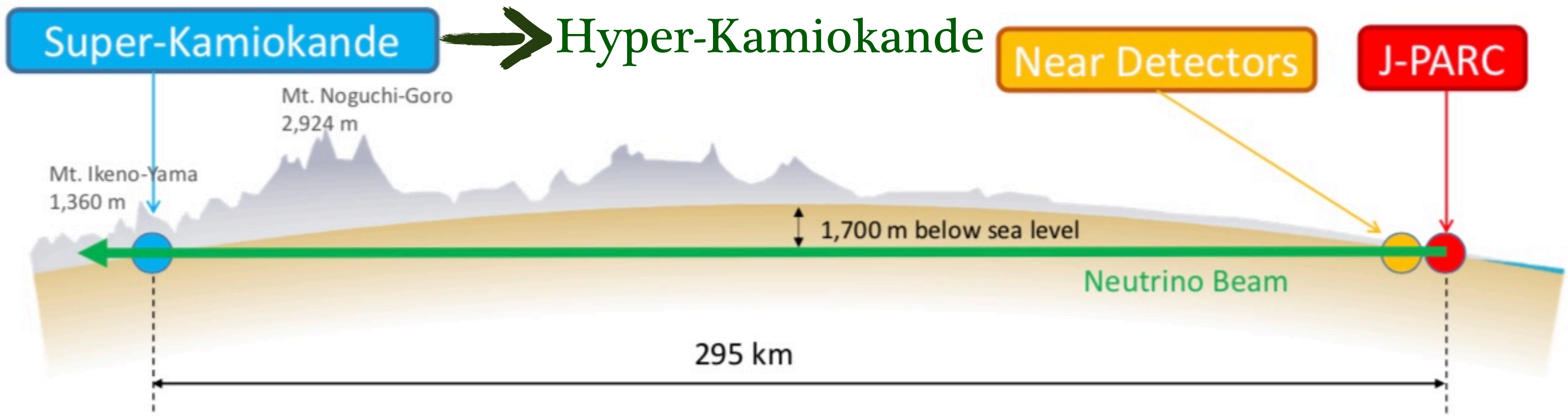
O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)

O. T. and Ivan Vitev, Phys. Rev. D 108 9, 9 (2023)

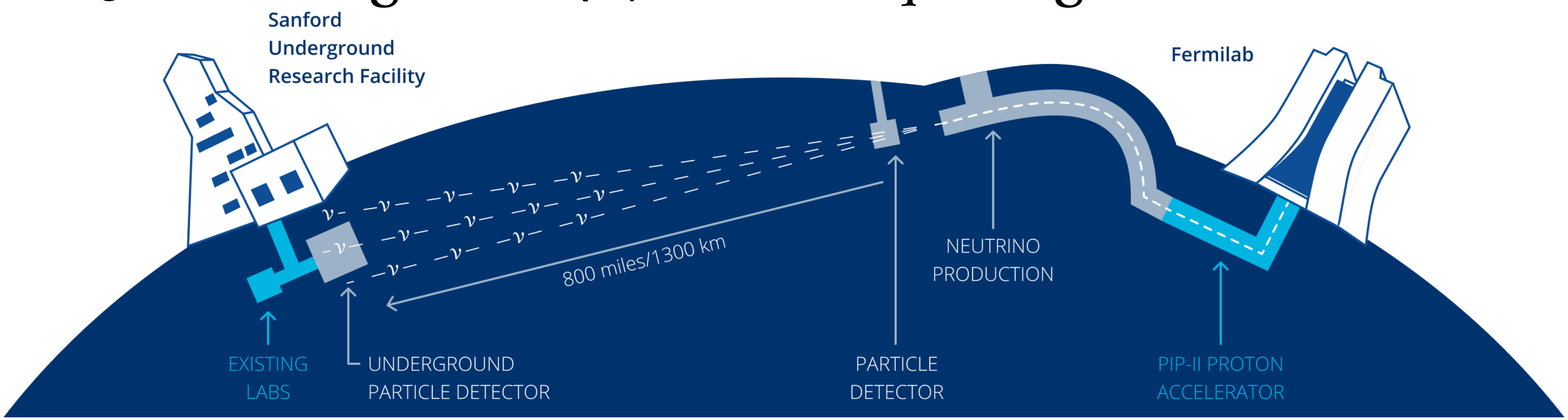
O. T. and Ivan Vitev, Phys. Rev. D 109 7, (2024)

CP violation and mass hierarchy@laboratory

650 m under rocks in mountain, 260000t of pure water

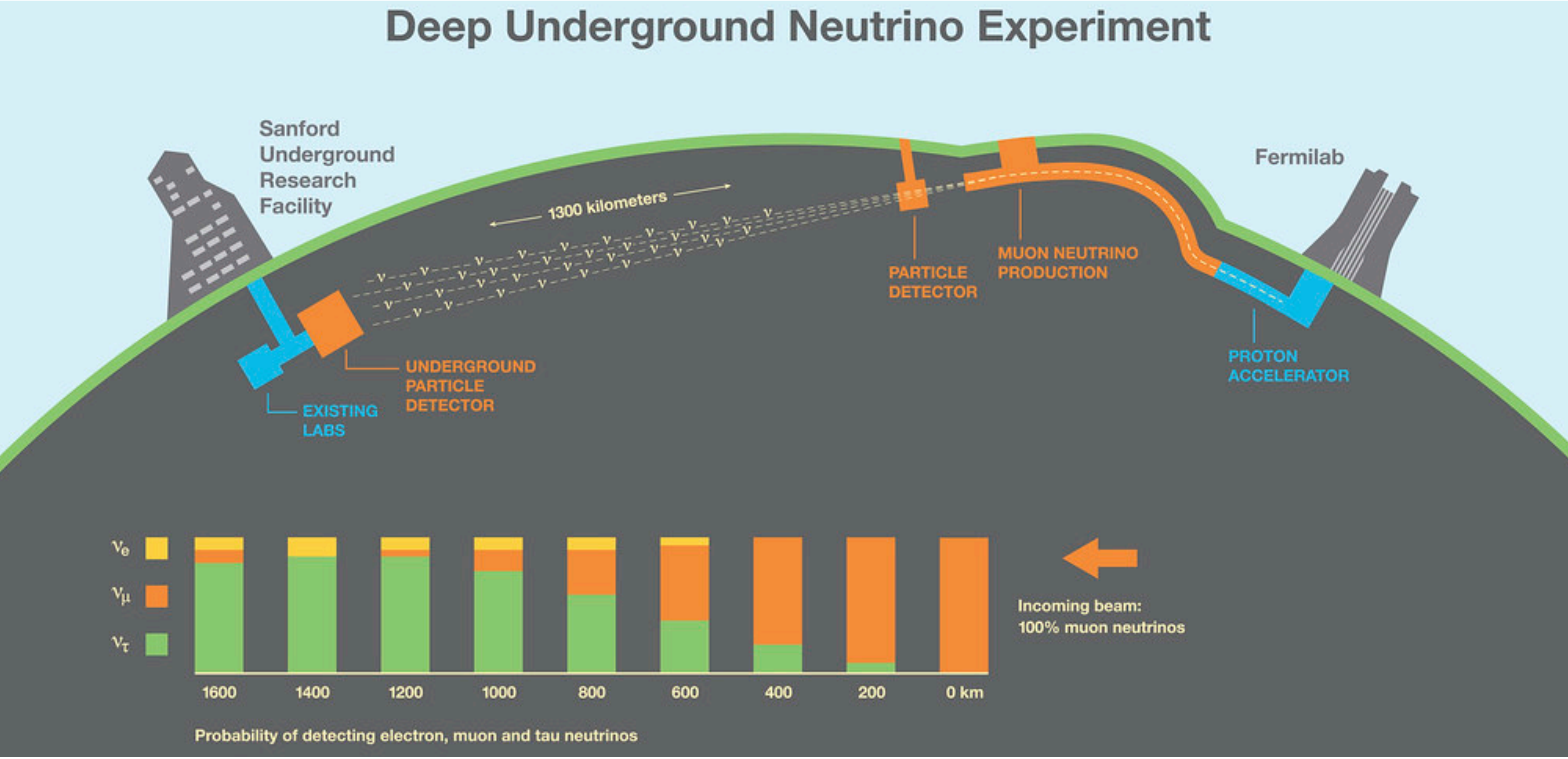


1.5 km underground, 4x70000t of liquid argon



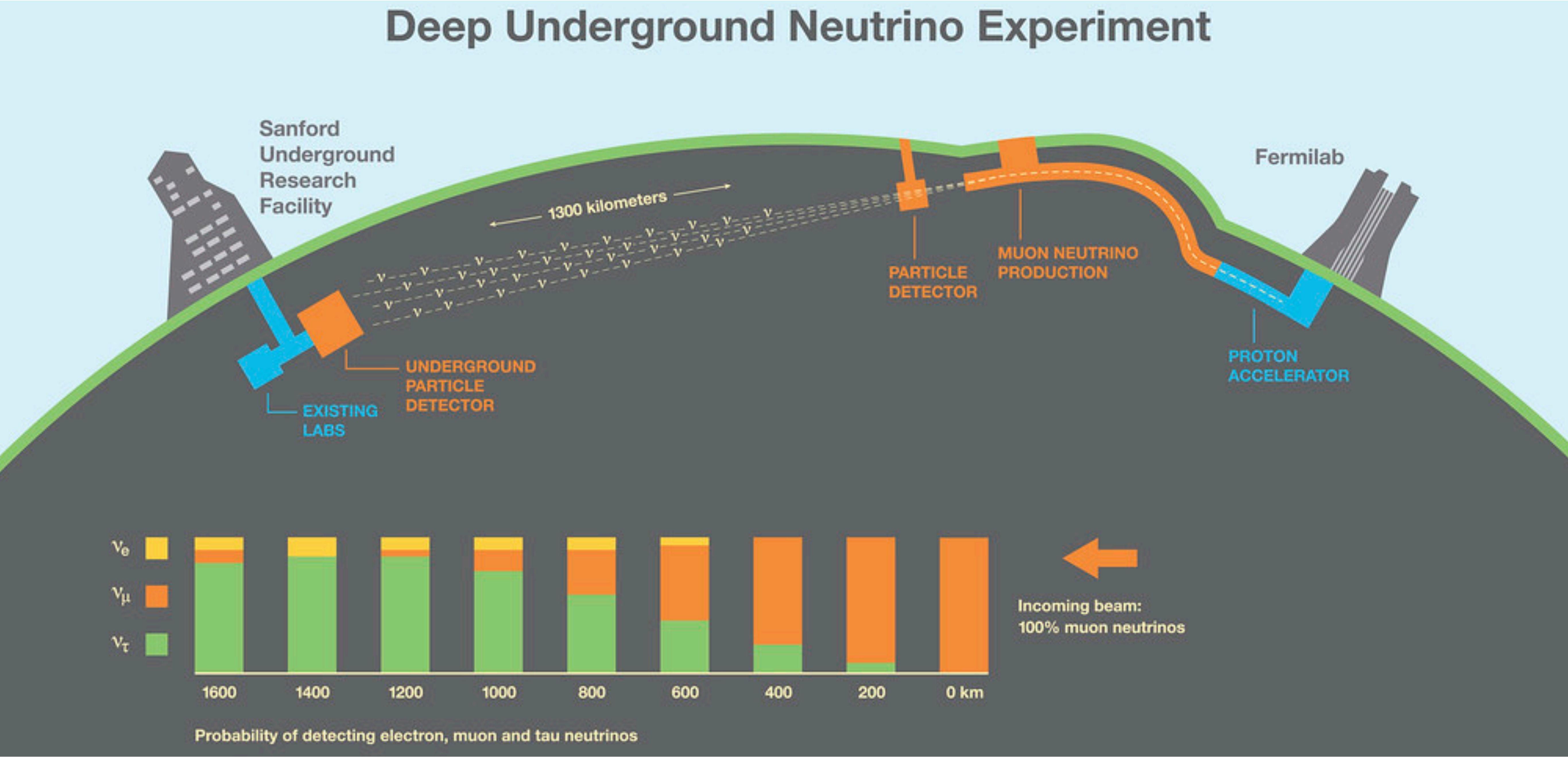
- CP violation in PMNS and mass hierarchy in next 10-20 years !!!

DUNE



- muon neutrinos oscillate to tau and electron flavors

Neutrino experiments

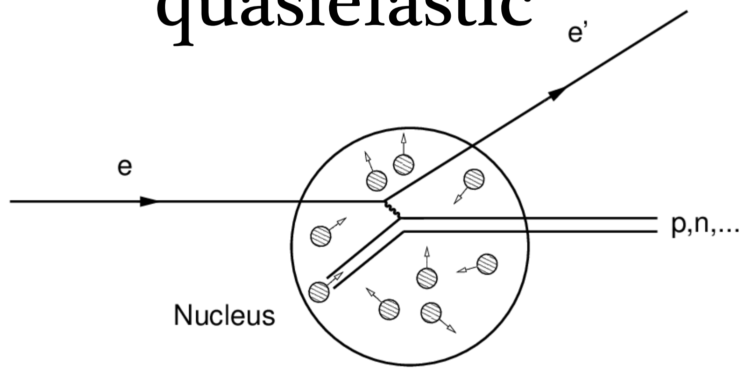


$$N_\nu \sim \int \mathrm{d}E_\nu \Phi_\nu \left(E_\nu\right) \times \sigma\left(E_\nu\right) \times R\left(E_\nu, E_\nu^{\mathrm{rec}}\right)$$

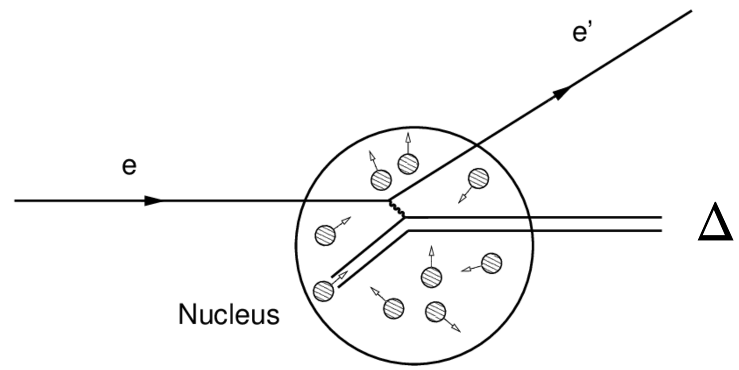
- precise neutrino physics: need in cross sections

Interaction mechanisms

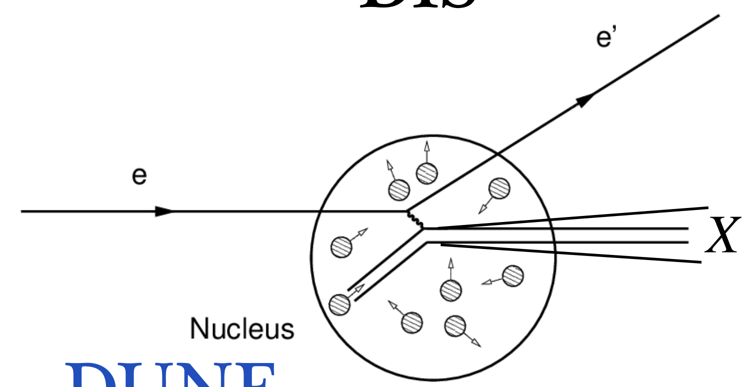
quasielastic



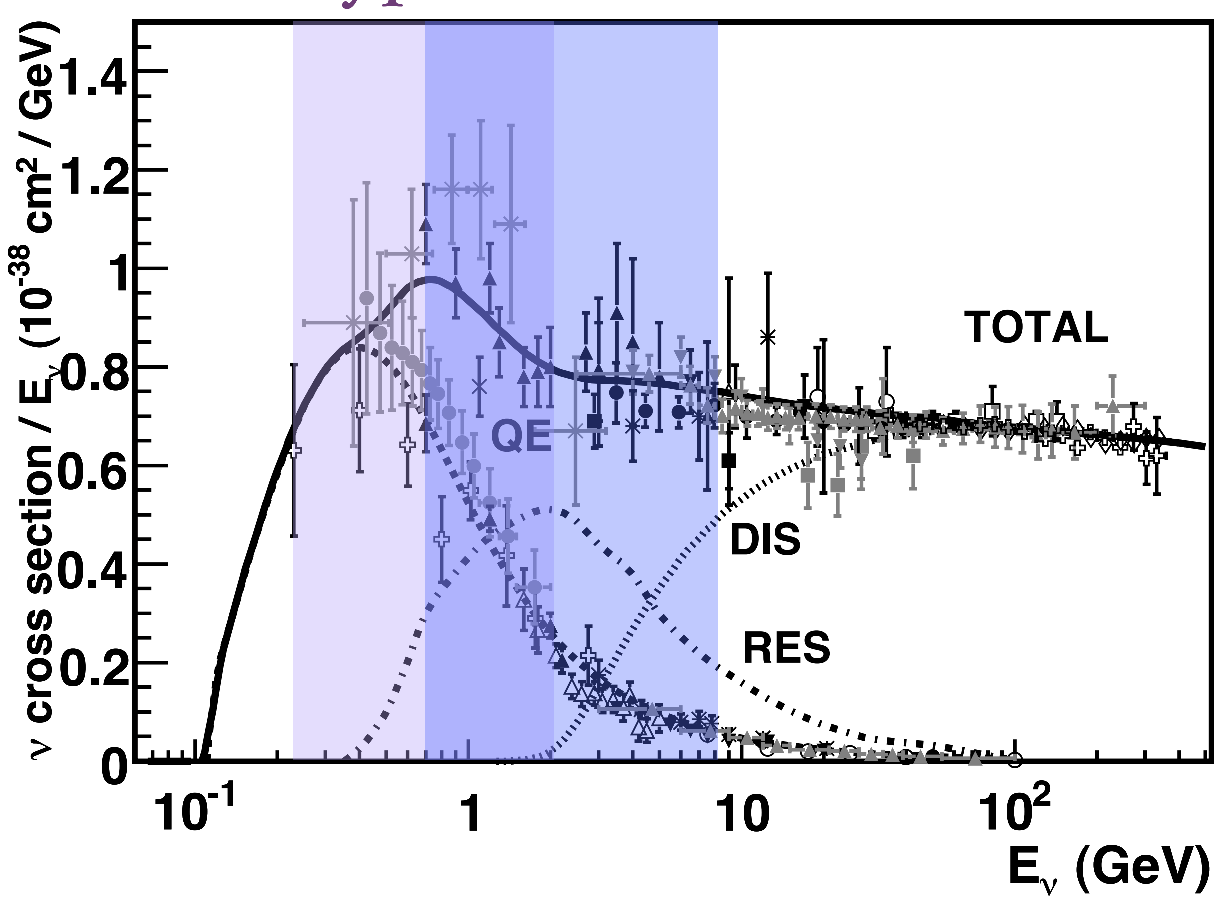
resonance



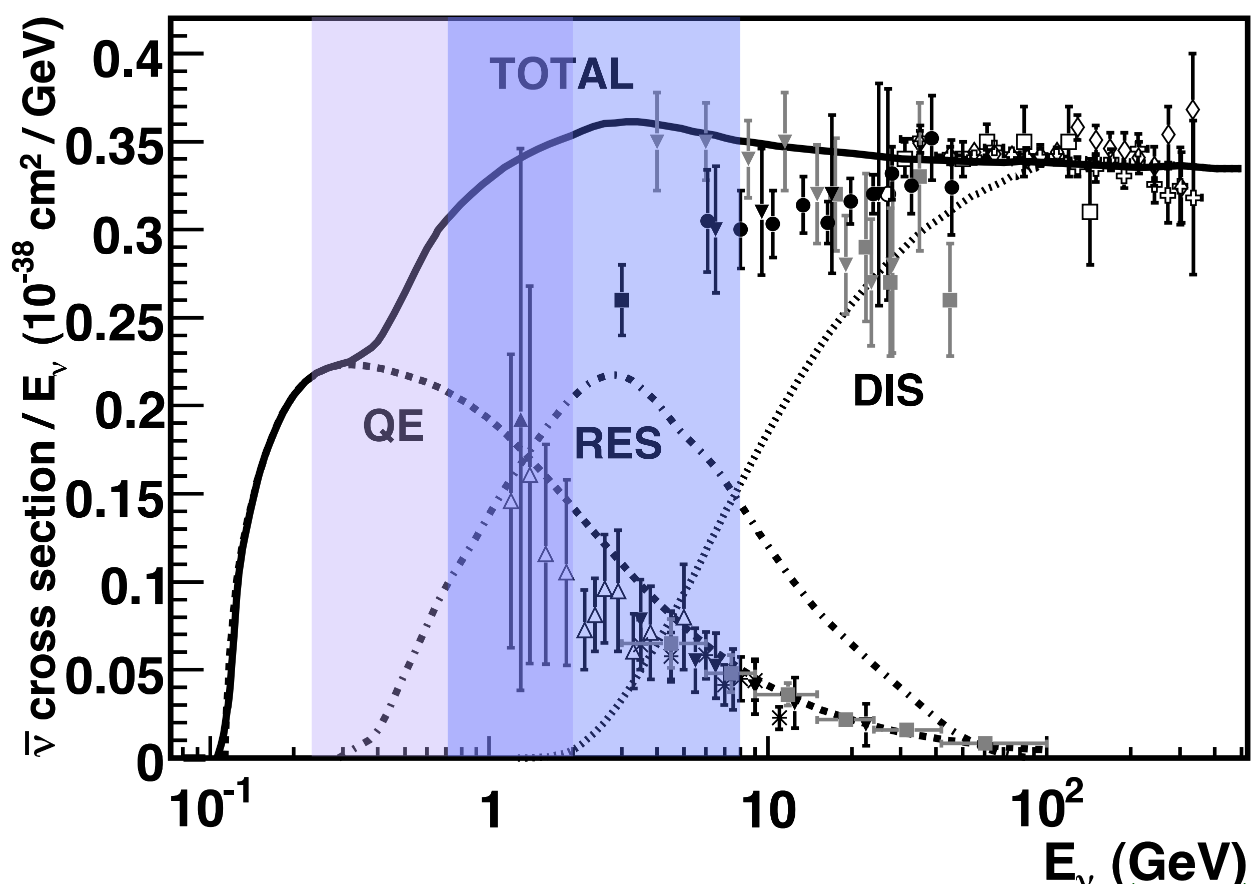
DIS



Hyper-K



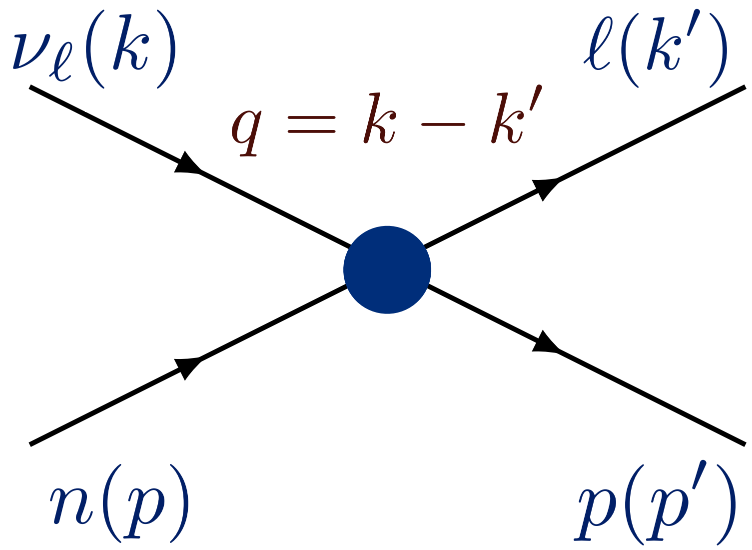
DUNE



Formaggio and Zeller (2013)

- significant overlap with prior and modern JLab energy range
- DUNE experimental program requires 3-5 % precise cross sections

CCQE scattering on free nucleon



neutrino energy E_ν

momentum transfer $Q^2 = -q^2$

contact interaction at GeV energies

- assuming isospin symmetry, nucleon current:

$$\Gamma^\mu\left(Q^2\right) =< p|\bar{u}\left(\gamma^\mu - \gamma^\mu\gamma_5\right)d|n >$$

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D^V(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M}F_P^V(Q^2) + \gamma^\mu\gamma_5 F_A(Q^2) + \frac{q^\mu}{M}\gamma_5 F_P(Q^2)$$

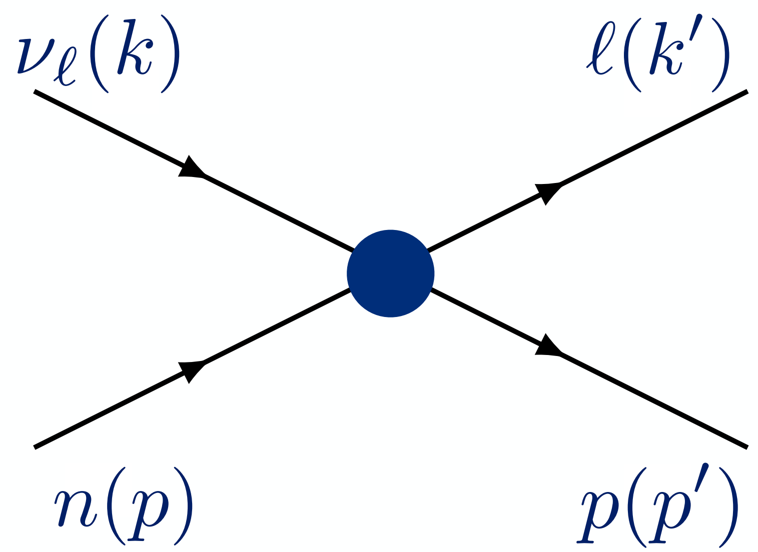
form factors: isovector Dirac and Pauli axial and pseudoscalar

$$F_{D,P}^V = F_{D,P}^p - F_{D,P}^n$$

tree-level amplitude

$$\mathrm{T} = \frac{\mathrm{G_F} V_{ud}}{\sqrt{2}}\left(\bar{\ell}(k')\gamma_\mu\left(1-\gamma_5\right)\nu_\ell(k)\right)\left(\bar{p}(p')\Gamma^\mu(Q^2)n(p)\right)$$

CCQE scattering on free nucleon



$$\nu = E_\nu/M - \tau - r^2$$

$$r = \frac{m_\ell}{2M} \qquad \tau = \frac{Q^2}{4M^2}$$

unpolarized cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} \sim \frac{M^2}{E_\nu^2}\left(\left(\tau + r^2\right) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1+\tau} C(Q^2)\right)$$

Llewellyn Smith (1972)

- structure-dependent functions

$$A = \tau\left(G_M^V\right)^2 - \left(G_E^V\right)^2 + (1+\tau)F_A^2 - r^2\left(\left(G_M^V\right)^2 + F_A^2 - 4\tau F_P^2 + 4F_A F_P\right)$$

$$B = \pm 4\tau F_A G_M^V \qquad C = \tau\left(G_M^V\right)^2 + \left(G_E^V\right)^2 + (1+\tau)F_A^2$$

