The best way to probe CP violation in the lepton sector is with long-baseline accelerator neutrino experiments in the appearance mode: the appearance of ν_e in predominantly ν_μ beams. Here we show that it is possible to discover