- pseudoscalar form factor contribution is suppressed by lepton mass
- cross section is sensitive to both vector and axial contributions

Elastic scattering on free nucleon

- only 3 experiments performed with deuterium bubble chamber

  direct access to form-factor shape

ANL 1982: 1737 events

BNL 1981: 1138 events

FNAL 1983: 362 events

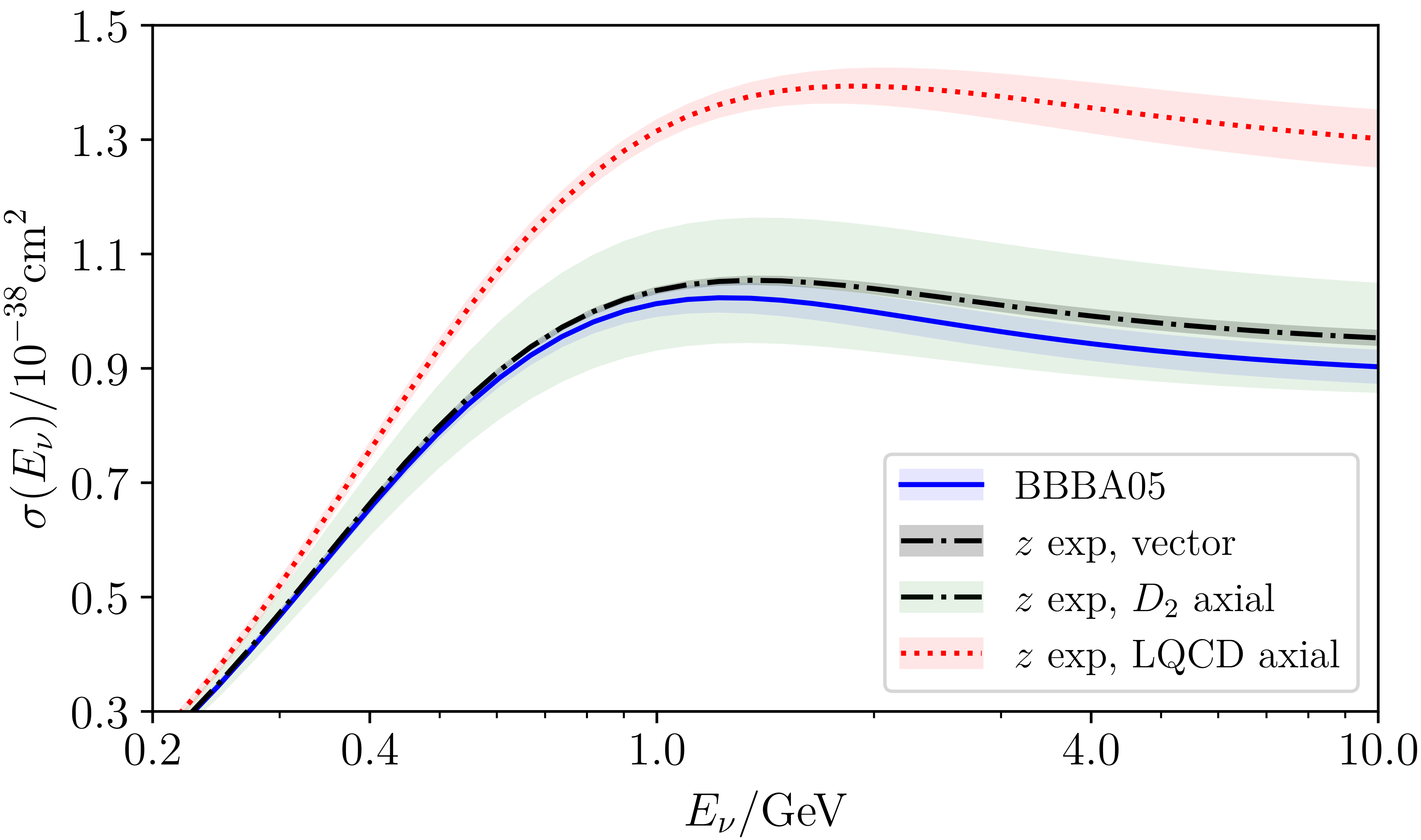
world data: ~3200 events

Fermilab bubble chamber, Richard Drew

- axial form factor extracted based on electromagnetic structure

A.S. Meyer, M. Betancourt, R. Gran and R.J. Hill (2016)

Neutrino-nucleon scattering (CC)



A.S. Meyer, A. Walker-Loud, C. Wilkinson, Ann. Rev. of 72, 010622-120608 (2022)
A.S. Meyer, M. Betancourt, R. Gran, and R.J. Hill, PRD (2016)
Kaushik Borah, Gabriel Lee, Richard J. Hill, and O. T. , PRD (2021)

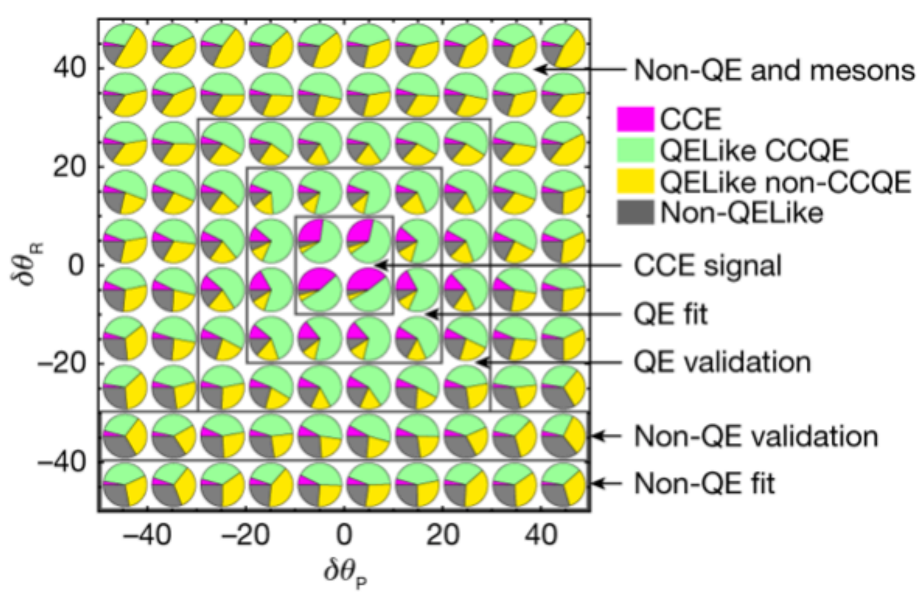
- knowledge of vector structure stops a progress in studies of axial
- acknowledged discrepancy: lattice QCD <-> experimental data

MINERvA result with free protons

- idea of scattering on molecular hydrogen realized !!!

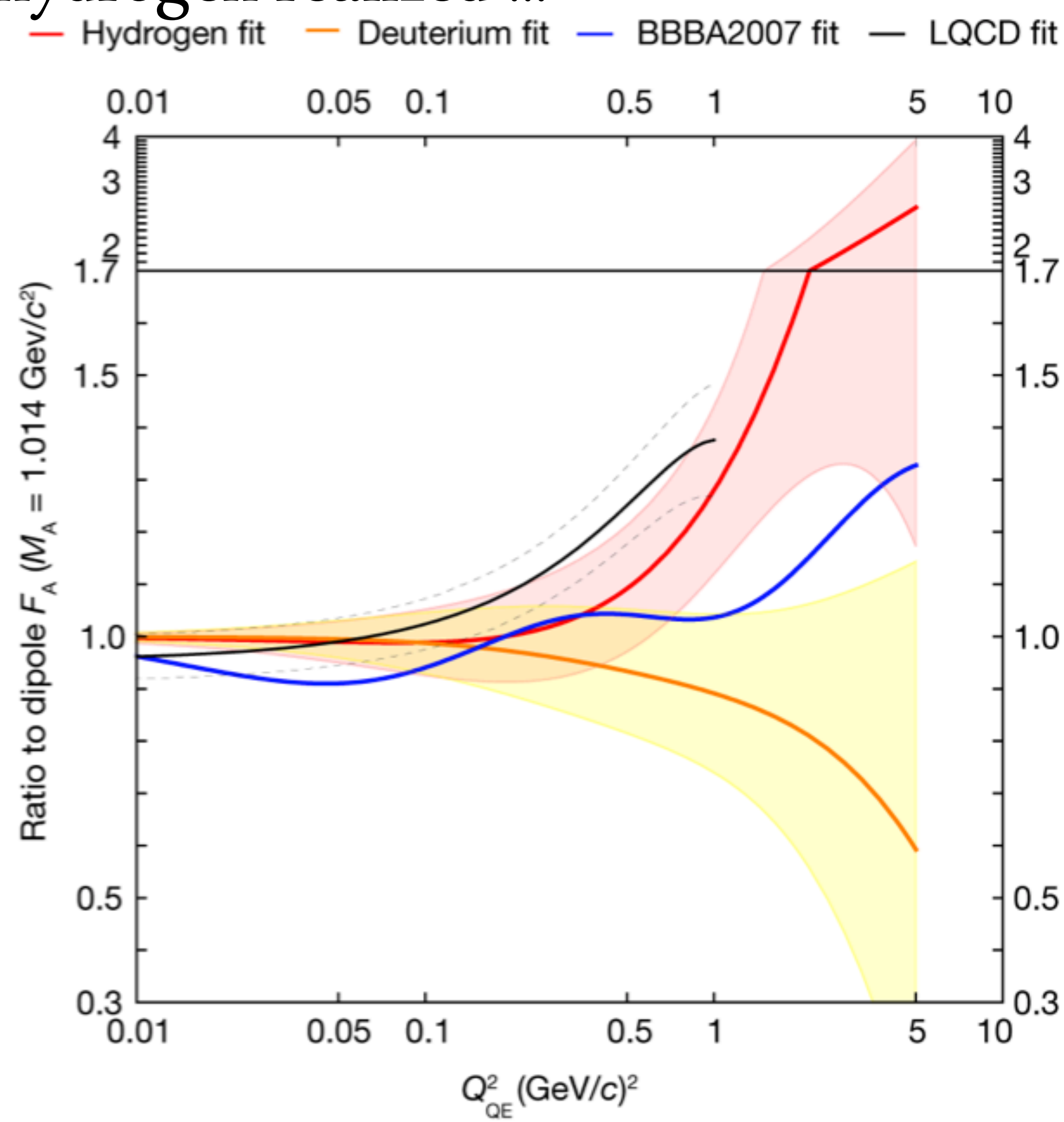
$$\overline{\nu}_\mu p \to \mu^+ n$$

kinematic selection



5580 events over
12500 background

background nuclear events
constrained by scattering of ν

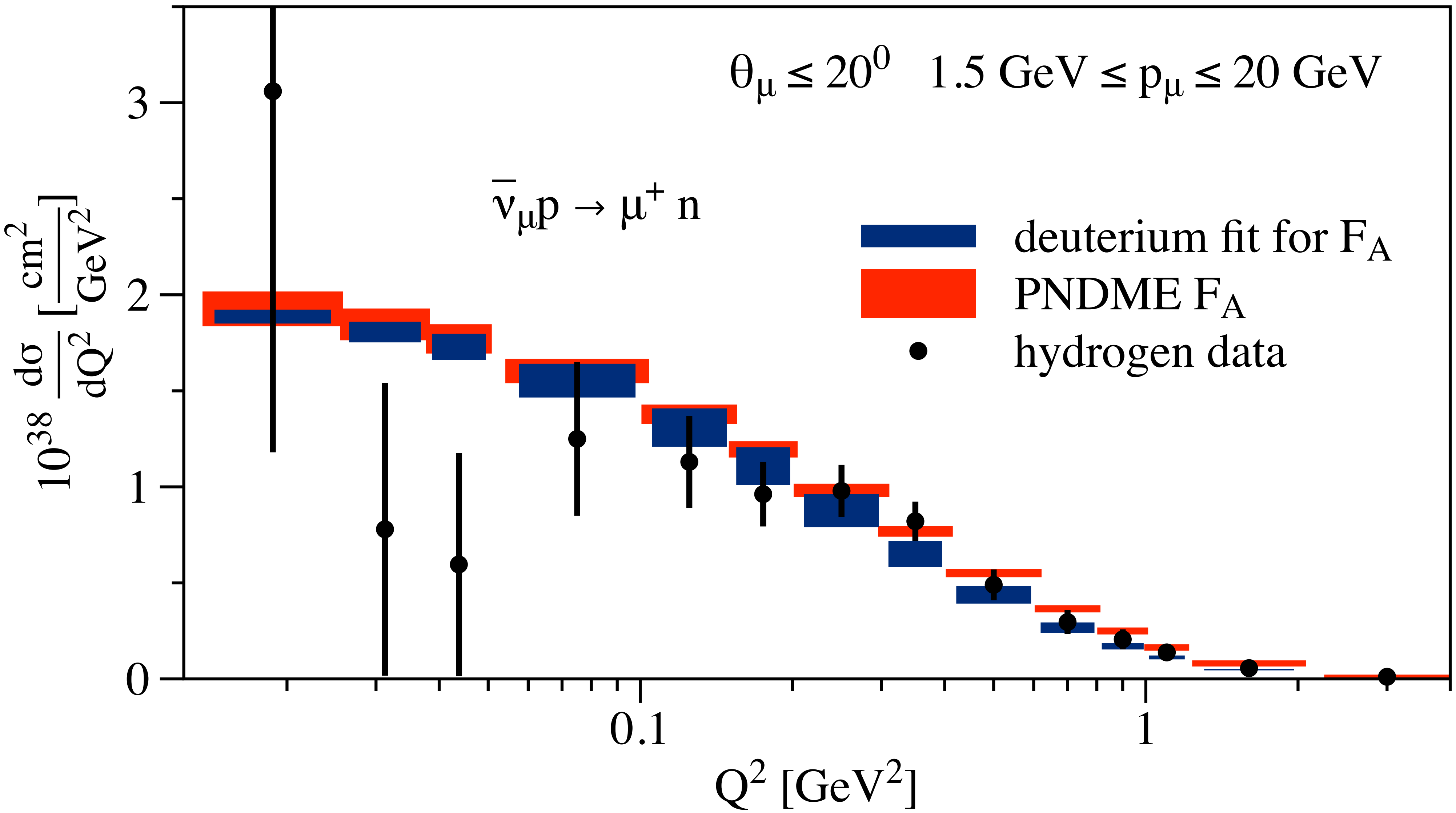


- 1st measurement of axial form factor on "free" protons $\overline{\nu}_\mu p \to \mu^+ n$

T. Cai et al., MINERvA Collaboration, Nature (2023), 614, 48-53

Lattice QCD vs MINERvA

- PNDME 2023 axial-vector form factor as representative of lattice QCD



- $\lesssim 1\sigma$ agreement for each bin besides two at small Q^2

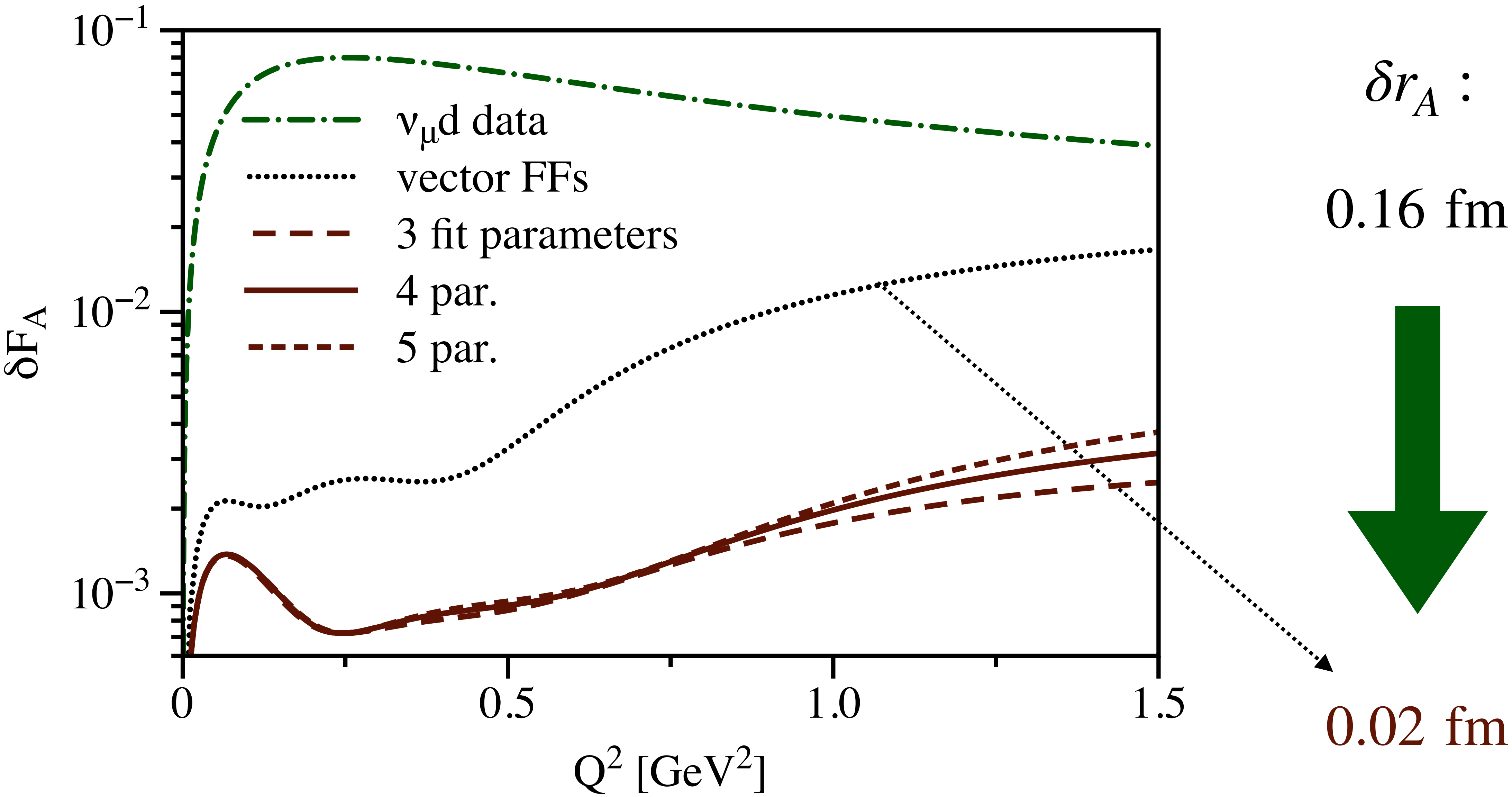
- 2-3σ tension between lattice QCD and deuterium data
- MINERvA hydrogen data consistent with LQCD and deuterium

O. T., Rajan Gupta, and Tanmoy Bhattacharya, Phys. Rev. D 108 (2023) 7, 074514

DUNE projections

- estimates for 700 kg of H in Straw Tube Tracker at near detector

H. Duyang, B. Guo, S. R. Mishra, and R. Petti (2016)



- order of magnitude improvement in axial form factor and radius
- DUNE will probe vector form factors and isospin symmetry

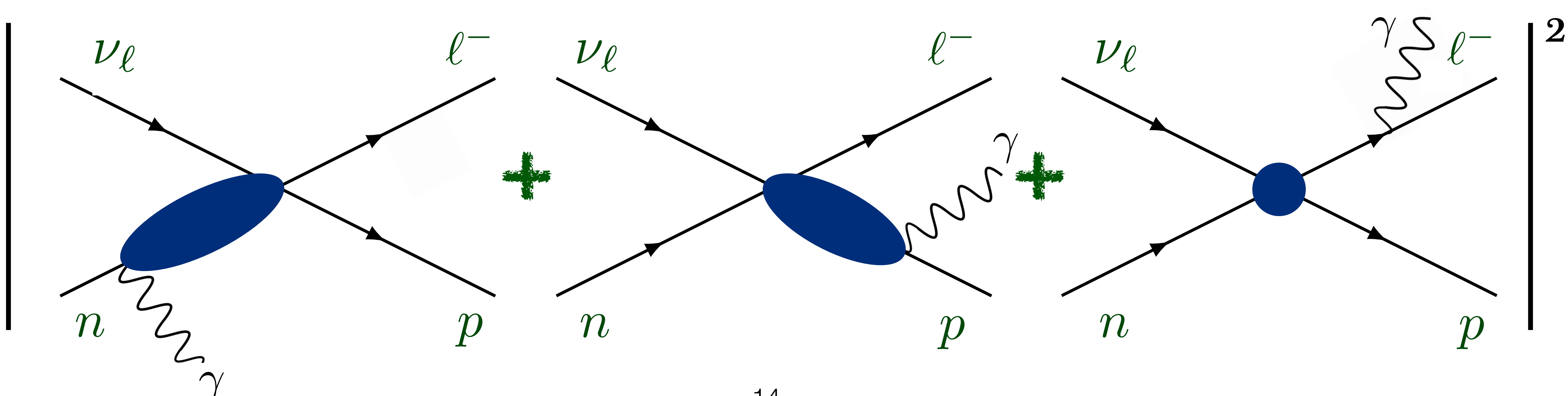
Roberto Petti, O. T., and Richard J. Hill, letter in Phys. Rev. D 109 5 (2024) factorization for radiative corrections with model for hard function

Charged-current elastic scattering on nucleons

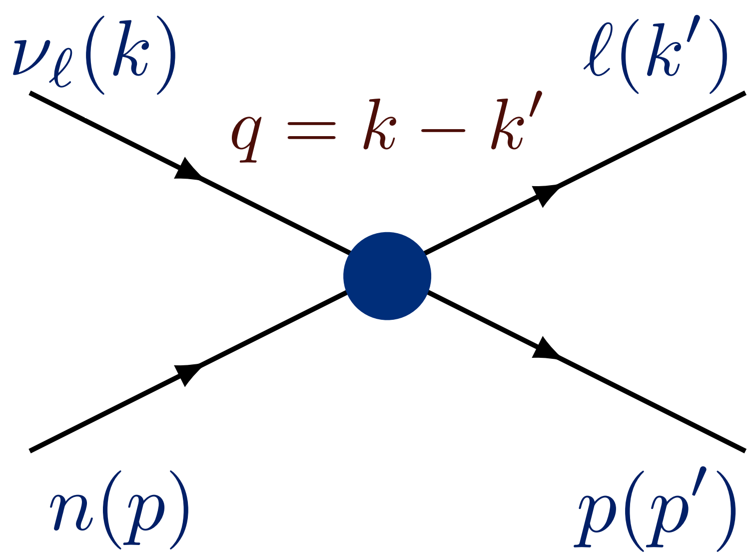
O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion in Phys. Rev. D (2022)

precise predictions for flavor ratios and radiative corrections



Invariant amplitudes



averaged lepton momentum $K_\mu = \dfrac{k_\mu + k'_\mu}{2}$

averaged nucleon momentum $P_\mu = \dfrac{p_\mu + p'_\mu}{2}$

- four amplitudes for massless charged lepton

$$T^{m_\ell=0}_{\nu_\ell n \to \ell^- p} = \sqrt{2} \mathrm{G_F} V_{ud}\, \bar{\ell}^- \gamma^\mu \mathrm{P_L} \nu_\ell\, \bar{p} \left(\gamma_\mu \left(g_M + f_A \gamma_5 \right) - \left(f_2 + f_A^3 \gamma_5 \right) \frac{K_\mu}{M} \right) n$$

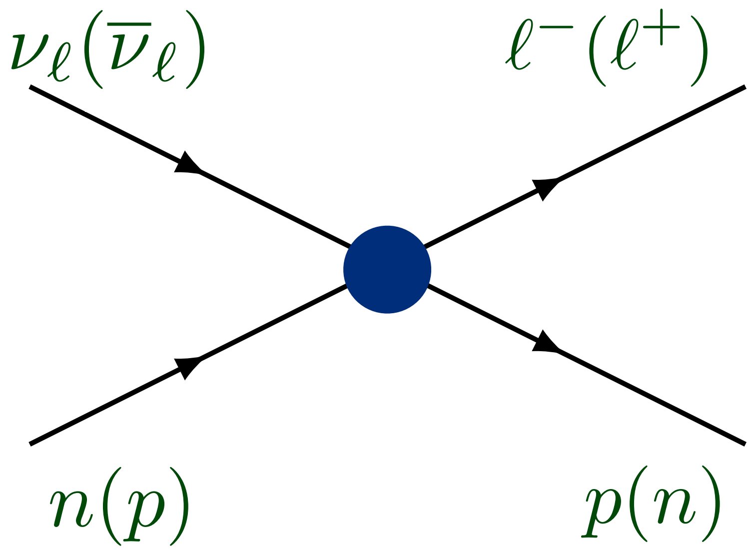
- four extra amplitudes for massive charged lepton

$$T^{m_\ell \neq 0}_{\nu_\ell n \to \ell^- p} = -\sqrt{2} \mathrm{G_F} V_{ud} \frac{m_\ell}{M}\, \bar{\ell}^- \mathrm{P_L} \nu_\ell\, \bar{p} \left(f_3 + f_P \gamma_5 - \frac{f_R}{4} \frac{P_\mu}{M} \gamma^\mu \gamma_5 \right) n$$

$$+ \sqrt{2} \mathrm{G_F} V_{ud} \frac{m_\ell}{M} \frac{f_T}{4}\, \bar{\ell}^- \sigma^{\mu\nu} \mathrm{P_L} \nu_\ell\, \bar{p} \sigma_{\mu\nu} n$$

- 8 invariant amplitudes for charged-current elastic scattering

Polarization observables



polarized target and/or recoil

$$A = \frac{\mathrm{d}\sigma\left(S\right) - \mathrm{d}\sigma\left(-S\right)}{\mathrm{d}\sigma\left(S\right) + \mathrm{d}\sigma\left(-S\right)}$$

S. M. Bilenky et al. (2013), A. Fatima et al (2018), J. Sobczyk et al (2019), B. Kowal et al (2019), Tomalak (2020)

- spin asymmetries are not suppressed by m_ℓ or coupling constant: comparable to unpolarized cross section rates

$$\mathrm{T},\mathrm{R},\mathrm{L} = \frac{\left(\tau + r^2\right) A^{\mathrm{T},\mathrm{R},\mathrm{L}}(Q^2) - \nu B^{\mathrm{T},\mathrm{R},\mathrm{L}}(Q^2) + \frac{\nu^2}{1+\tau} C^{\mathrm{T},\mathrm{R},\mathrm{L}}(Q^2)}{\left(\tau + r^2\right) A(Q^2) - \nu B(Q^2) + \frac{\nu^2}{1+\tau} C(Q^2)}$$

- target, recoil, and lepton asymm.: spin in and normal to scattering plane

- alternative way to access nucleon structure at GeV energies

Beta decay constraints

scalar → J. C. Hardy and I. S. Towner, Phys. Rev. C 79, 055502 (2009)

scalar → A. Kozela, Phys. Rev. C 85, 0445501 (2012)

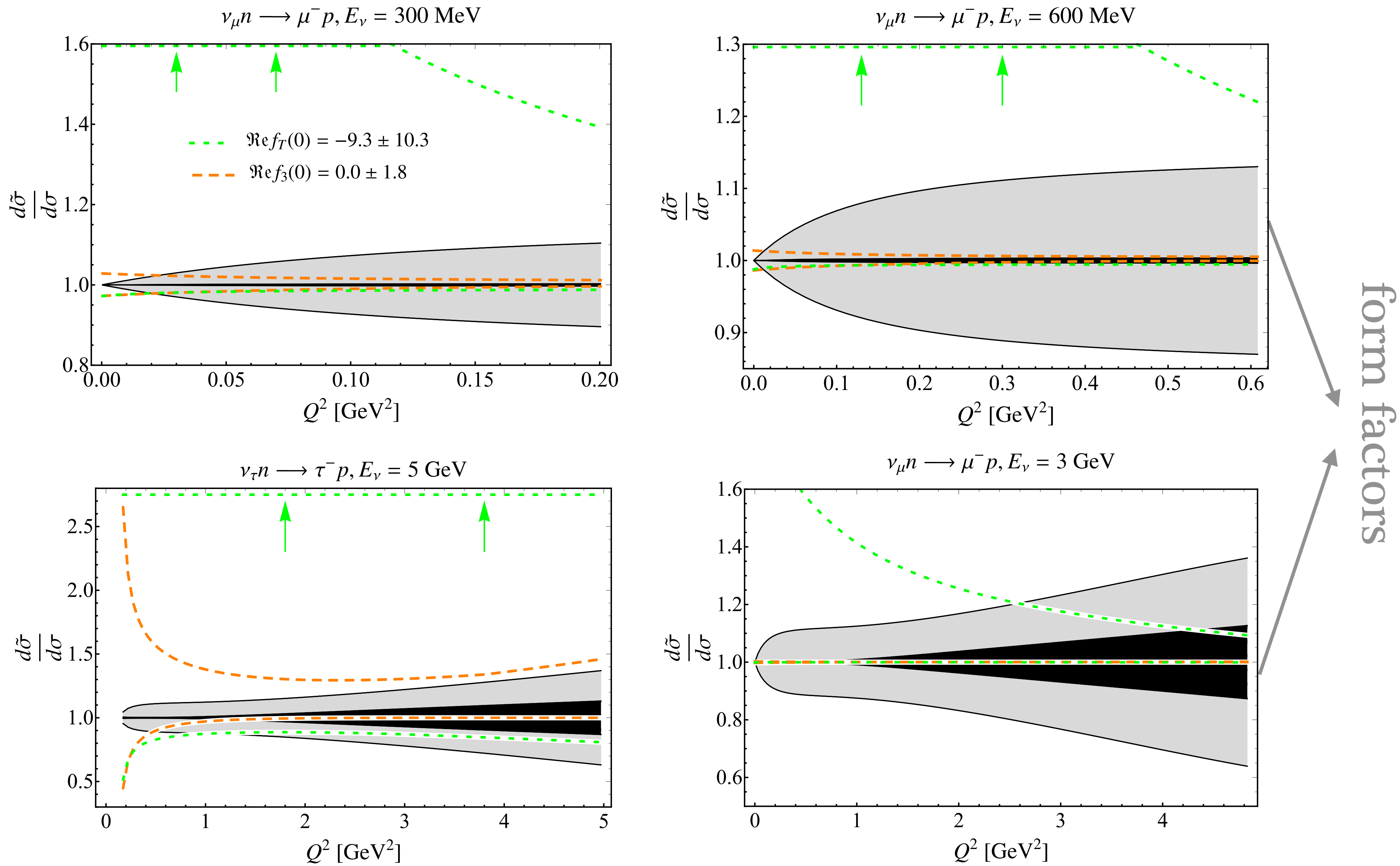
tensor → N. Severijins et al., Rev. Mod. Phys. 78, 991 (2006)

tensor, other → M. Gonzales-Alonso et al., Prog. Part. Nucl. Phys. 104, 165 (2019)

other → M. Day and K. S. McFarland, Phys. Rev. D 86, 053003 (2012)

Unpolarized cross section

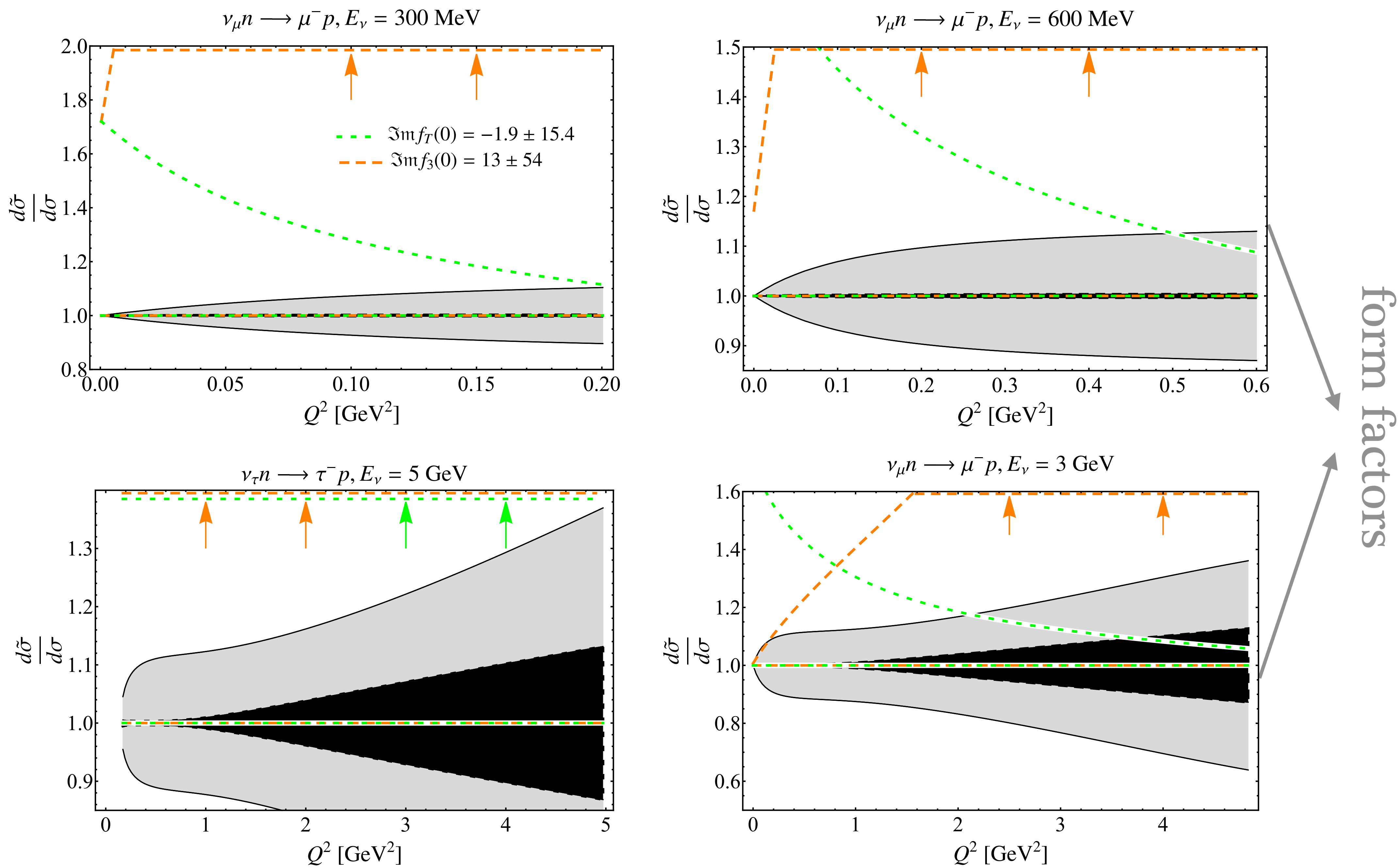
- Standard Model scaling with lepton mass, dipole Q^2 dependence



- sensitivity to tensor interactions for muon and tau neutrinos

Unpolarized cross section

- Standard Model scaling with lepton mass, dipole Q^2 dependence



- sensitivity to scalar and tensor interactions

Radiative corrections

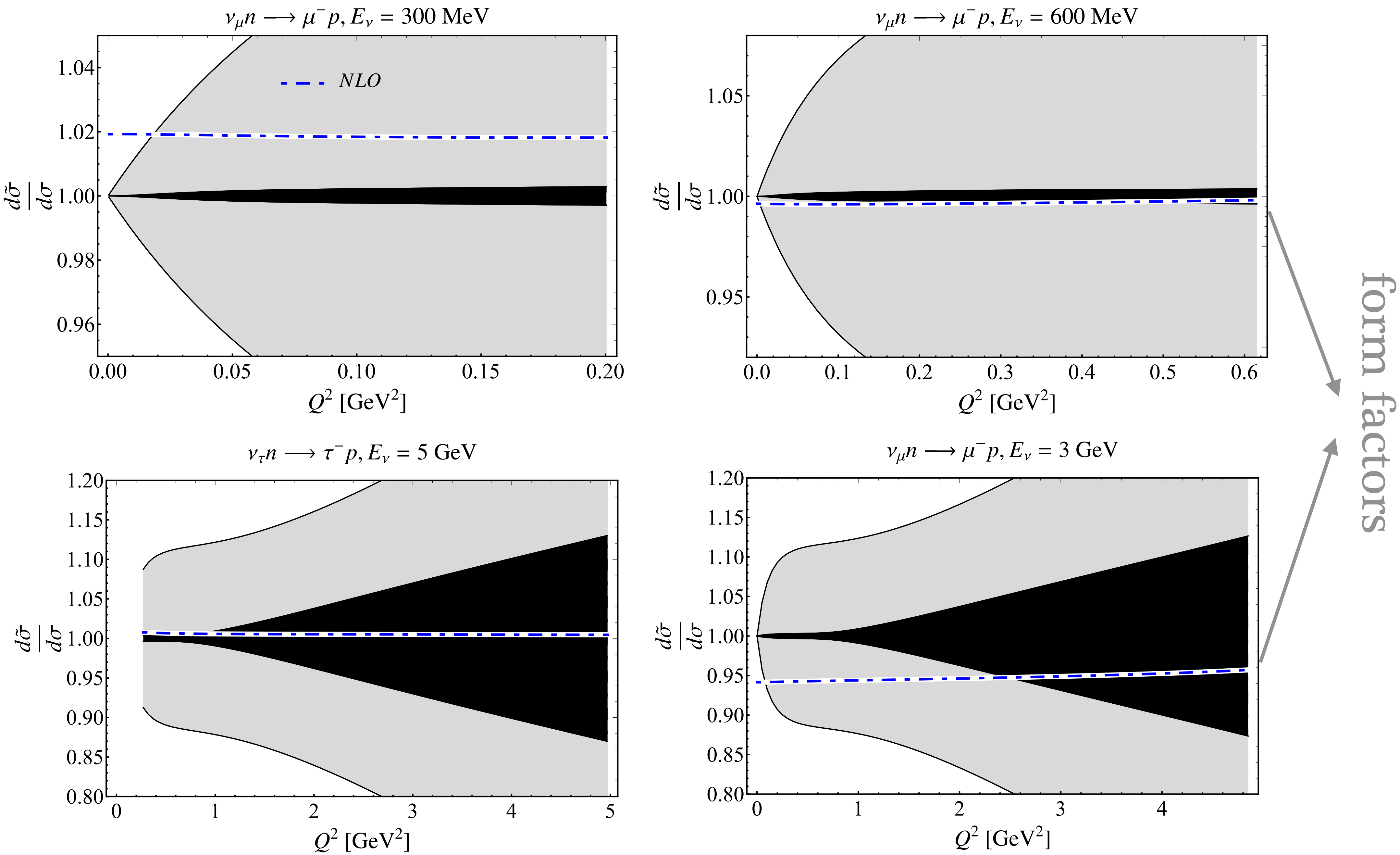
O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion in Phys. Rev. D (2022)

QED virtual and real

Unpolarized cross section

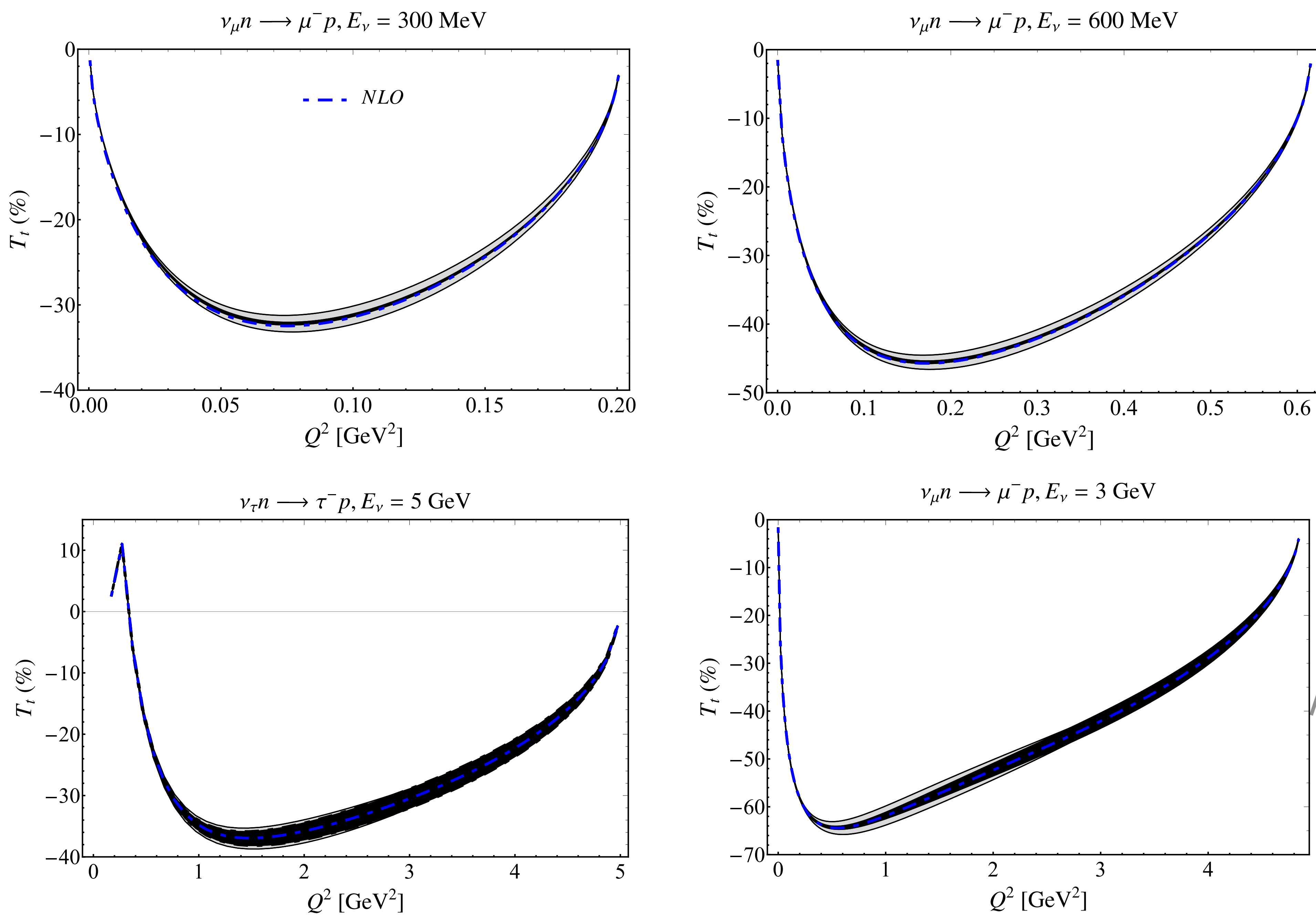
- QED virtual corrections + 1 soft photon with energy below 10 MeV



- muon flavor: sizable corrections at low and high energies

Transverse target asymmetry

- QED virtual corrections + 1 soft photon with energy below 10 MeV

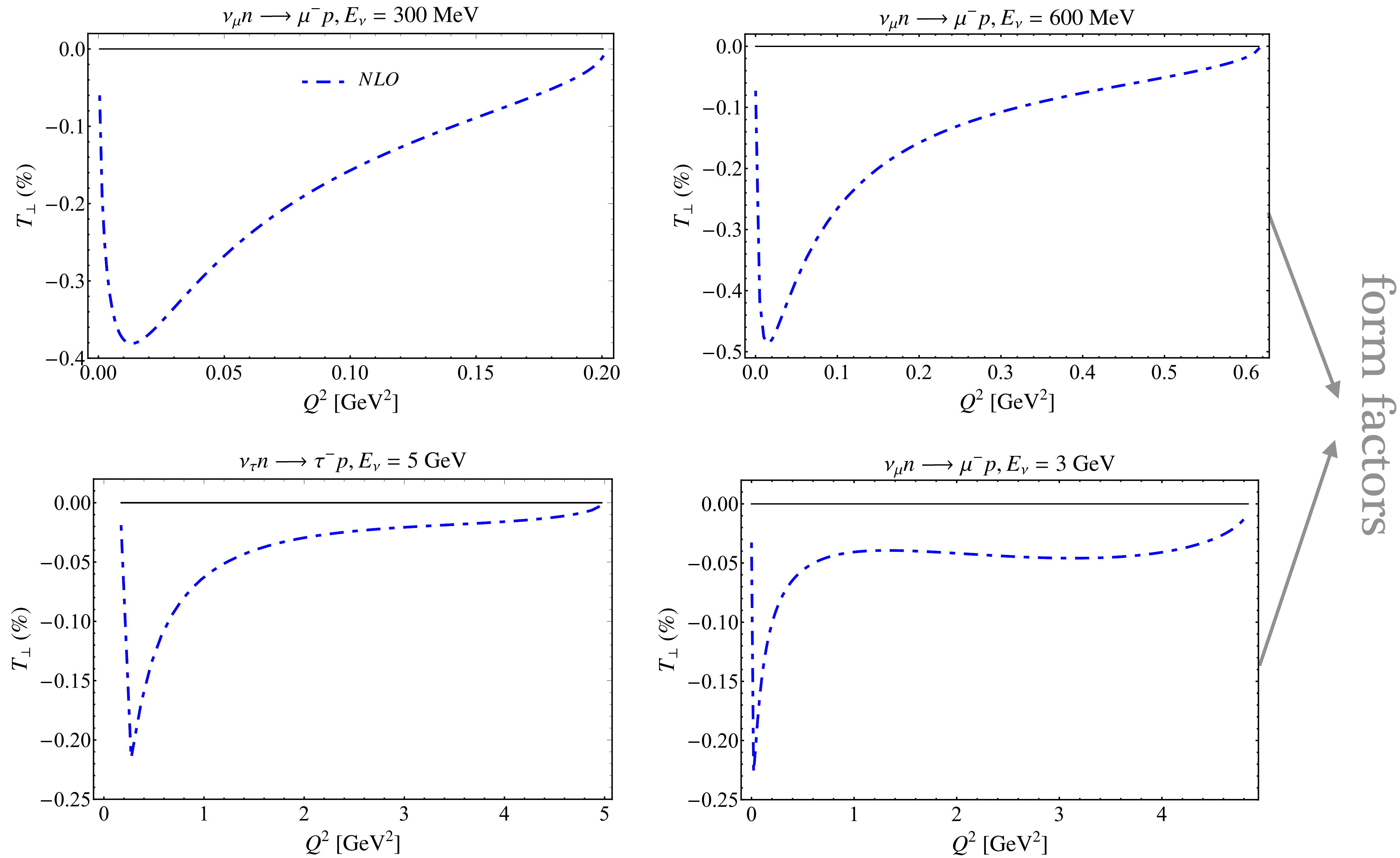


form factors

- negligible radiative corrections

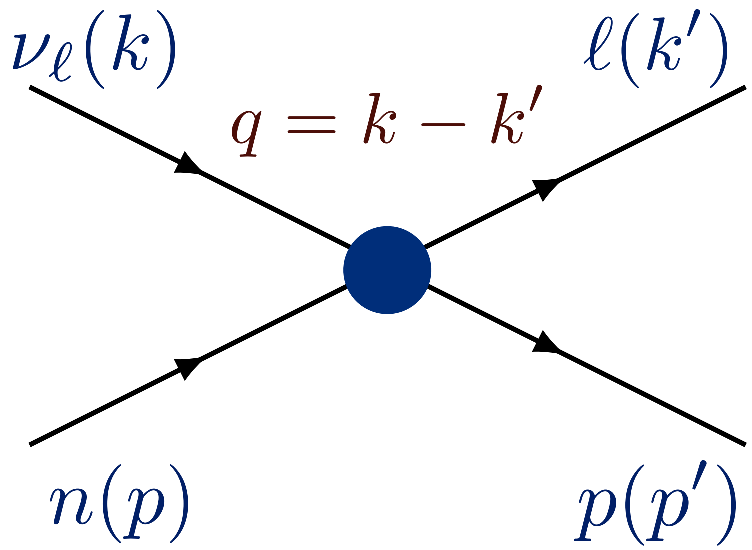
Transverse target asymmetry 2

- QED virtual corrections + 1 soft photon with energy below 10 MeV



- radiative corrections: SSA with normal to the scattering plane spin

Invariant amplitudes



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- four extra amplitudes for massive charged lepton

$$T^{m_\ell \neq 0}_{\nu_\ell n \to \ell^- p} = -\sqrt{2} \mathrm{G_F} V_{ud} \frac{m_\ell}{M} \, \bar{\ell}^- \mathrm{P_L} \nu_\ell \, \bar{p} \left(f_3 + f_P \gamma_5 - \frac{f_R}{4} \frac{P_\mu}{M} \gamma^\mu \gamma_5 \right) n$$

$$+ \sqrt{2} \mathrm{G_F} V_{ud} \frac{m_\ell}{M} \frac{f_T}{4} \, \bar{\ell}^- \sigma^{\mu\nu} \mathrm{P_L} \nu_\ell \, \bar{p} \sigma_{\mu\nu} n$$

- 8 invariant amplitudes for charged-current elastic scattering

Constraints from MINERvA data

- Standard Model scaling with lepton mass, dipole Q^2 dependence

- nucleon form factors from experimental data

A.S. Meyer, M. Betancourt, R. Gran, and R.J. Hill, PRD (2016)
Kaushik Borah, Gabriel Lee, Richard J. Hill, and O. T. , PRD (2021)

- rigorous error propagation to 68% confidence intervals

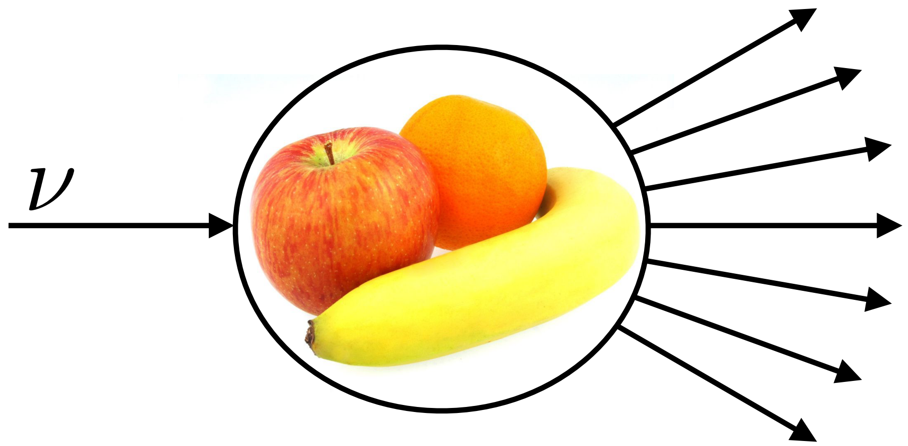
	$\mathfrak{Re}\,\bar{f}_3$	$\mathfrak{Re}\,\bar{f}_T$	$\mathfrak{Re}\,\bar{f}_{A3}$	$\mathfrak{Re}\,\bar{f}_R$
$\bar{\nu}p$ scattering	$88.4^{+33.5}_{-58.0}$	$-0.5^{+5.0}_{-4.8}$	$-1.0^{+0.4}_{-0.3}$ & $1.0^{+0.3}_{-0.4}$	$-80.1^{+40.6}_{-26.0}$
beta decay	0.0 ± 1.8	-9.3 ± 10.3	0.0 ± 0.075	

	$\mathfrak{Im}\,\bar{f}_3$	$\mathfrak{Im}\,\bar{f}_T$	$\lvert\mathfrak{Im}\,\bar{f}_{A3}\rvert$	$\lvert\mathfrak{Im}\,\bar{f}_R\rvert$
$\bar{\nu}p$ scattering	$-82.1^{+34.6}_{-23.8}$ & $82.1^{+23.8}_{-34.6}$	0.0 ± 4.9	$1.00^{+0.29}_{-0.43}$	$69.9^{+20.9}_{-30.9}$
beta decay	13.0 ± 54.0	-1.9 ± 15.4		

- improvements for tensor and scalar interactions

- first constraints on muon-specific interactions at quark level

Conclusions



neutrino interactions

for precise measurements

- MINERvA data vs deuterium data and lattice QCD: within 1σ
- axial-vector form factor and radius at DUNE: subpercent precision
- unpolarized cross sections and single-spin asymmetries (SSA)
- SSA with spin in the scattering plane: radiative corrections negligible
- MINERvA hydrogen data: update on tensor and scalar interaction

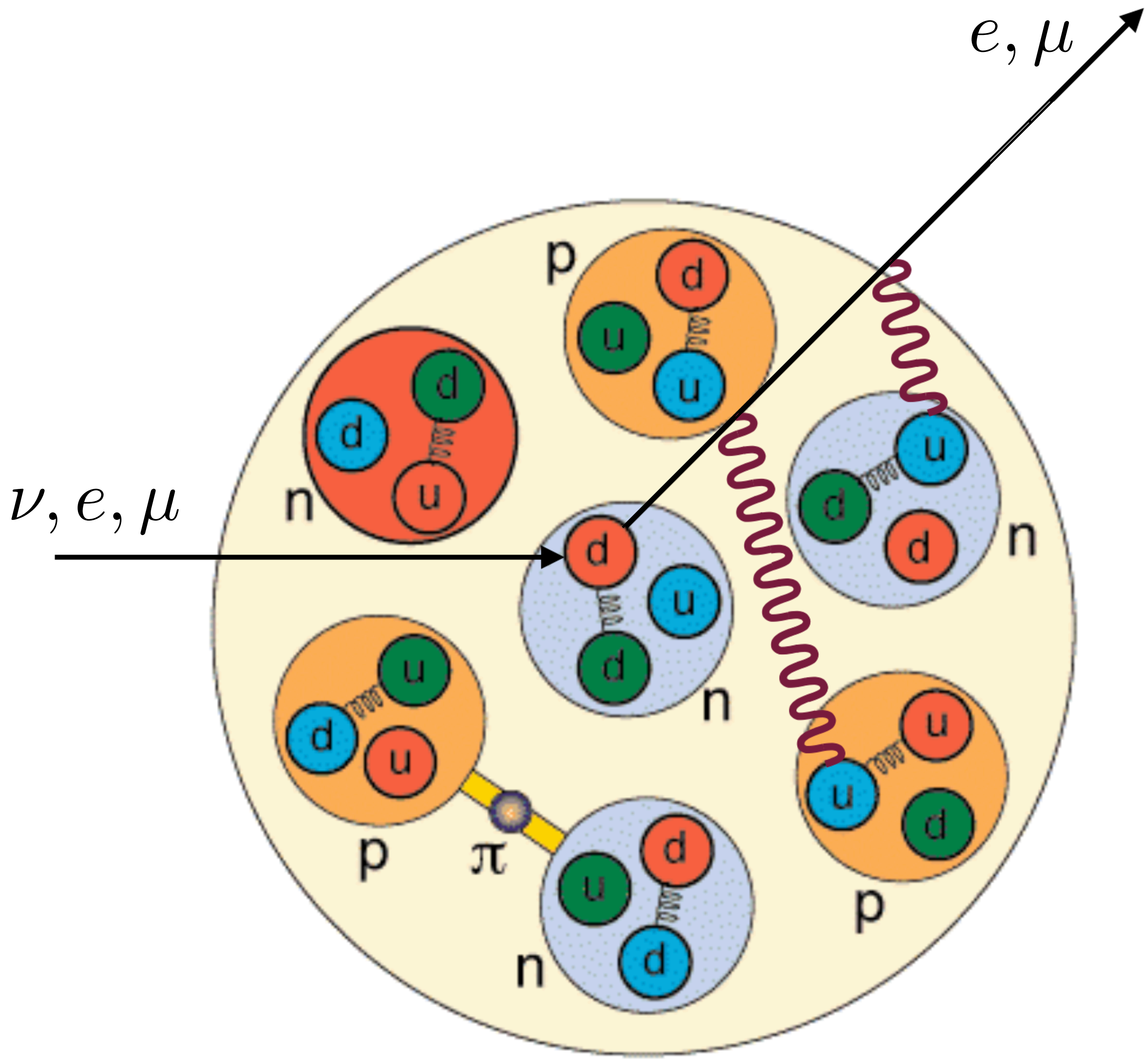
QED Nuclear Medium Effects

O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)

O. T. and Ivan Vitev, Phys. Rev. D 108 9, 9 (2023)

O. T. and Ivan Vitev, Phys. Rev. D 109 7, (2024)

QED medium effects



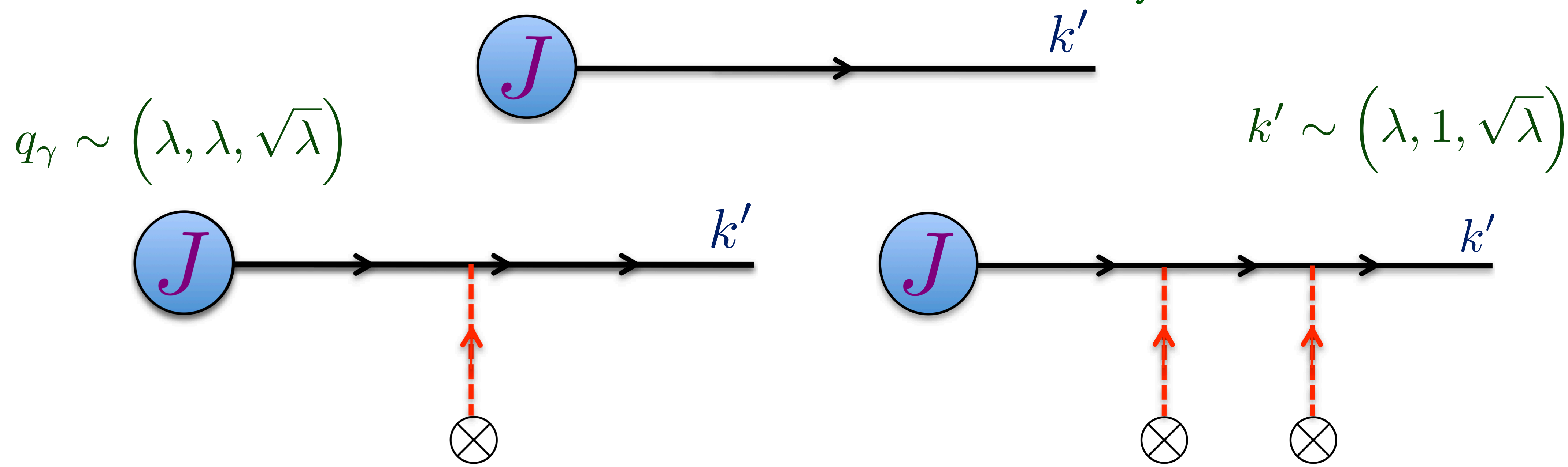
- charged lepton exchanges photons with nuclear medium

SCET$_G$ formulation

- forward scattering is dominant process

- Glauber photons exchanged with a nuclear charge distribution

QCD: G. Ovanesyan and I. Vitev, JHEP (2011)



- change: integral along final lepton direction over charge and potential

$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho\left(z\right) \mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{\left(2\pi\right)^2} |v\left(\vec{q}_\perp\right)|^2 \left(\sigma_0\left(\vec{k}, \vec{k}' - \vec{q}_\perp\right) - \sigma_0\left(\vec{k}, \vec{k}'\right)\right)$$

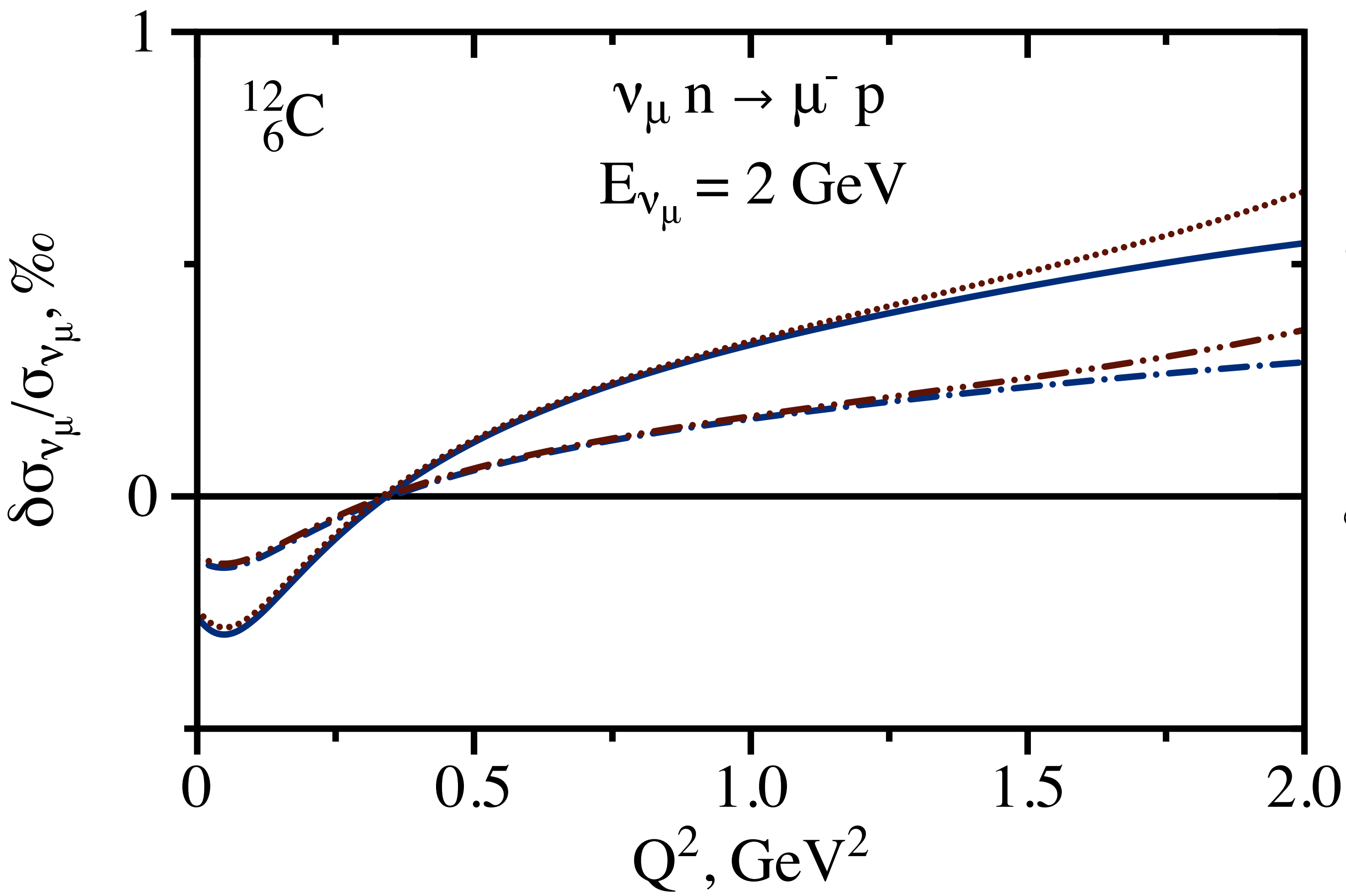
- leading-order cross sections are distorted
- EFT and full QED calculations are performed

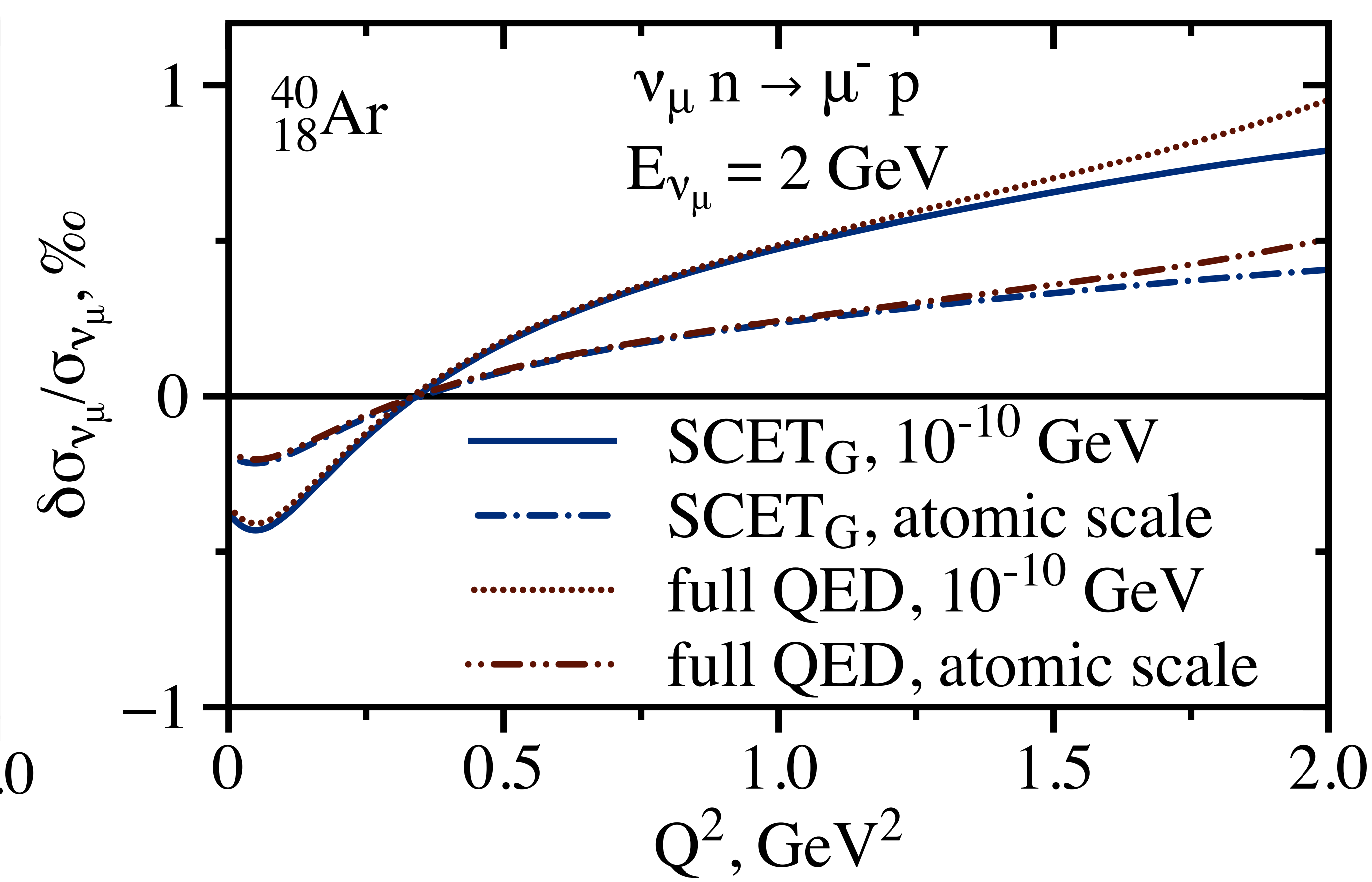
Neutrino scattering

- relative correction per nucleon

IR regularization

$$v\left(q_\perp^2\right) = \frac{e^2}{q_\perp^2 + \lambda^2}$$





flavor-independent at GeV energies

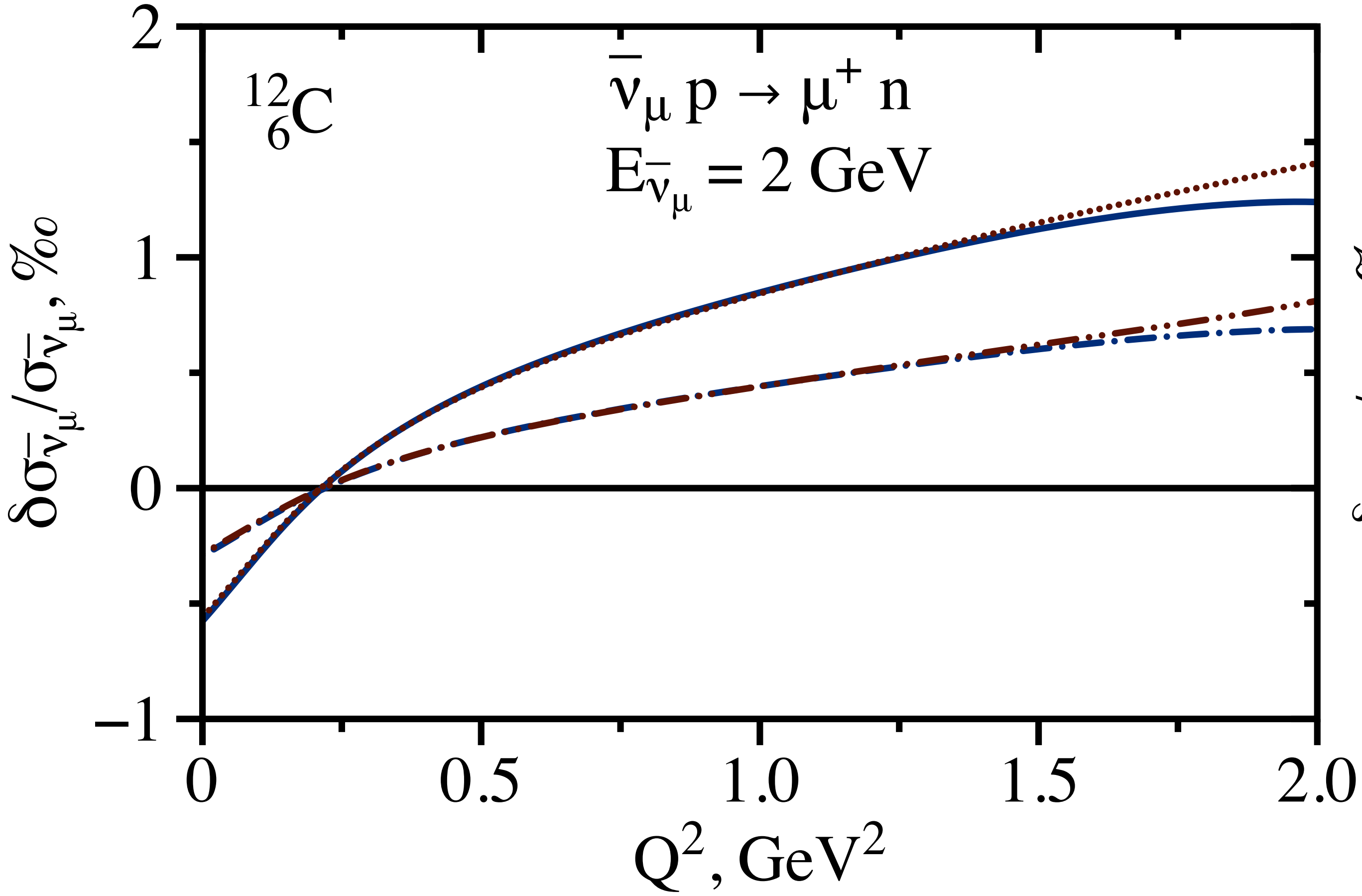
- permille-level distortion of cross sections: $O(\alpha^2)$ correction
- smaller correction to inclusive cross section

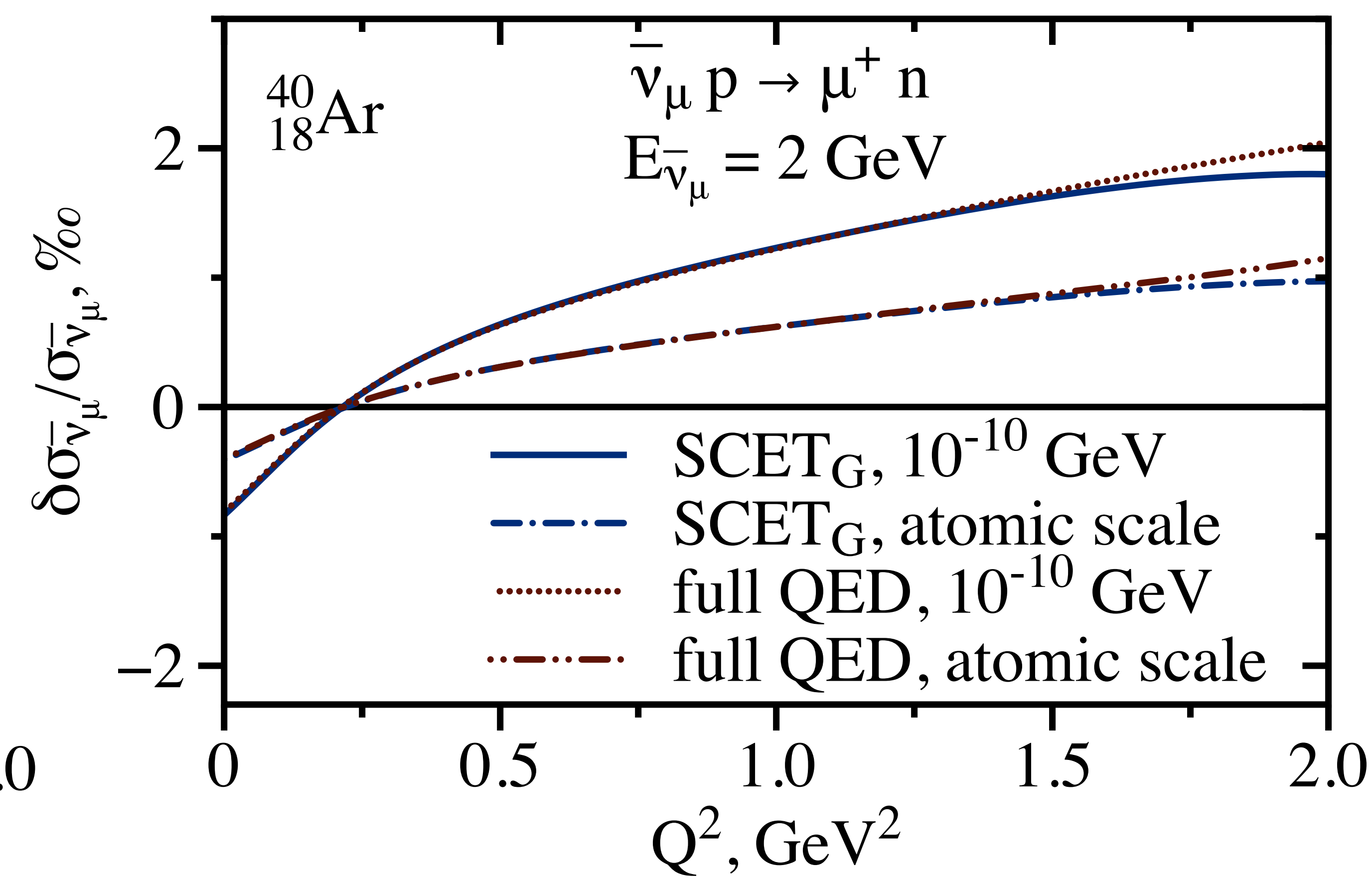
Antineutrino scattering

- relative correction per nucleon

IR regularization

$$v\left(q_\perp^2\right) = \frac{e^2}{q_\perp^2 + \lambda^2}$$





flavor-independent at GeV energies

- permille-level distortion of cross sections: $O(\alpha^2)$ correction
- larger correction than for neutrino scattering

SCET$_G$ formulation

- forward scattering is dominant process

- Glauber photons exchanged with a nuclear charge distribution

- add initial-state exchanges, no interference with final-state exchanges

- change: integral along initial lepton direction over charge and potential

$$\delta\sigma_i \sim \int_{\text{lepton line}}^{\text{initial}} \rho(z)\,\mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0\left(\vec{k}+\vec{q}_\perp, \vec{k}'\right) - \sigma_0\left(\vec{k}, \vec{k}'\right)\right)$$

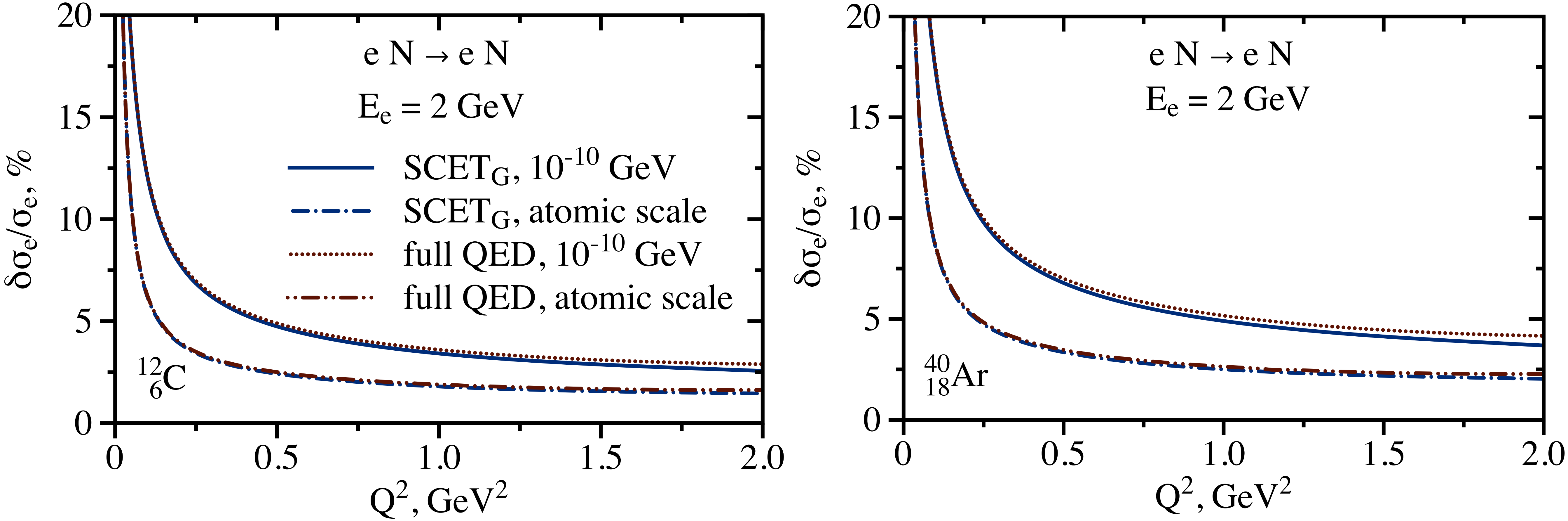
- change: integral along final lepton direction over charge and potential

$$\delta\sigma_f \sim \int_{\text{lepton line}}^{\text{final}} \rho(z)\,\mathrm{d}z \int \frac{\mathrm{d}^2\vec{q}_\perp}{(2\pi)^2} |v(\vec{q}_\perp)|^2 \left(\sigma_0\left(\vec{k}, \vec{k}'-\vec{q}_\perp\right) - \sigma_0\left(\vec{k}, \vec{k}'\right)\right)$$

- leading-order cross sections are distorted
- EFT and full QED agree above the lepton mass scale

Electron scattering

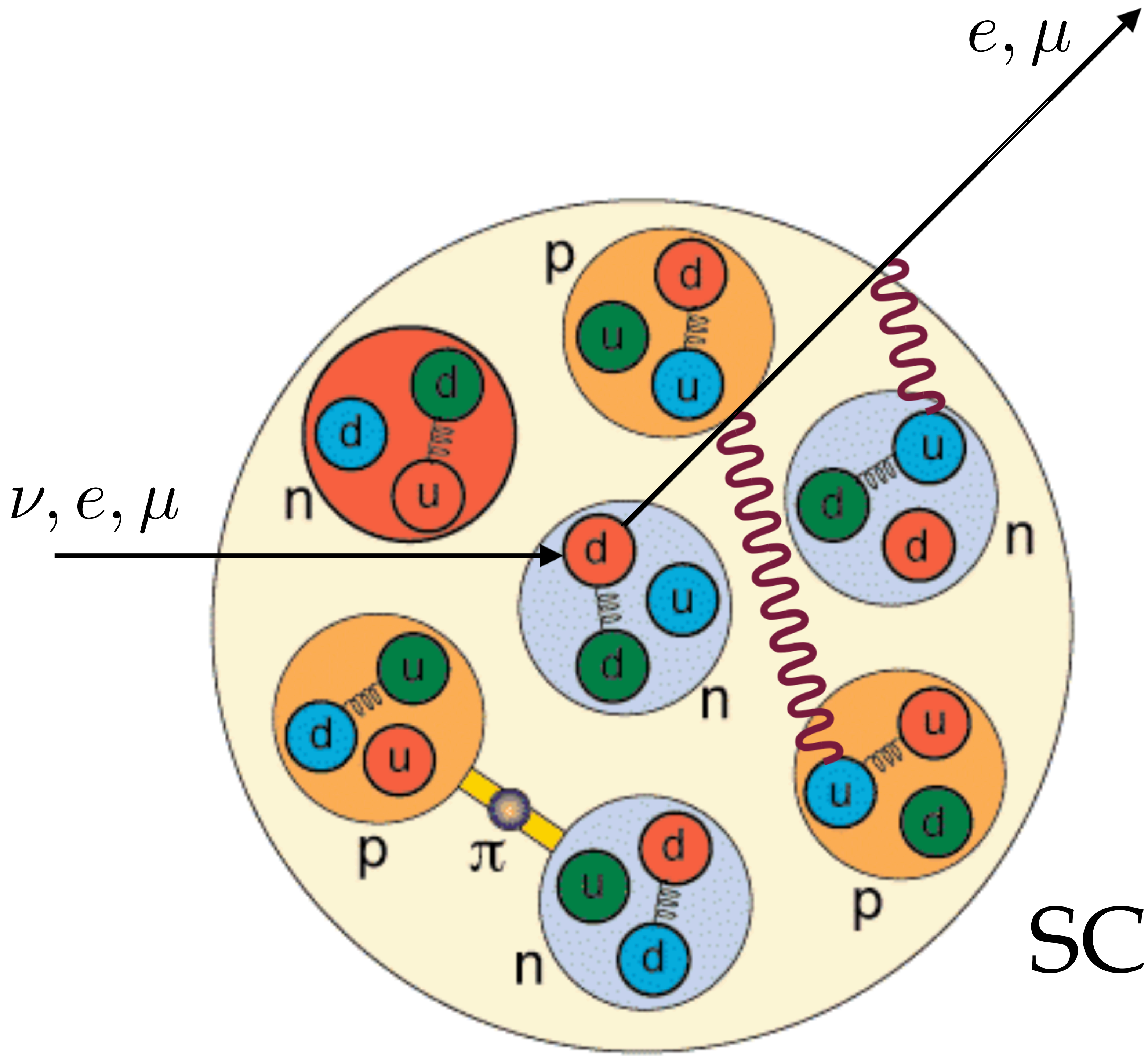
- relative correction per nucleus after incoherent sum over nucleons



O. T. and Ivan Vitev, Phys. Lett. B 805, 135466 (2022)

- percent-level at low momentum transfers: $O(\alpha^2)$ correction
- critical new effect for electron scattering experiments

QED medium effects

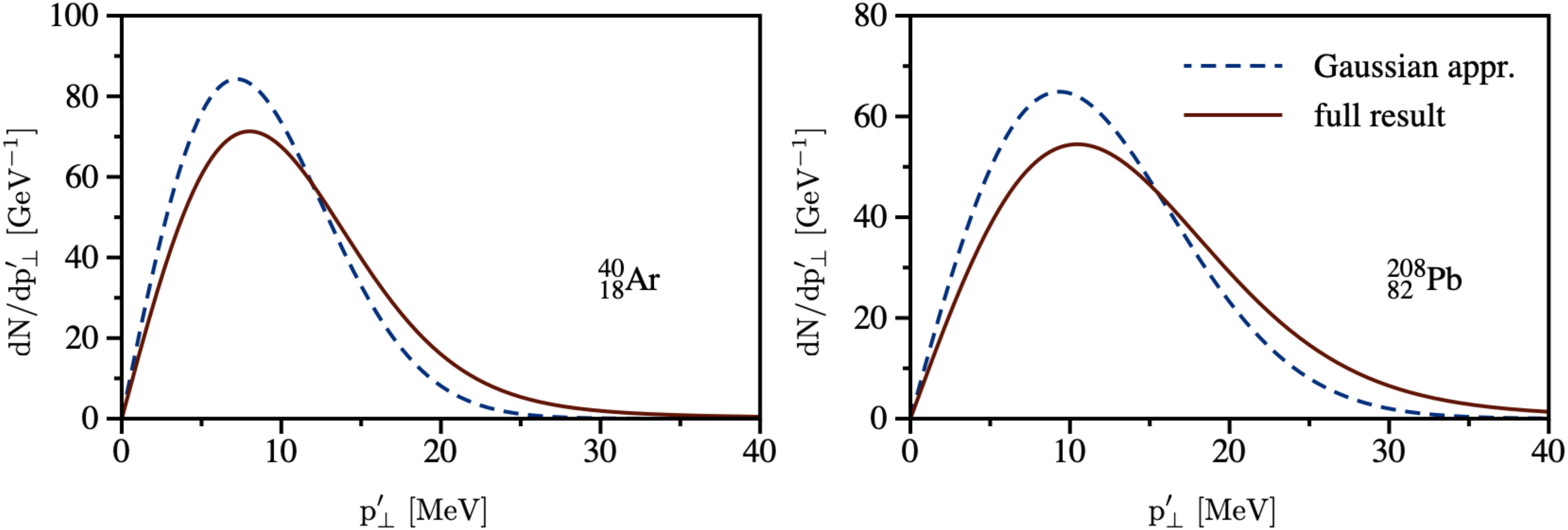


SCET$_G$ only

- >10000 interactions along the lepton trajectory resumed

Broadening of electron tracks

- multiple re-scattering generates transverse momentum



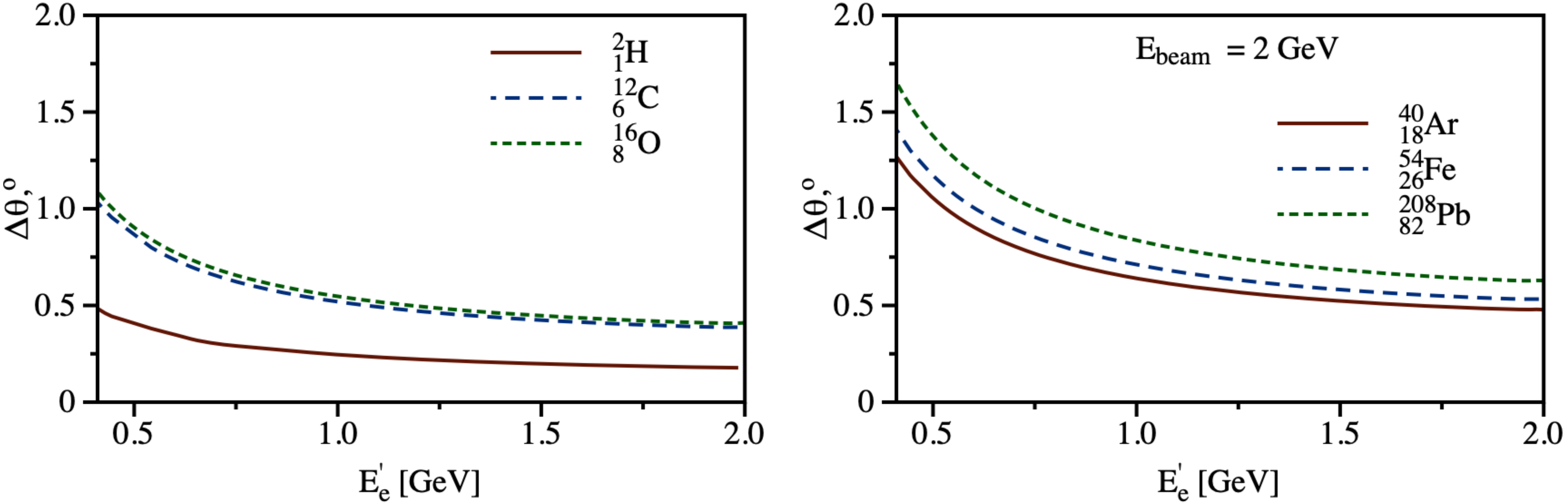
O. T. and Ivan Vitev, Phys. Rev. D 108 9, 9 (2023)

- exact resummation vs Gaussian approximation: nuclear size scale

- Glauber exchange induces 10-30 MeV transverse momentum

Broadening of electron tracks

- r. m. s. deflection angle after multiple rescattering



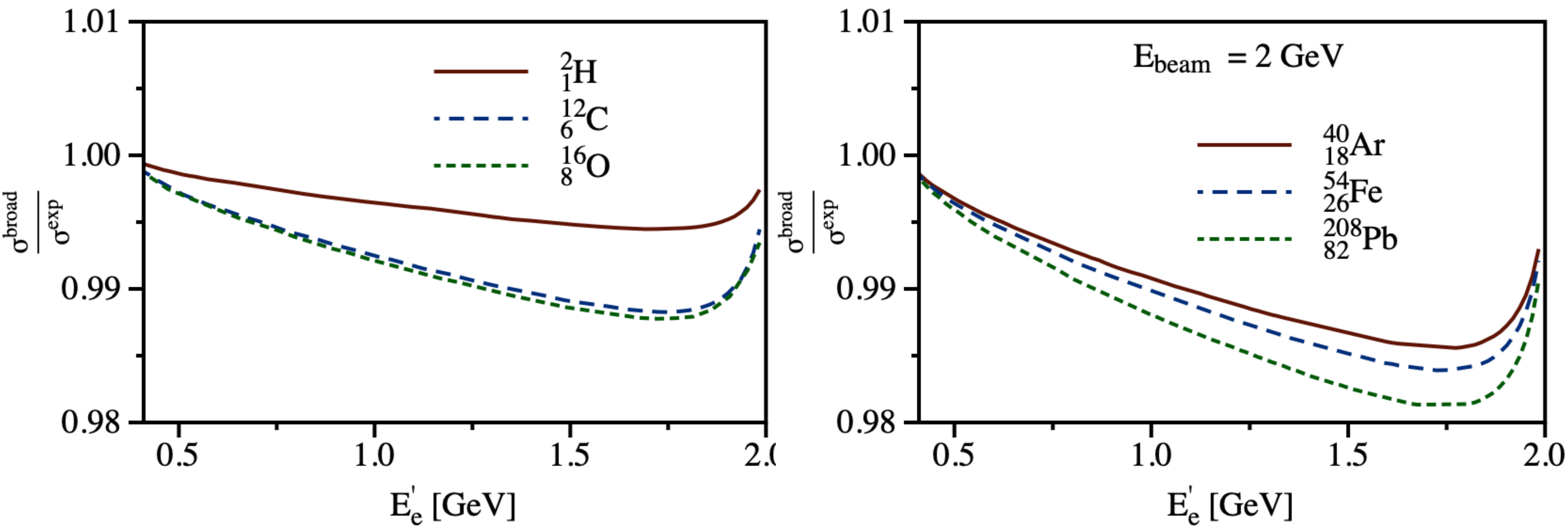
- nucleus approximated as sphere of constant density

- sizable deflection of electron tracks $\sqrt{< (\Delta\theta)^2 >} \sim 1/E$

Effect on unpolarized cross section

- initial and final re-scattering is taken into account

- momentum transfer from electron kinematics



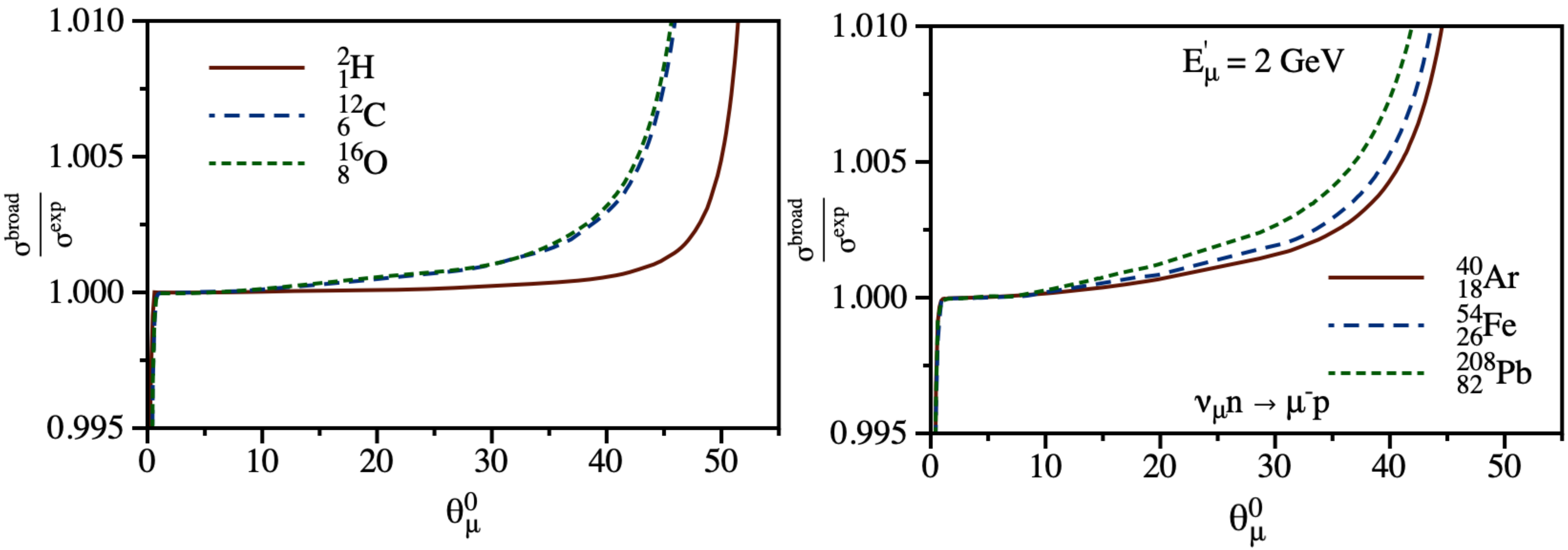
- nucleus approximated as sphere of constant density

- percent-level electron-nucleus cross-section suppression

Effect on unpolarized cross section

- only final re-scattering present

$$E_\nu^r \approx \frac{E'_\mu - \frac{1}{2}\frac{E_B^2 - 2M_i E_B + m_\mu^2 + M_i^2 - M_f^2}{M_i - E_B}}{1 - \frac{E'_\mu}{M_i - E_B}\left(1 - \beta_\mu \cos\theta_\mu\right)}$$



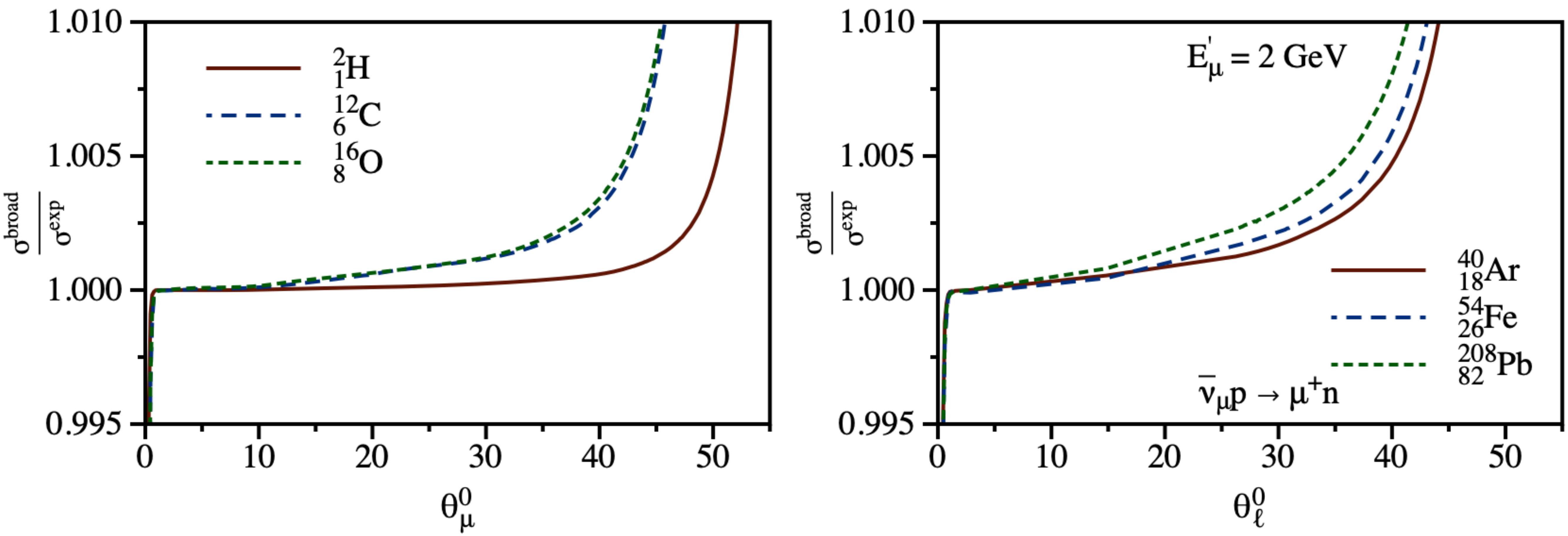
- nucleus approximated as sphere of constant density

- neutrino-nucleus: percent-level at kinematic endpoints

Effect on unpolarized cross section

- only final re-scattering present

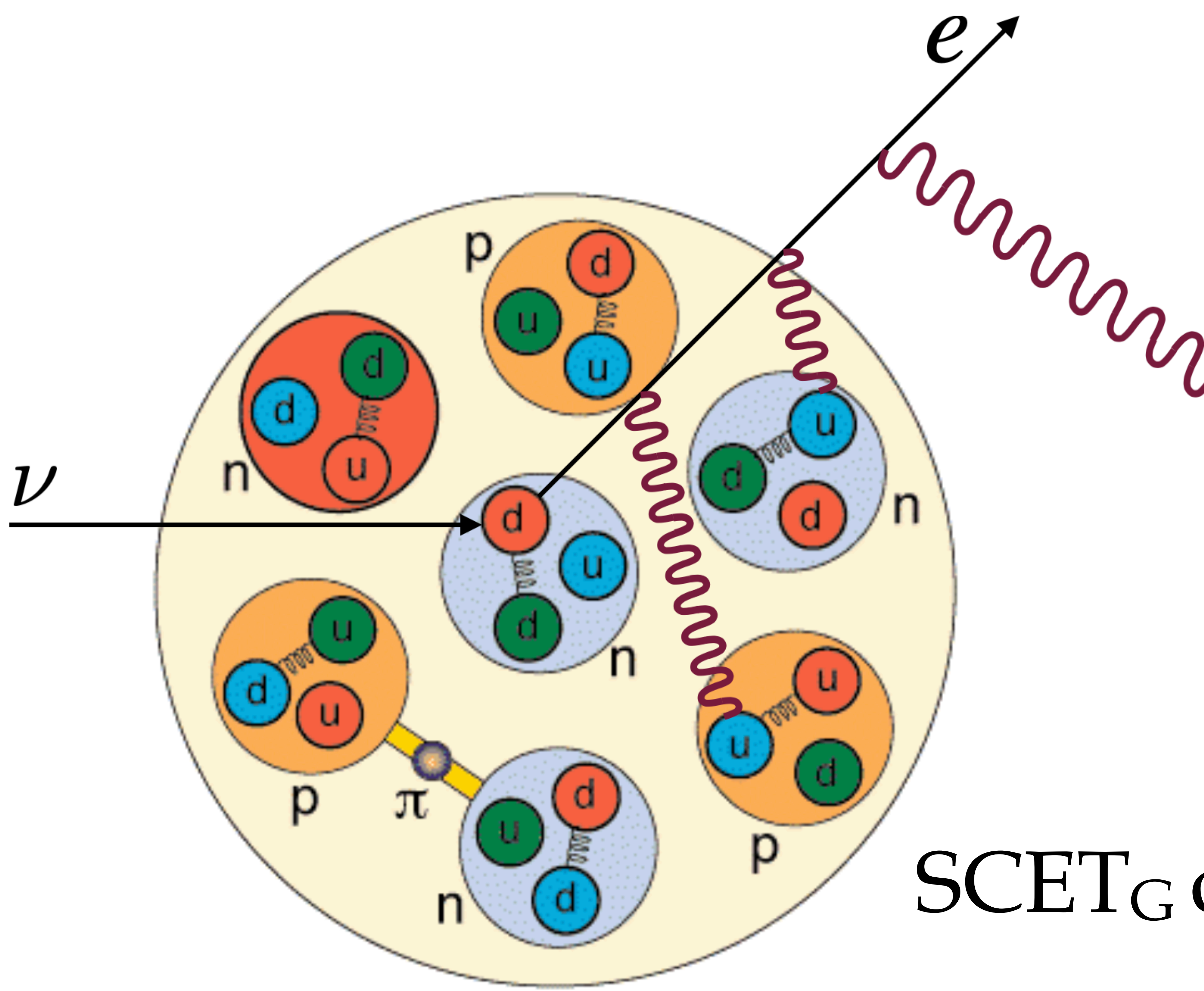
$$E_\nu^r \approx \frac{E'_\mu - \frac{1}{2}\frac{E_B^2 - 2M_i E_B + m_\mu^2 + M_i^2 - M_f^2}{M_i - E_B}}{1 - \frac{E'_\mu}{M_i - E_B}\left(1 - \beta_\mu \cos\theta_\mu\right)}$$



- nucleus approximated as sphere of constant density

- antineutrino-nucleus: percent-level at kinematic endpoints

QED medium-induced radiation



SCET$_G$ only

- >10000 interactions along the lepton trajectory resumed

QED medium-induced radiation

broadening with radiation: p_T spectrum is multiplied with soft (collinear) function in vacuum for observables including soft (collinear) photons

soft (collinear) functions in vacuum:

O. T., Qing Chen, Richard J. Hill and Kevin S. McFarland, Nature Commun. 13 (2022), 1, 5286

O. T., Qing Chen, Richard J. Hill, Kevin S. McFarland and Clarence Wret
editors suggestion in Phys. Rev. D (2022)

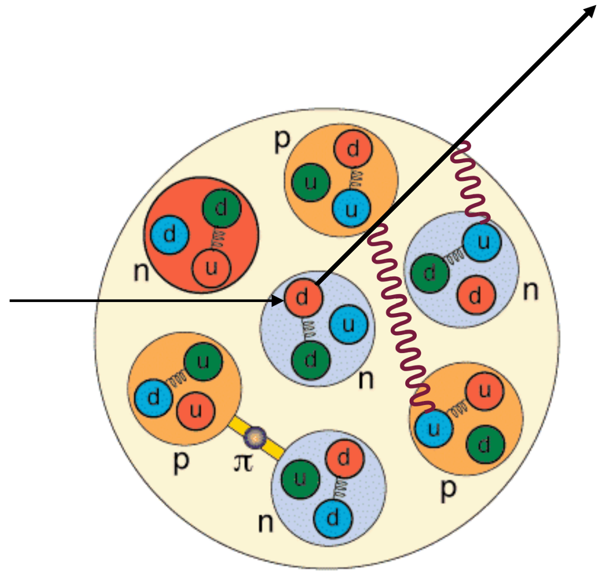
relative cross-section correction at each order in expansion on number of re-scatterings: the same for soft, collinear, and no-radiation cases

vanishing spectrum of soft or collinear medium-induced photons

O. T. and Ivan Vitev, Phys. Rev. D XXX X, X (2024)

- separation of scales: exact resummation of medium effects

Conclusions



formulation of

QED nuclear medium

effects

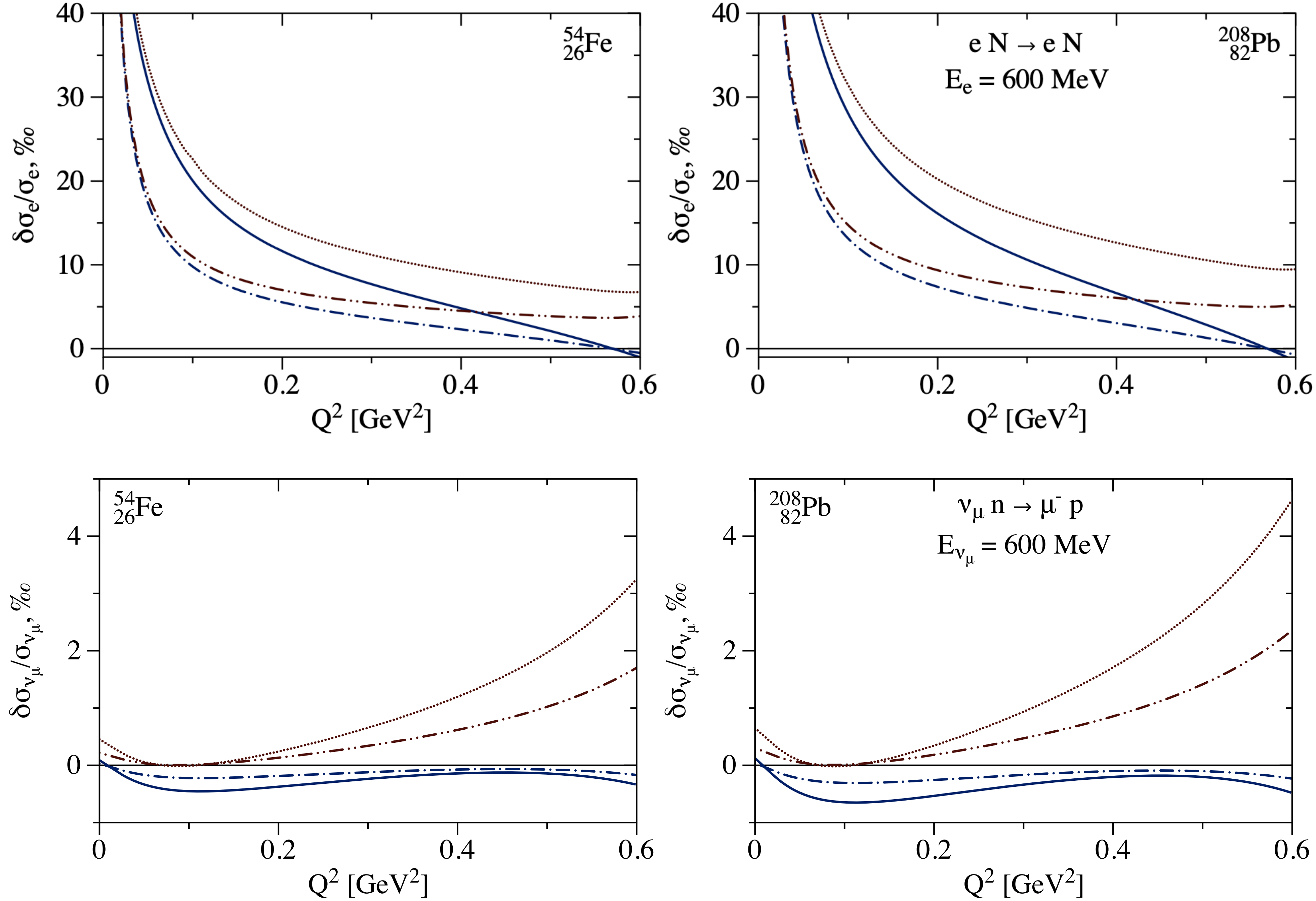
virtual corrections at 1st order in opacity: SCET_G and full QED

broadening and radiation: SCET_G

verified: SCET_G works perfectly at GeV energies

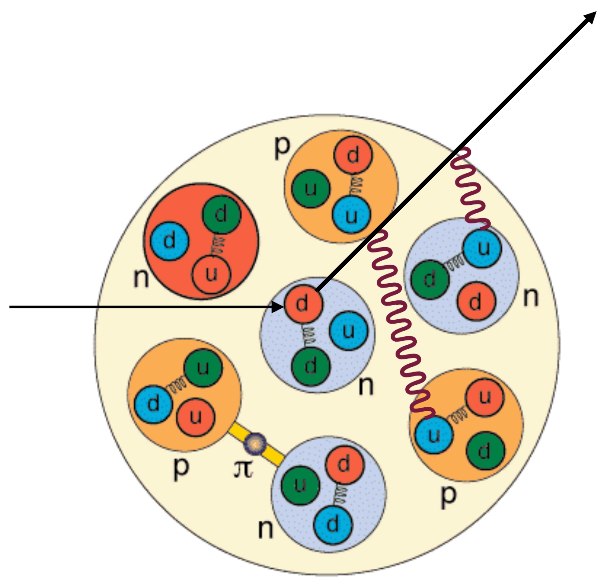
Cross sections at 600 MeV beam energy

1st order in opacity



- QED and $SCET_G$ significantly differ at 100th of MeV energy

Conclusions



formulation of

QED nuclear medium

effects

virtual corrections at 1st order in opacity: $SCET_G$ and full QED

broadening and radiation: $SCET_G$

verified: $SCET_G$ works perfectly at GeV energies but not for 100th MeV !!!

found: a) sizable deflection of charged lepton tracks

b) multiple rescattering: %-level corrections at GeV energies

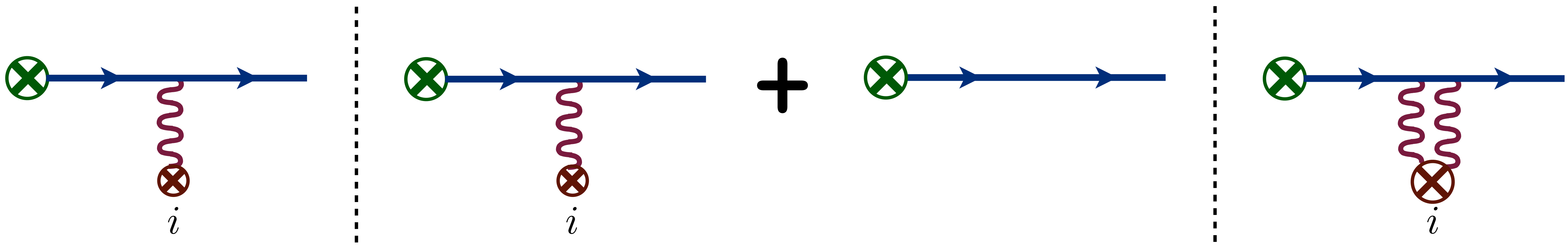
c) vanishing nuclear medium-induced photon energy spectra

d) radiation sizably (~10-20 %) modifies broadening

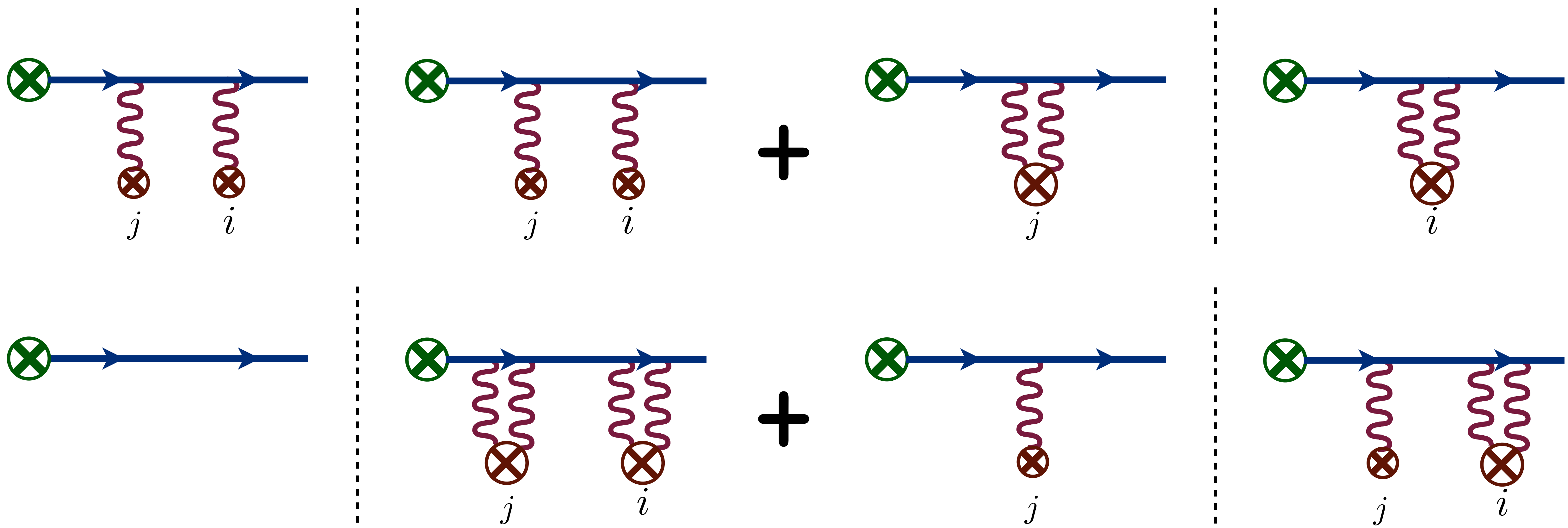
Thanks for your attention !!!

QED medium effects

1st order in opacity

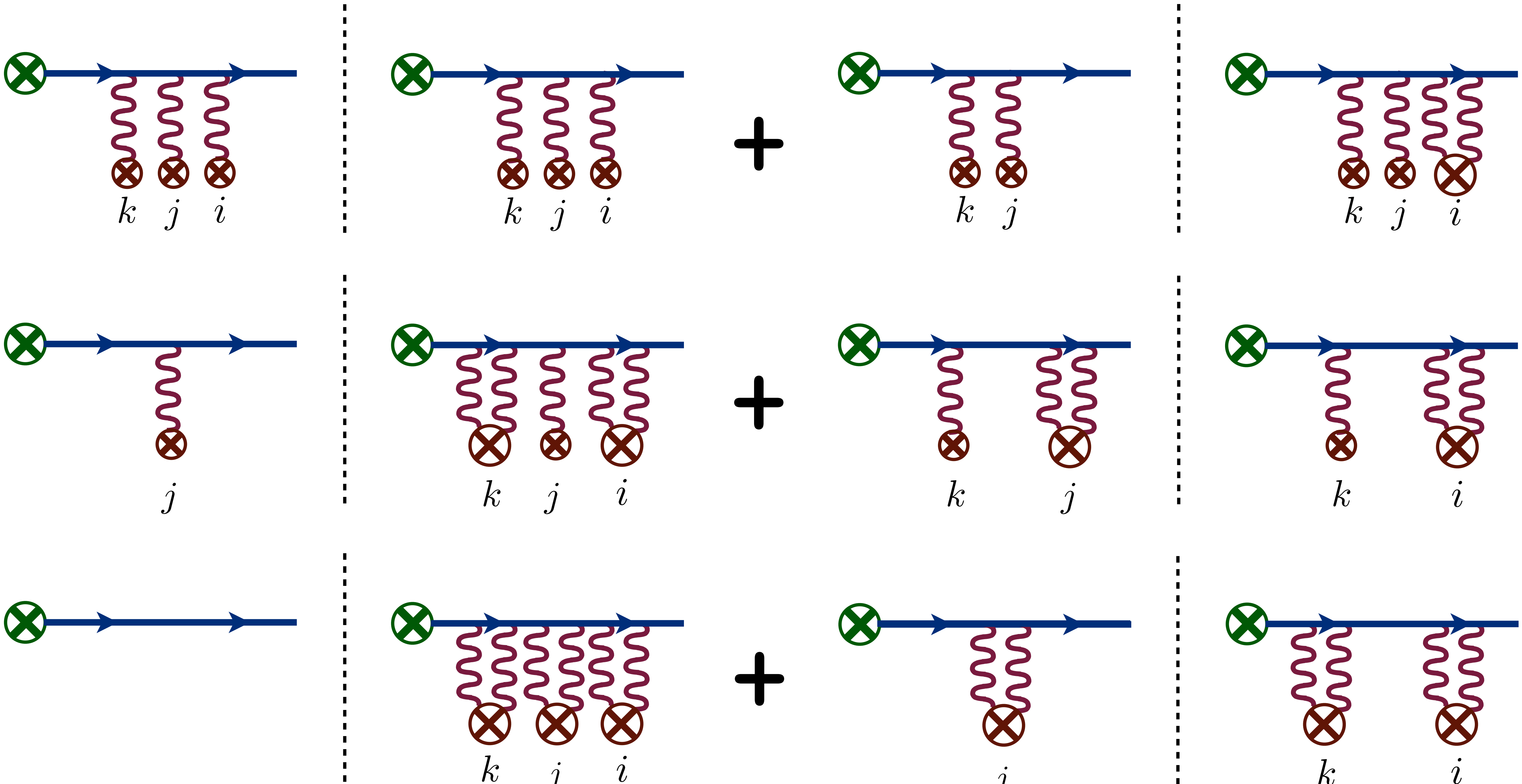


2nd order in opacity



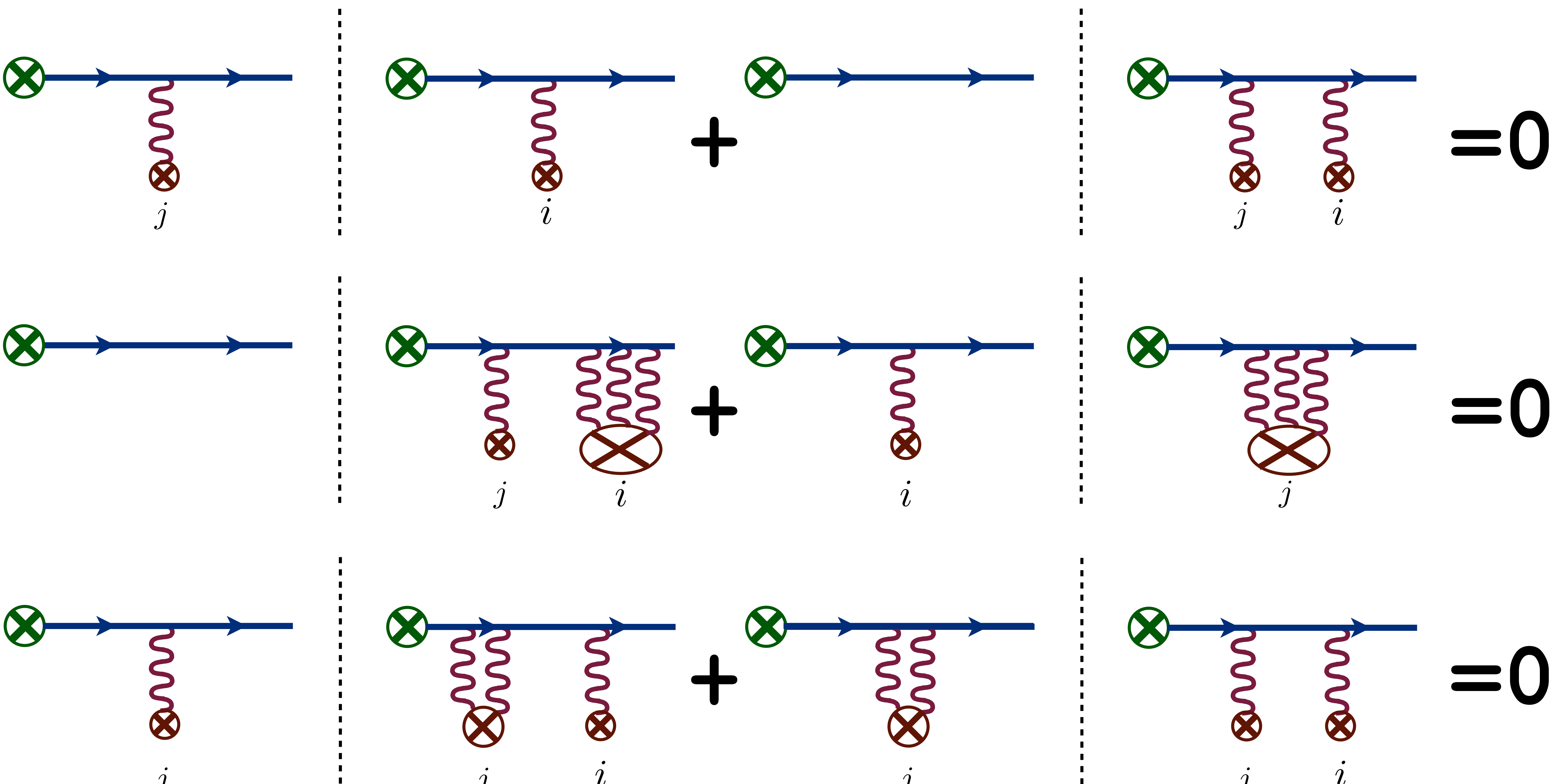
QED medium effects

3rd order in opacity



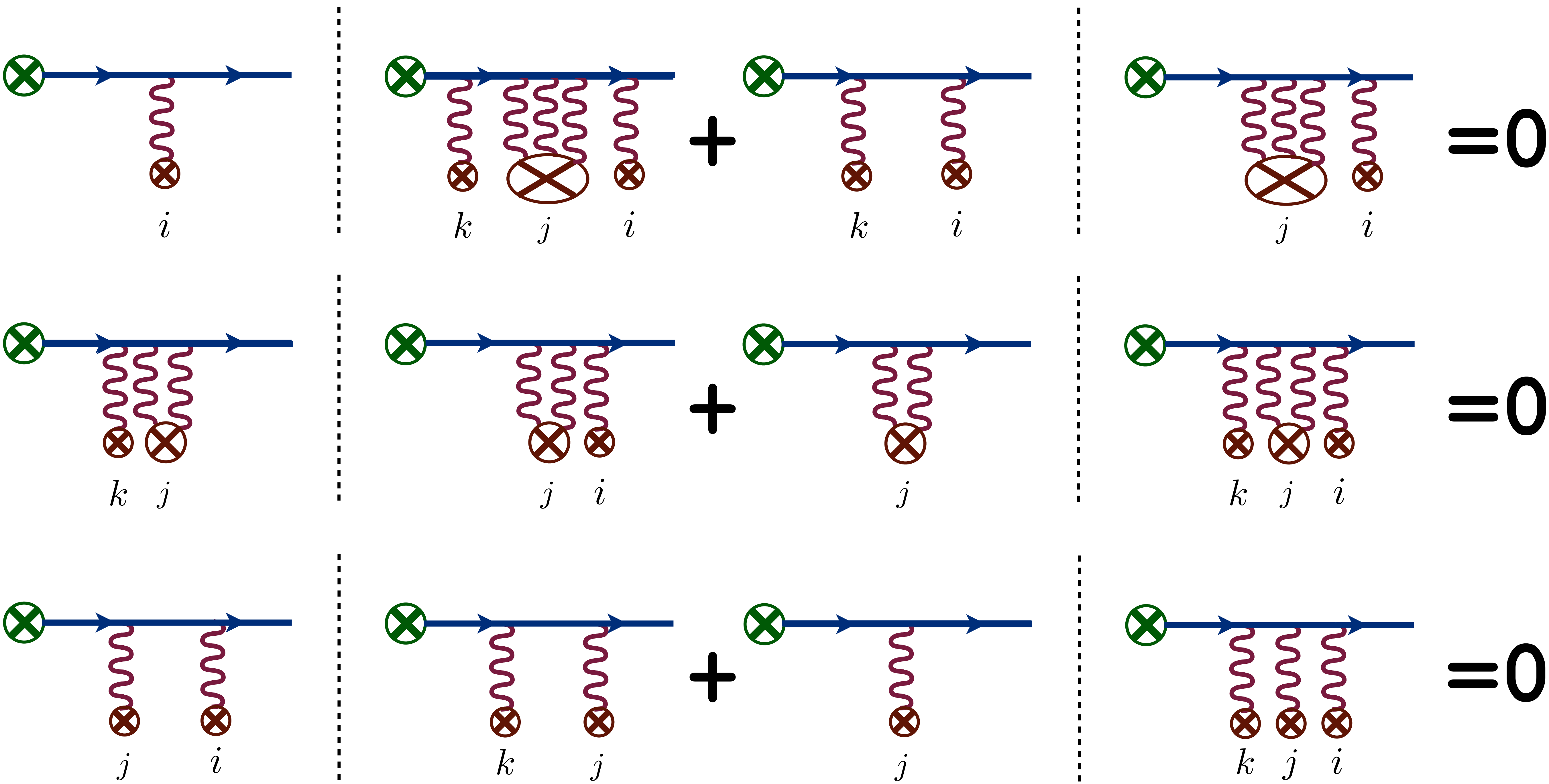
QED medium effects

2nd order in opacity



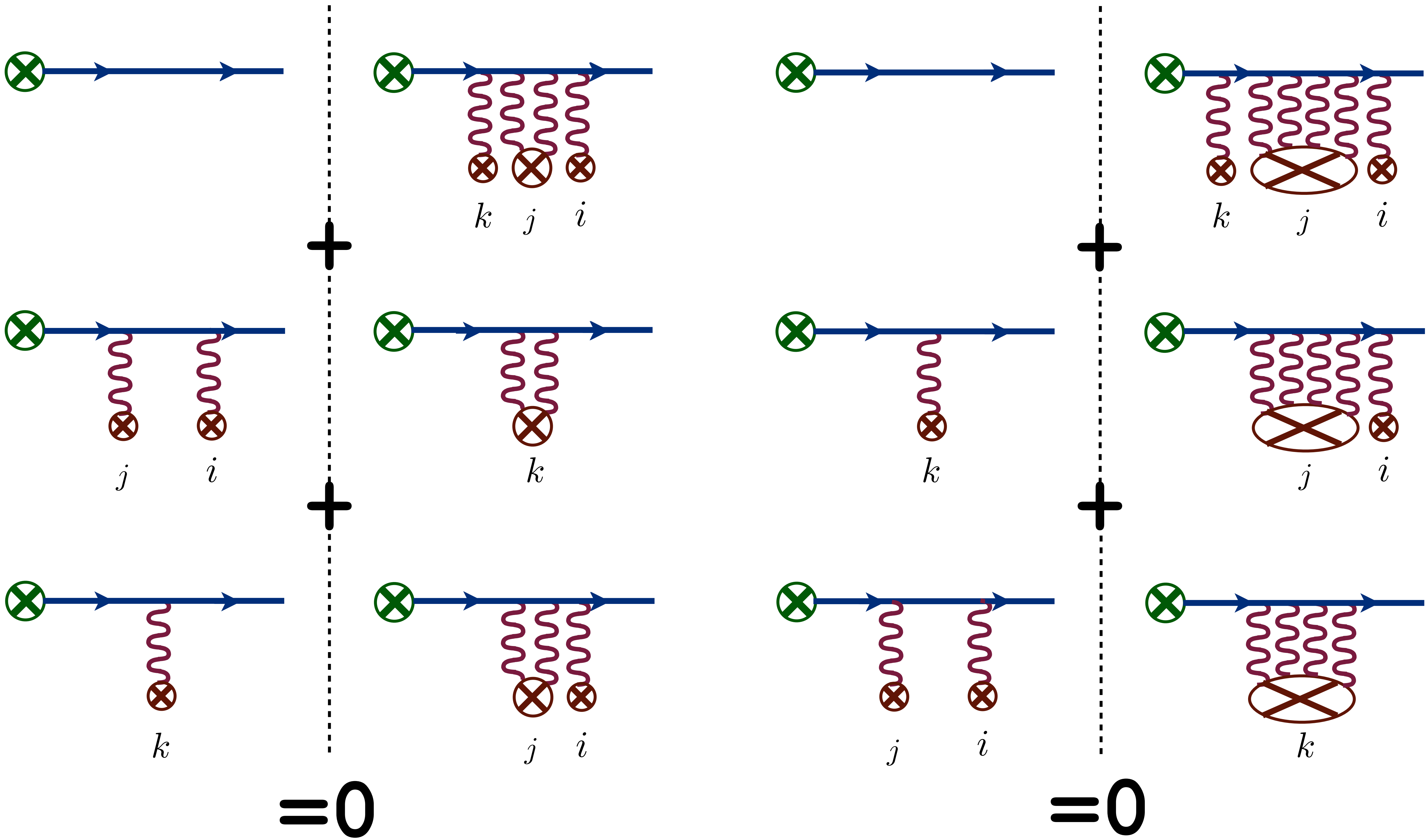
QED medium effects

3rd order in opacity



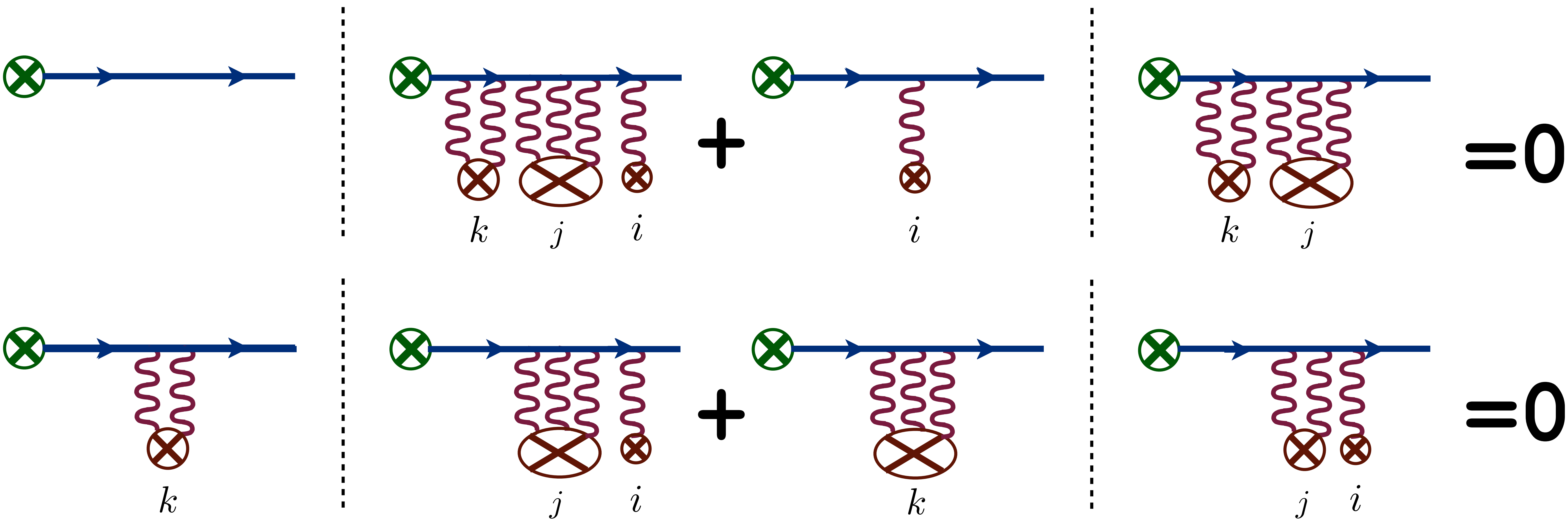
QED medium effects

3rd order in opacity



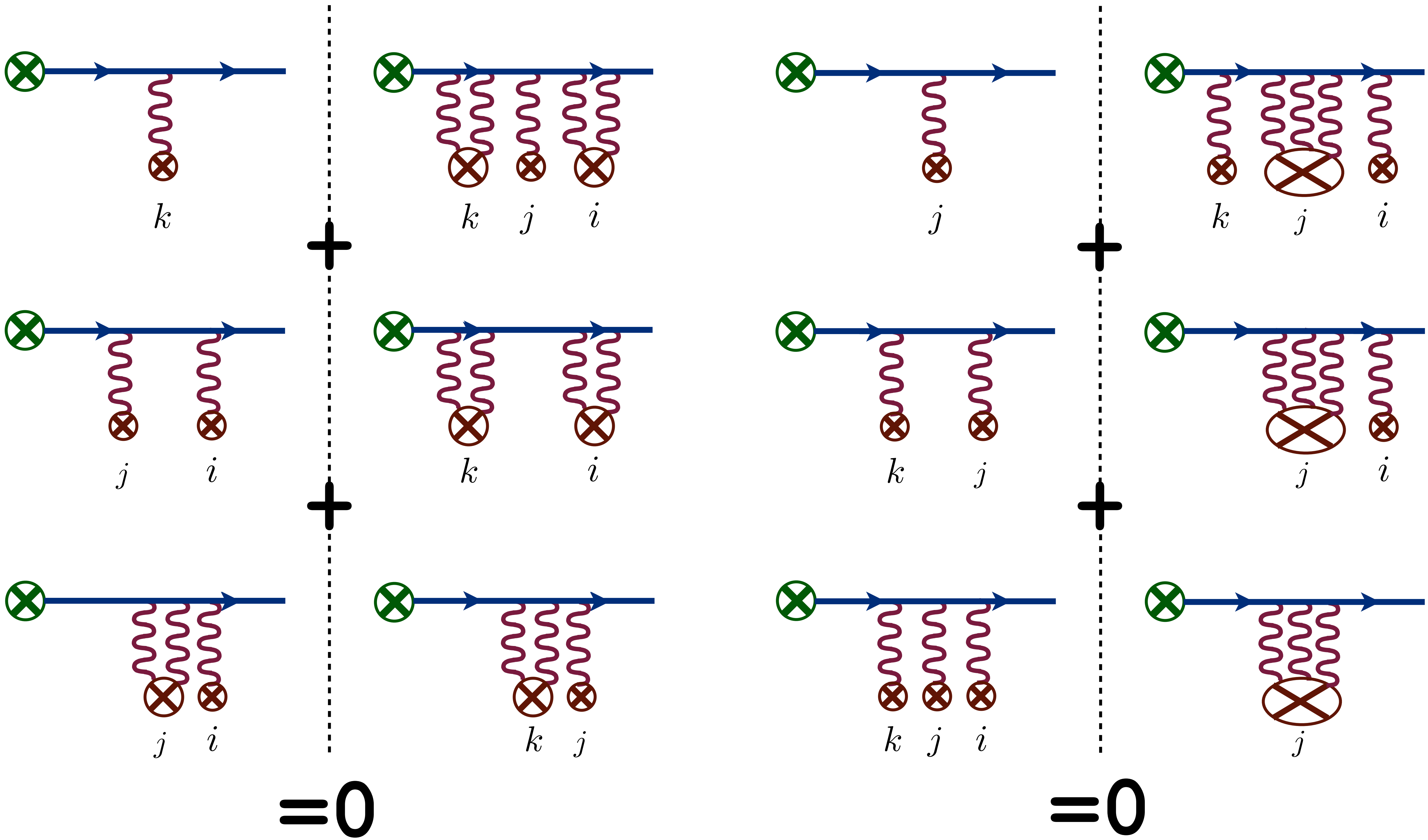
QED medium effects

3rd order in opacity



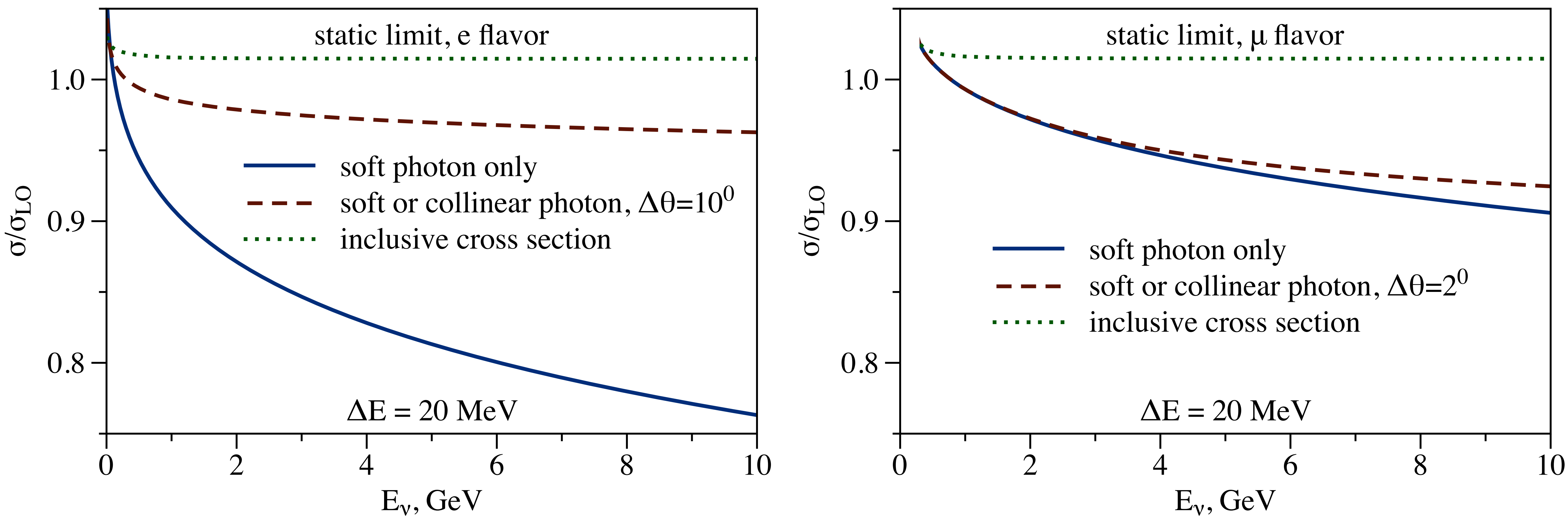
QED medium effects

3rd order in opacity



Static nucleon limit

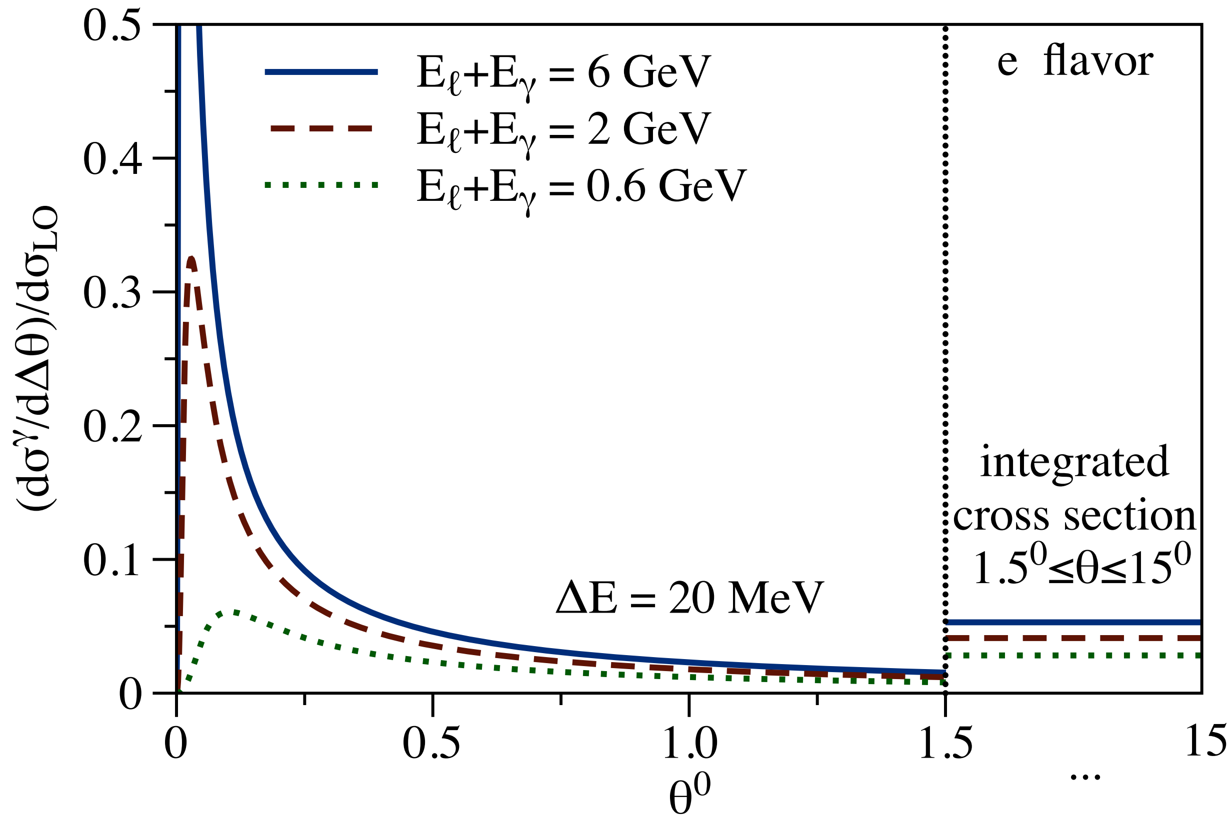
- formal limit of infinitely heavy nucleus $m_\ell \ll E_\ell \ll M$
- provides correct soft and collinear logarithms
- soft-photon energy < 20 MeV, jet size: 10° for electron and 2° for muon

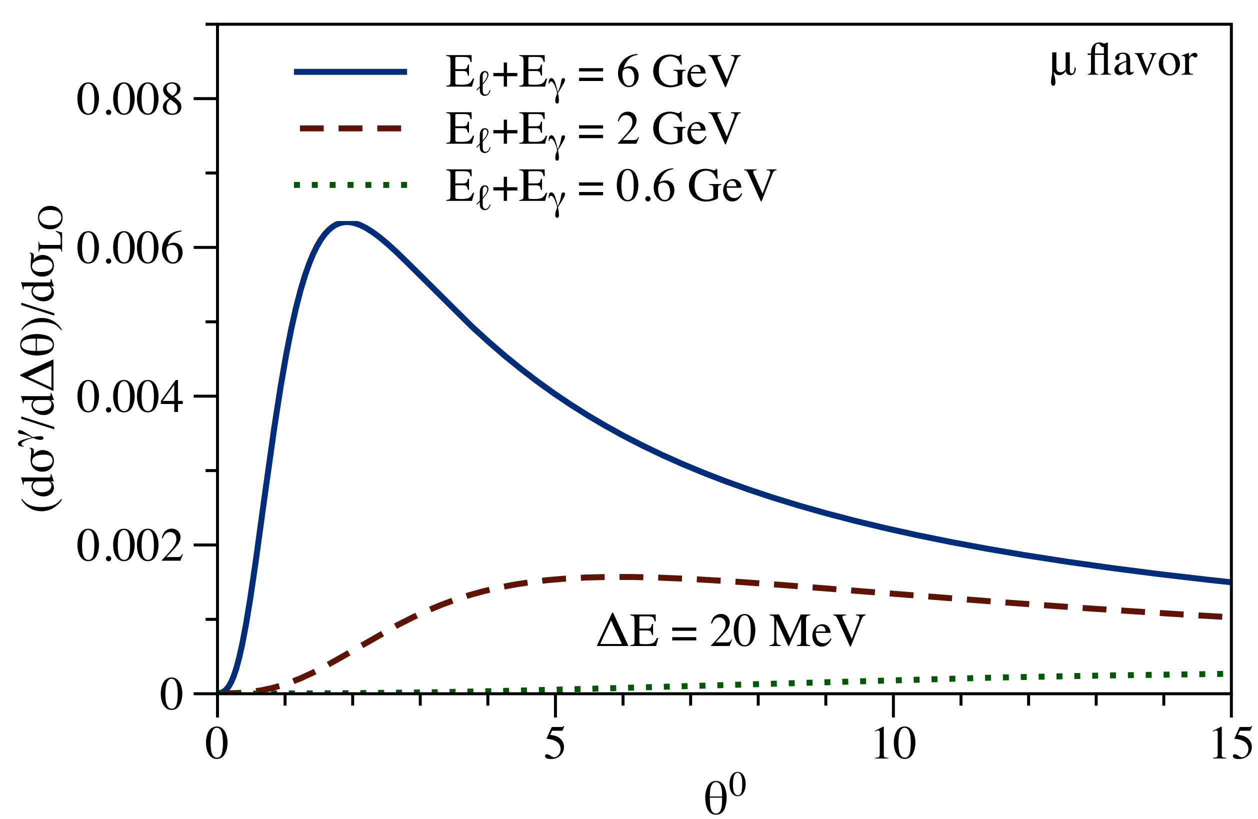


- flavor-dependent effect, same for $\nu_\ell n \to \ell^- p$ vs $\bar{\nu}_\ell p \to \ell^+ n$
- collinear observable: cancellation of virtual vs real logs
- inclusive observables (+γ): few % level, flavor independent

Electron vs muon jets

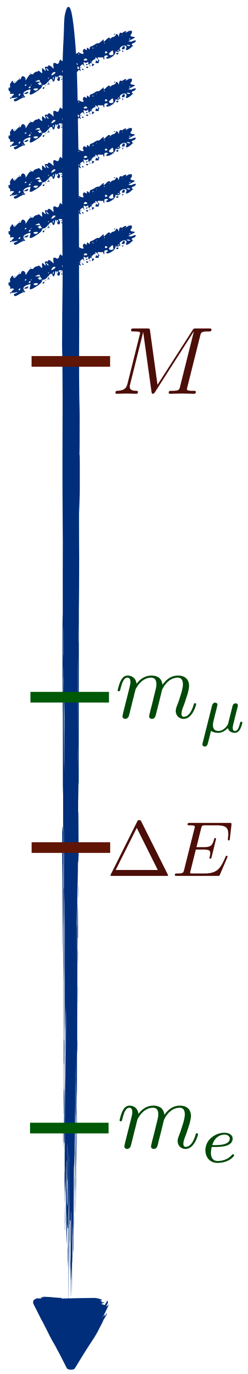
- factorization for radiation of collinear photons
- cone angle is defined to lepton direction
- photons of energy > 20 MeV, fixed energy in the cone





- flavor-dependent effect, same for $\nu_\ell n \to \ell^- p$ vs $\bar{\nu}_\ell p \to \ell^+ n$
- forward-peaked radiation for electron flavor
- negligible radiation for muons with shifted peak position

Factorization approach



- cross section is given by factorization formula

$$\mathrm{d}\sigma \sim S\left(\frac{\Delta E}{\mu}\right) J\left(\frac{m_\ell}{\mu}\right) H\left(\frac{M}{\mu}\right)$$

- determine hard function at hard scale by matching experiment or hadronic model to the theory with heavy nucleon

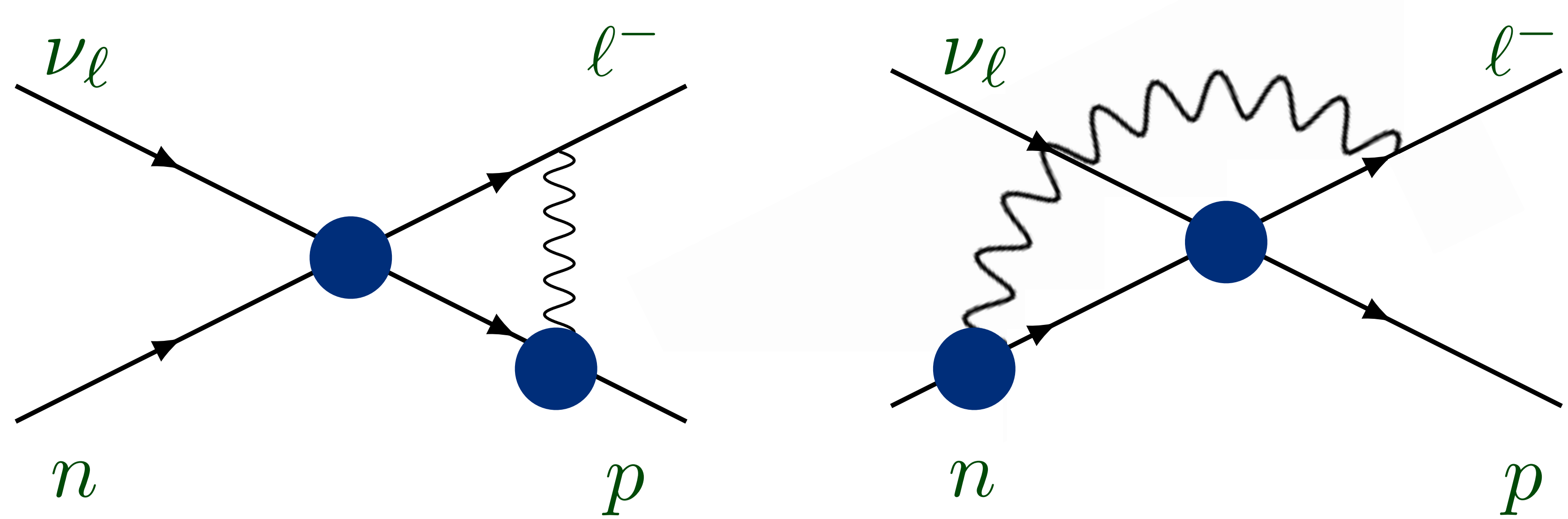
- soft and collinear functions are evaluated perturbatively

SCET power expansion parameter

$$\lambda \sim \frac{m_\mu^2}{E_\nu^2} \sim (\Delta\theta)^2 \sim \frac{\Delta E}{E_\nu}$$

$\ln \lambda$ enhancements

Hadronic model at GeV scale



- exchange of photon between the charged lepton and nucleons
- assume onshell form for each interaction with dipole form factors

discussed for neutrino-nucleon scattering: Graczyk (2013)

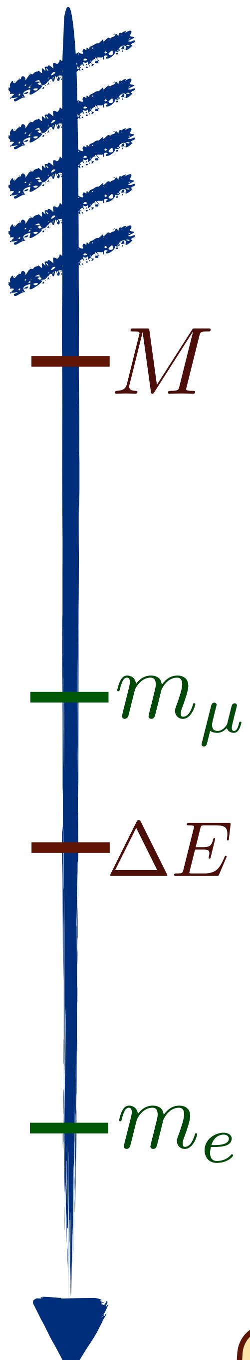
- add self energy for charged particles
- reproduce soft and collinear regions of SCET

- best determination of hard function

Factorization approach

- cross section is given by factorization formula

$$d\sigma \sim S\left(\frac{\Delta E}{\mu}\right) J\left(\frac{m_\ell}{\mu}\right) H\left(\frac{M}{\mu}\right)$$



- determine hard function at hard scale by matching experiment or <u>hadronic model</u> to the theory with heavy nucleon

- RGE evolution of the hard function to scales $\Delta E, m_\ell$

- soft and collinear functions are evaluated perturbatively

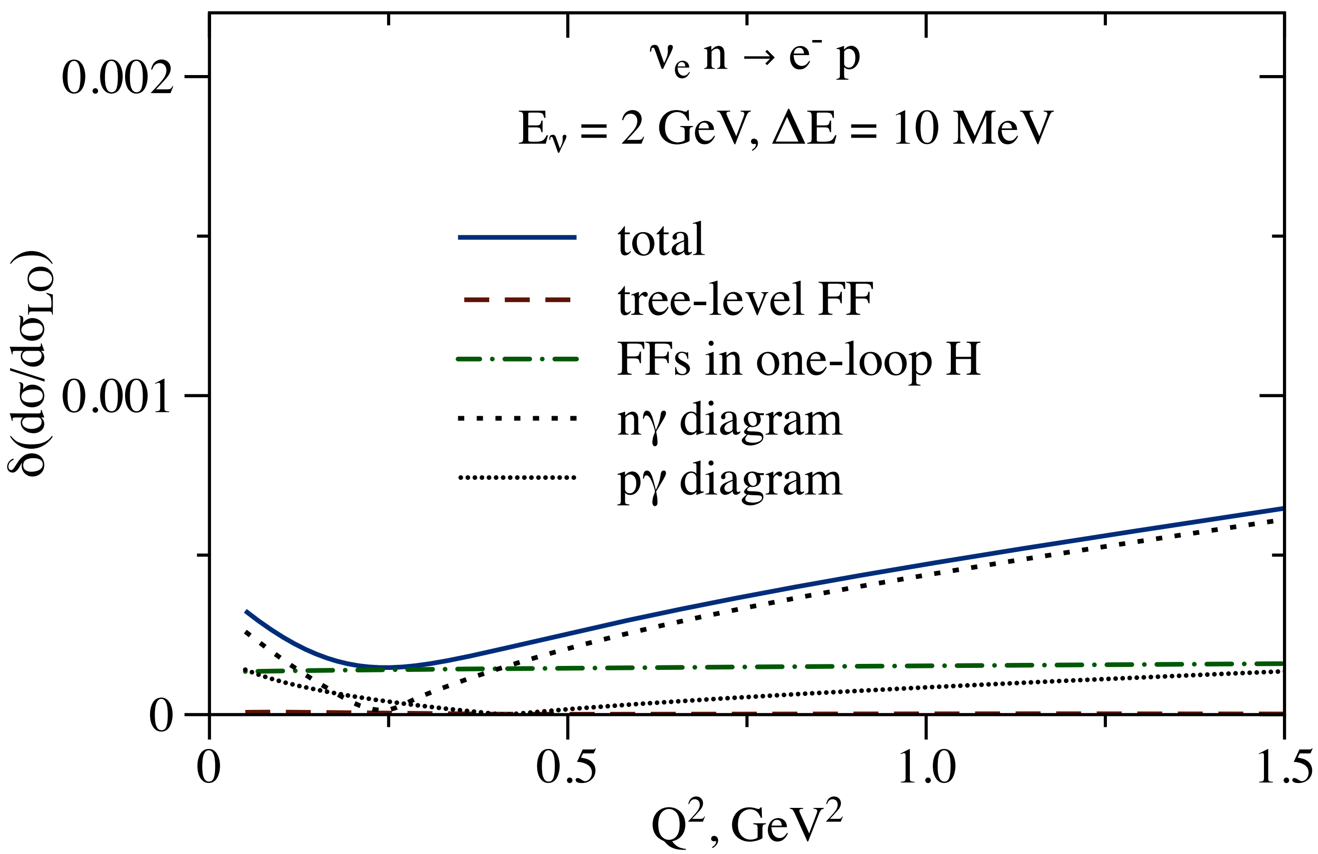
- calculate cross section at low energies accounting for all large logs

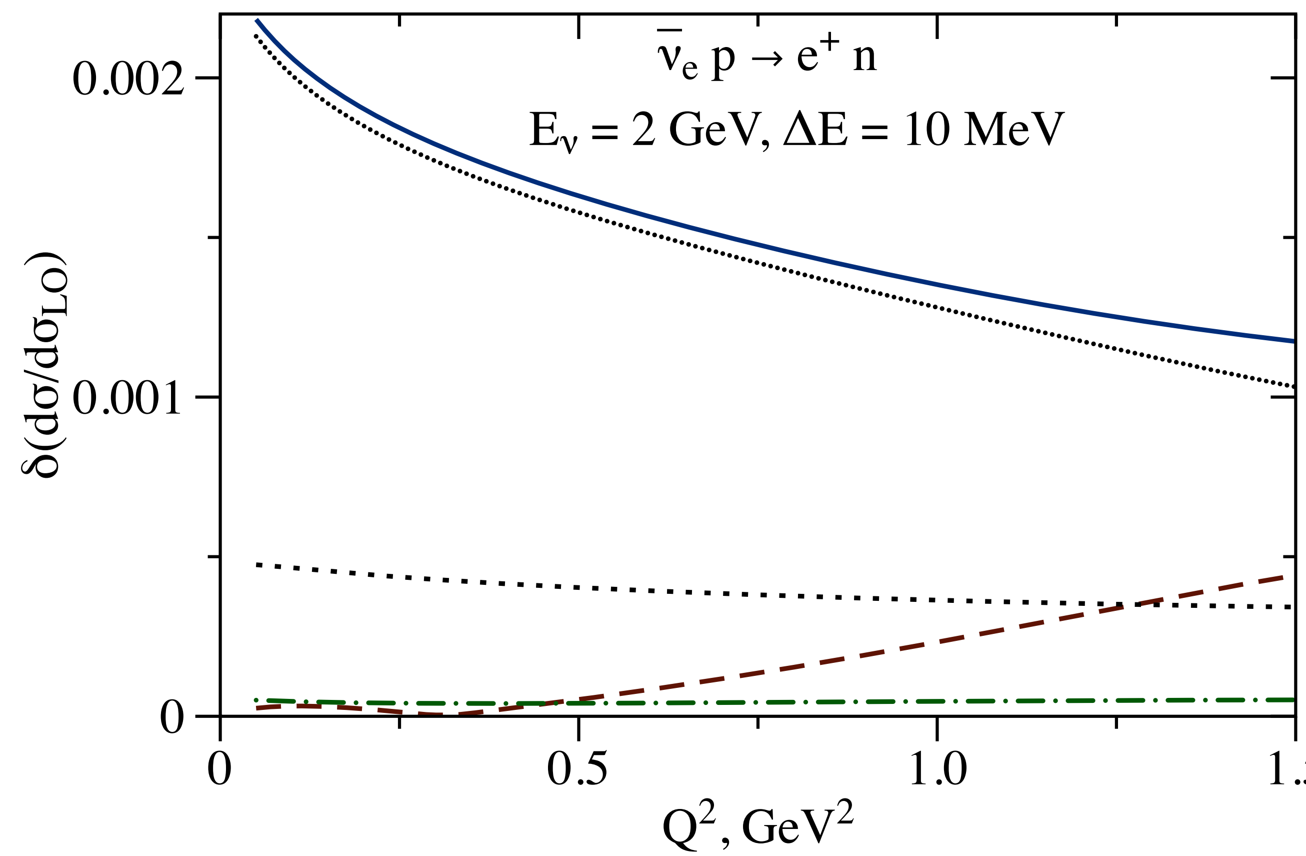
ep scattering with soft radiation only: Richard J. Hill (2016)

> - soft and collinear functions determined analytically
> - hard function describes physics at GeV energies

Error budget

- uncertainties from hard function





- nucleon form factors

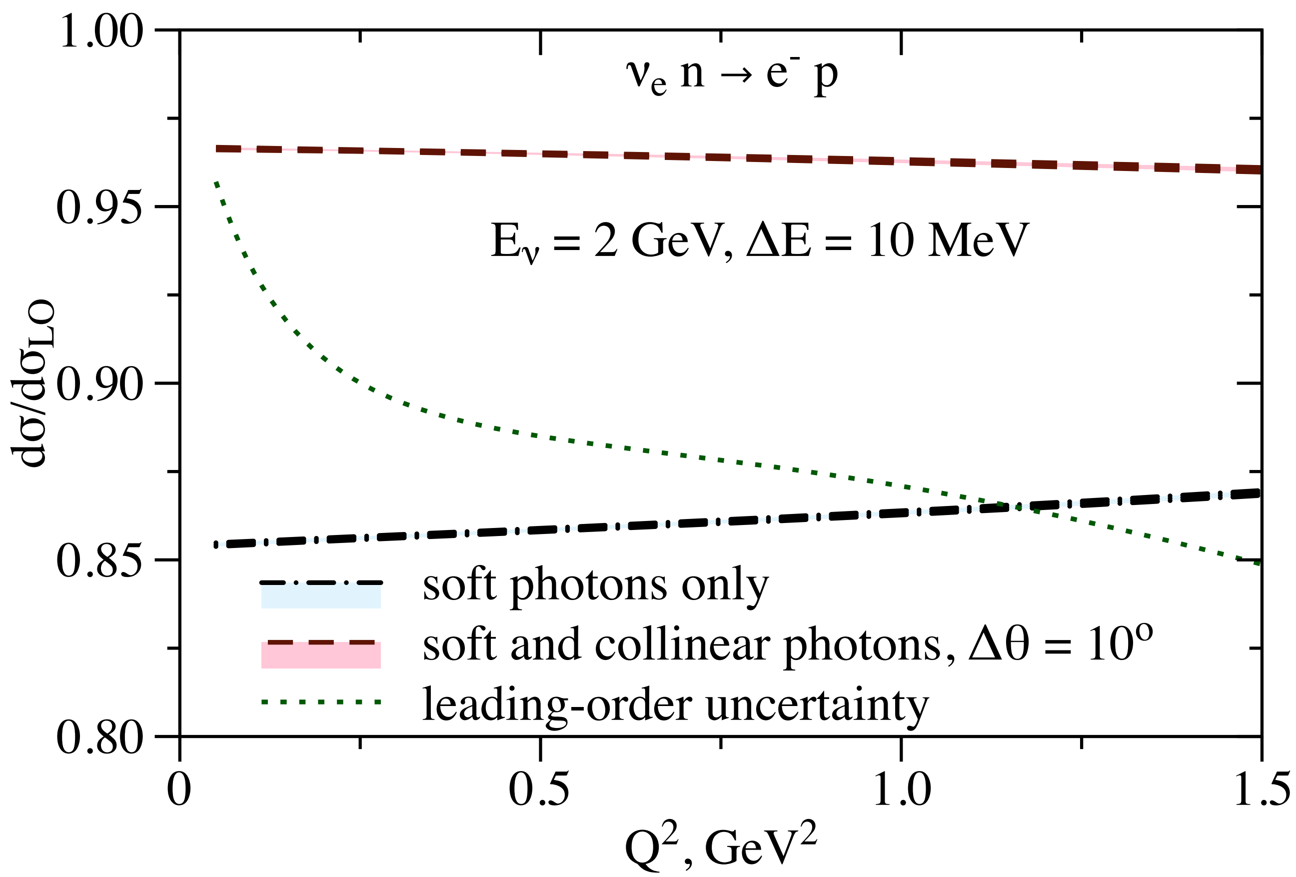
Meyer, Betancourt, Gran and Hill (2016)
Kaushik Borah, Gabriel Lee, Richard J. Hill and O.T. (2020)

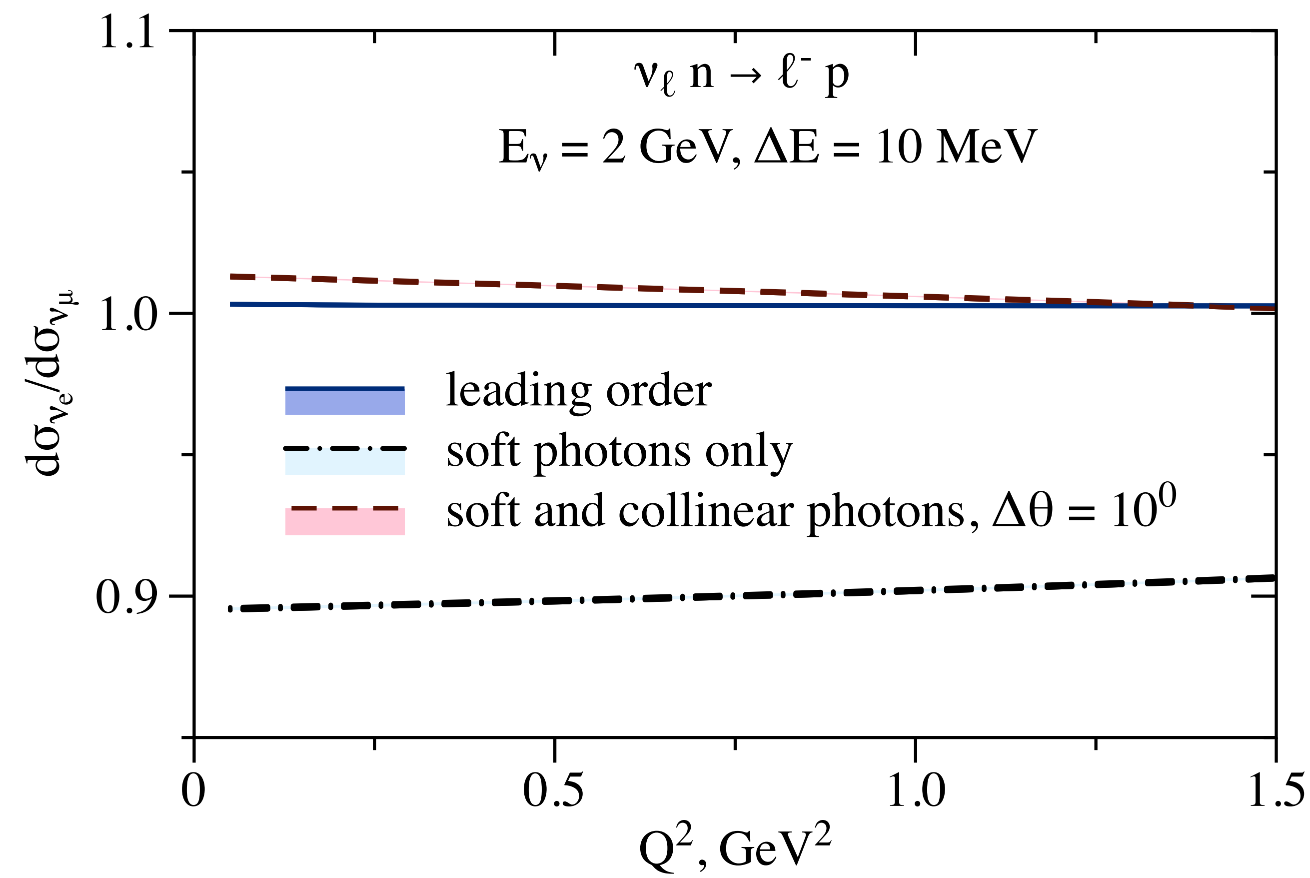
- add perturbative uncertainty by variation of scale

- uncertainty of permille level for the ratio to LO result

Exclusive observables

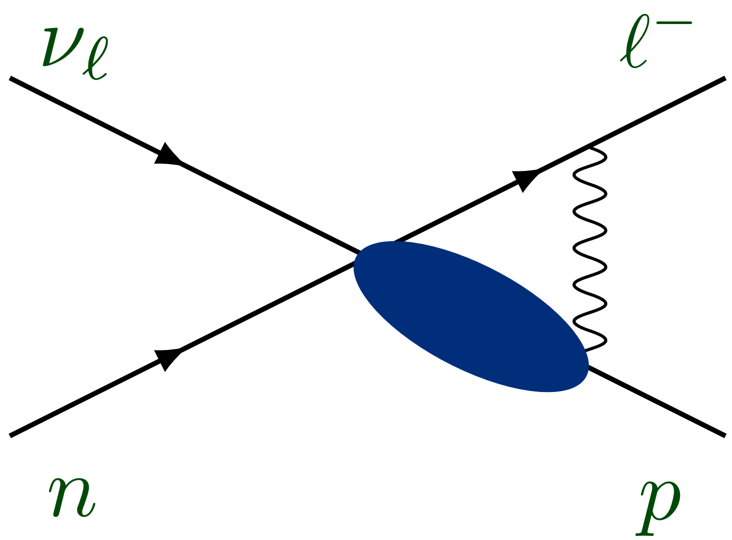
- cancellation of uncertainties from hard function for e/μ and ratio to LO

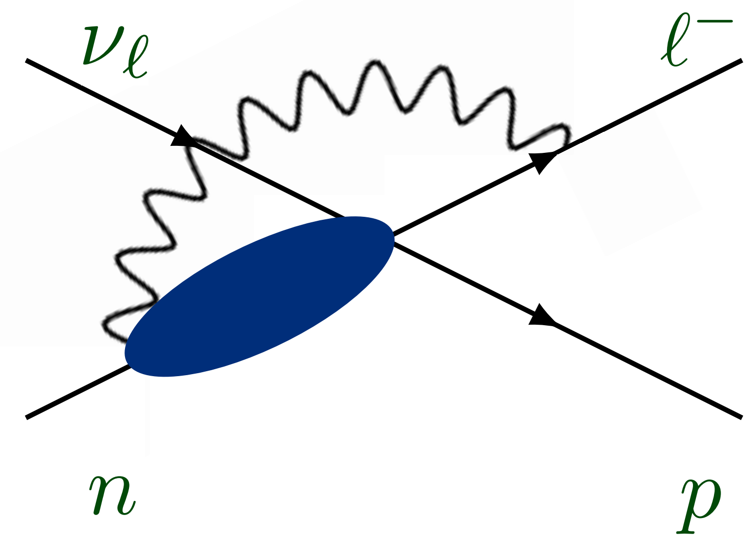




- ratios: cancellation of uncertainty from hard function

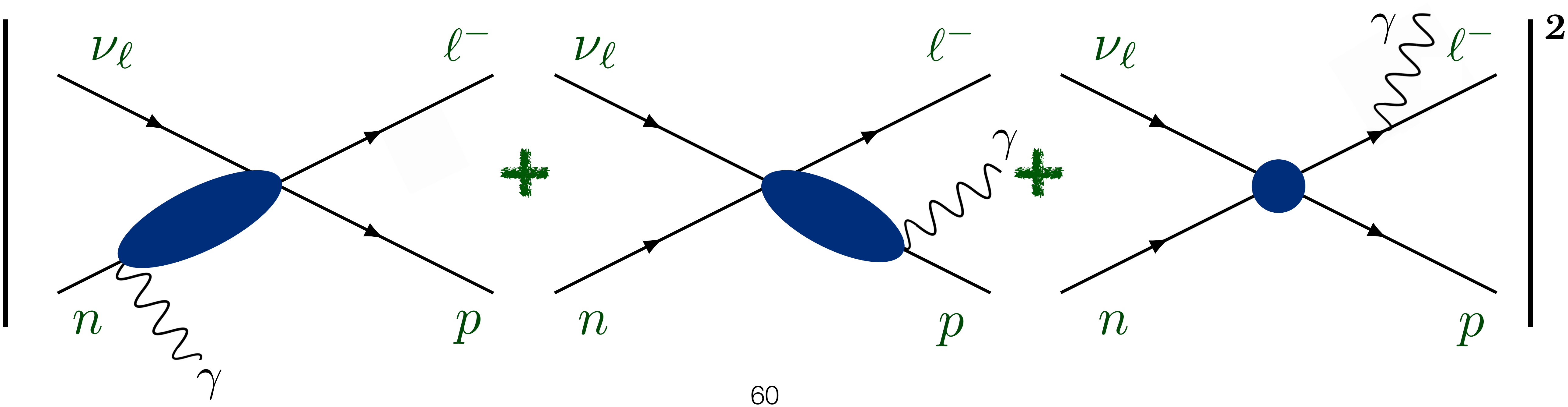
Inclusive observables





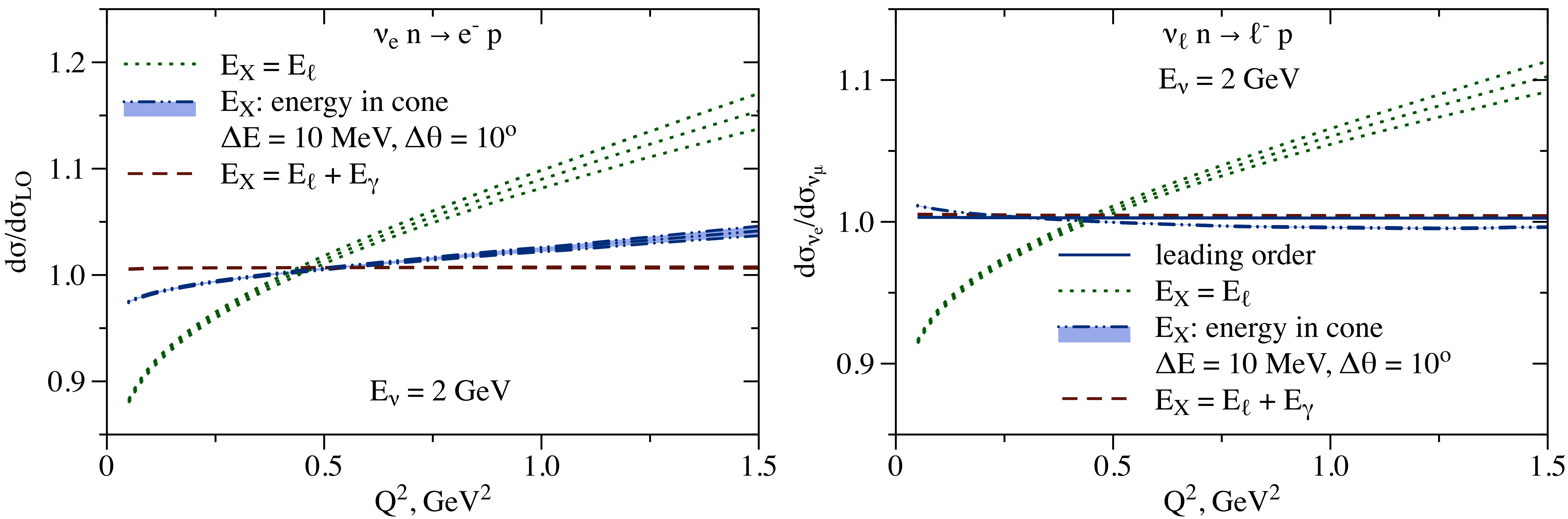
- the same gauge-invariant model for the real radiation

- arbitrary hard photons are part of the observable



Inclusive observables

- kinematics $Q^2 = 2M\,(E_\nu - E_X)$ is reconstructed with 3 different E_X



- dependence on reconstruction of kinematics and cuts
- predict σ_{ν_e} from σ_{ν_μ} measurements with neutrino beam ✓

Electron/muon ratio

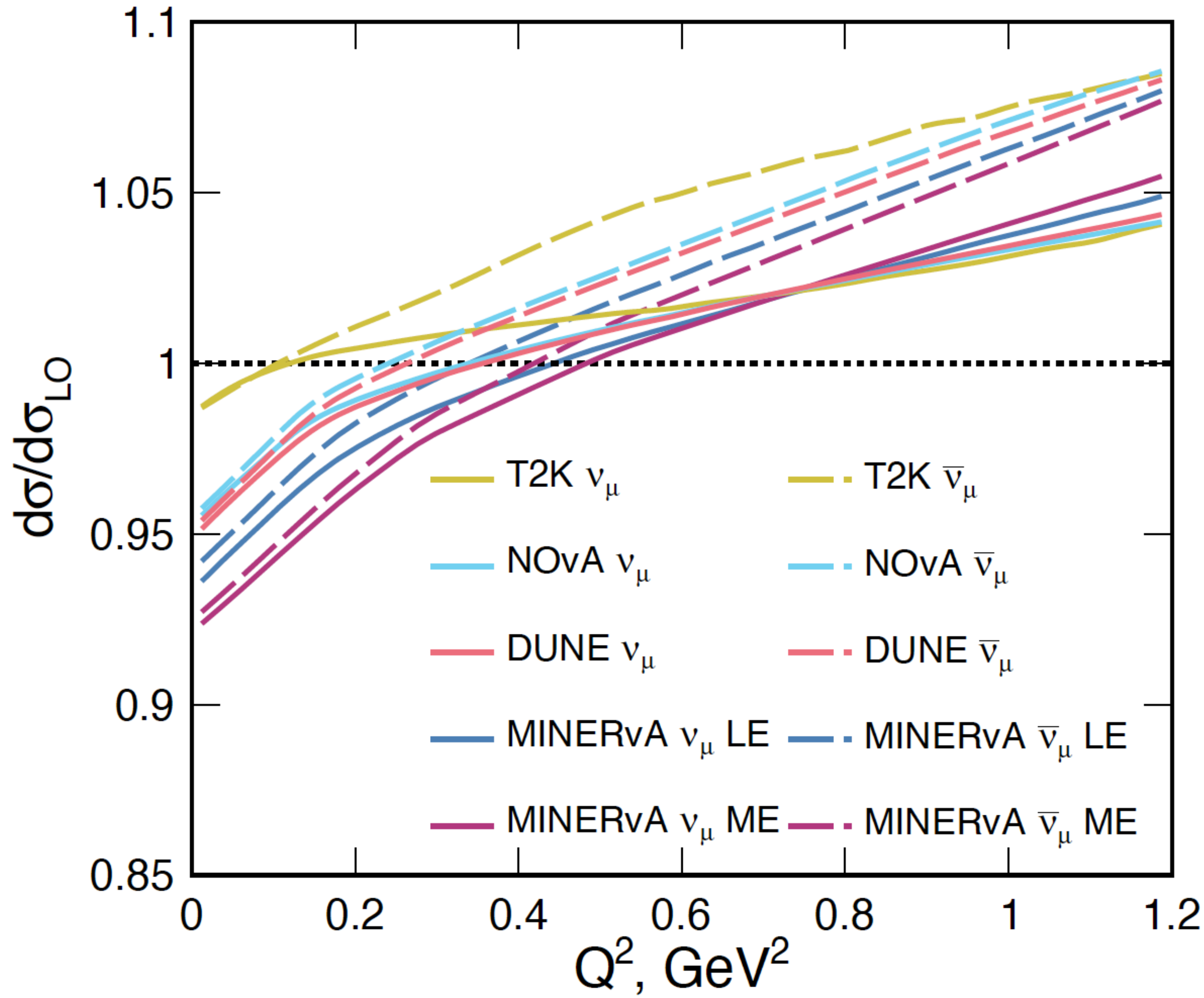
	E_ν, GeV		$\left(\frac{\sigma_e}{\sigma_\mu} - 1\right)_{\rm LO}$, %	$\frac{\sigma_e}{\sigma_\mu} - 1$, %
T2K/HyperK	0.6	ν	2.47 ± 0.06	$2.84 \pm 0.06 \pm 0.37$
		$\bar{\nu}$	2.04 ± 0.08	$1.84 \pm 0.08 \pm 0.20$
NOvA/DUNE	2.0	ν	0.322 ± 0.006	$0.54 \pm 0.01 \pm 0.22$
		$\bar{\nu}$	0.394 ± 0.003	$0.20 \pm 0.01 \pm 0.19$

TABLE II: Inclusive electron-to-muon cross-section ratios for neutrinos and antineutrinos without kinematic cuts. Uncertainties at leading order are from vector and axial nucleon form factors. For the final result, we include an additional hadronic uncertainty from the one-loop correction to the first uncertainty, and provide a second uncertainty as the magnitude of the radiative correction.

$$\frac{\sigma\left(m_\ell \to 0\right)}{\sigma\left(m_\ell = 0\right)} \approx 1 + A m_\ell^2 + \alpha B m_\ell^2 \ln m_\ell$$

- inclusive cross sections and flavor ratios determined by KLN
- nuclear effects: suppressed by expansion parameters squared

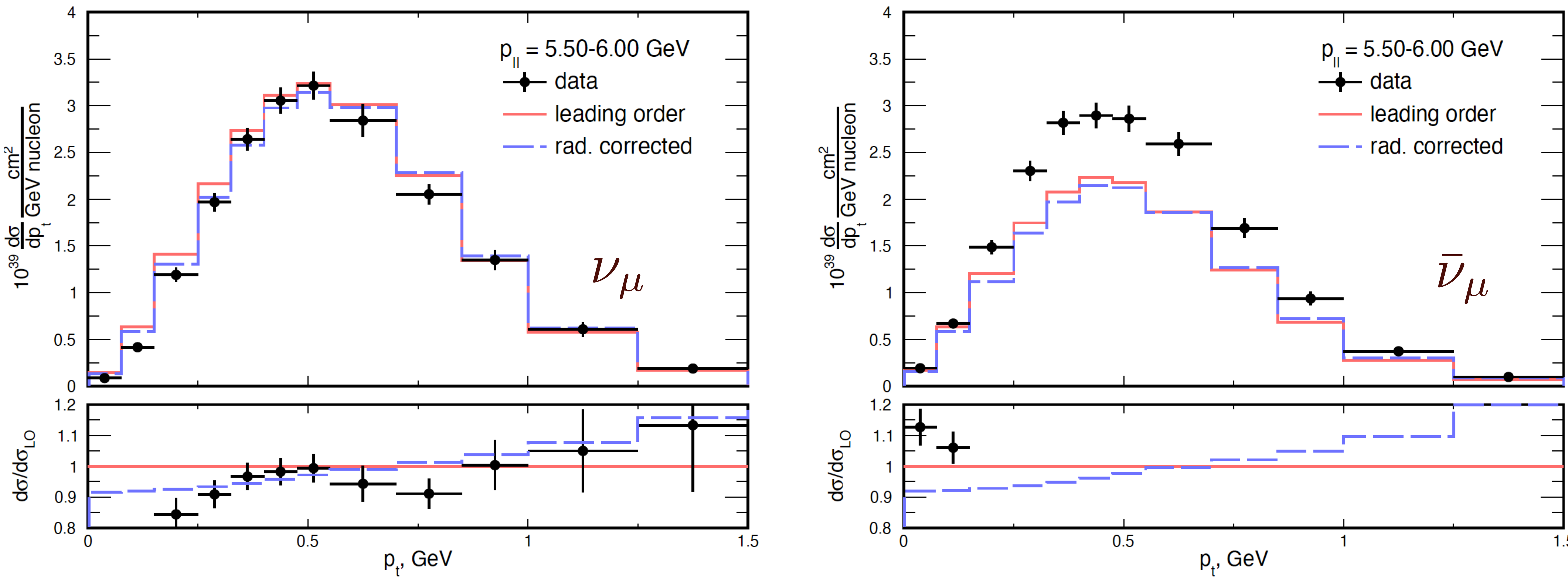
Comparison to data



- lepton energy spectra with lepton kinematics
- NEUT generator + flux averaging

Comparison to data

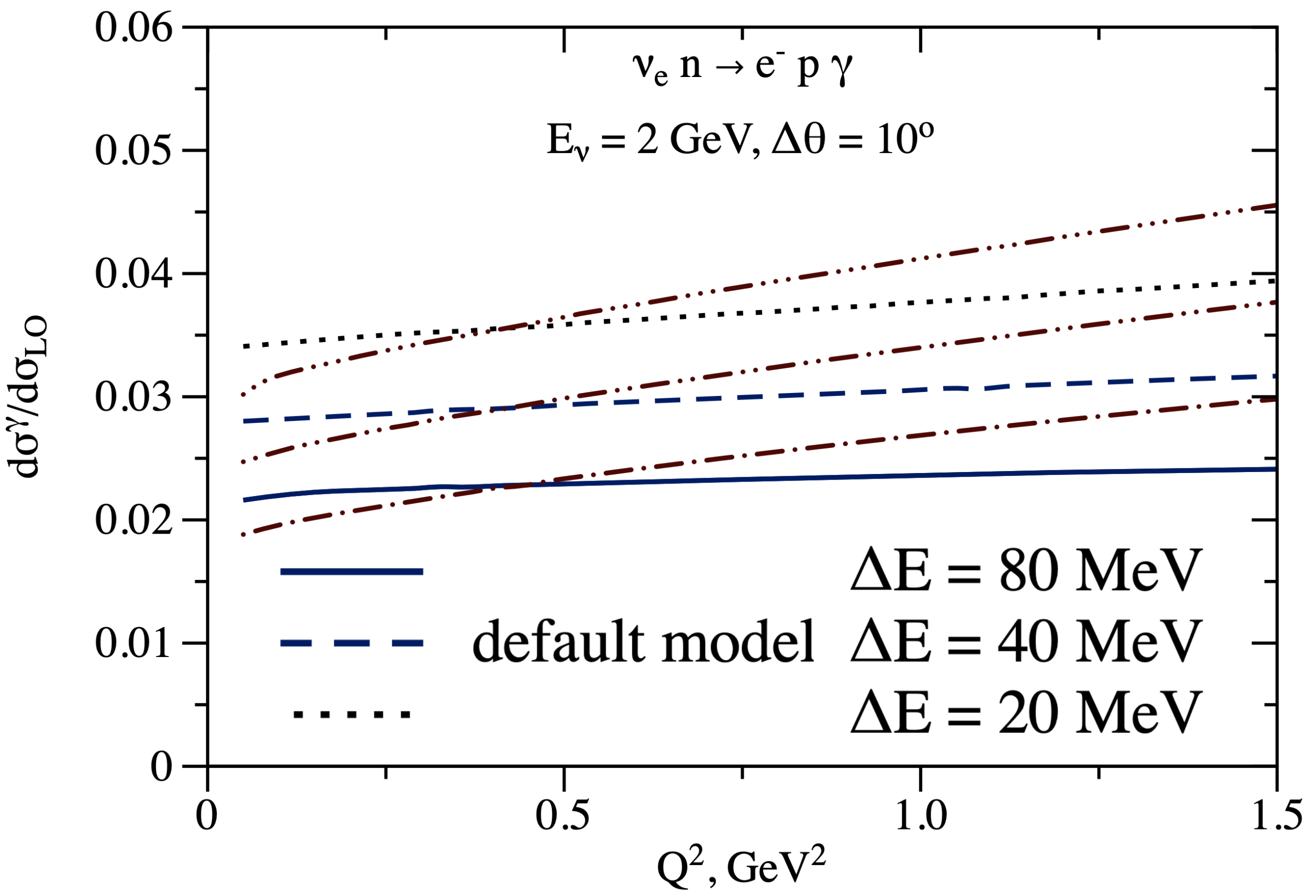
- medium-energy flux data from MINERvA@FERMILAB

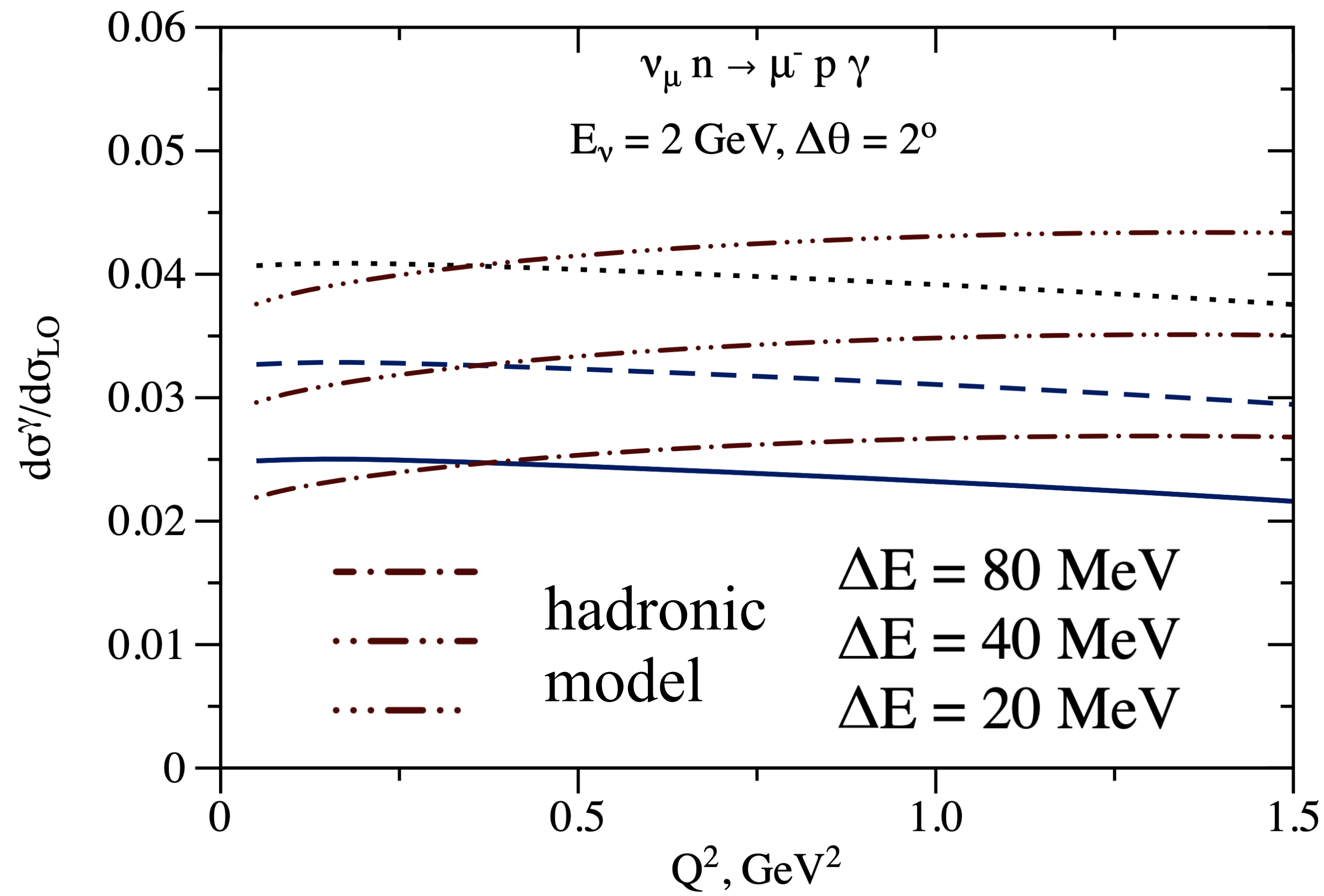


- electron flavor: measurements are uncertain
- muon flavor: comparable to experimental precision

Radiation of hard photons

- model-dependent description for radiation of hard photons





"Blunden calculation"

- photon energies are above 20, 40, and 80 MeV: default vs "SIFF"

"hadronic model"

- %-level radiation of non-collinear hard photons
- 10^{-4} flavor misidentification rate for NOvA&T2K kinematics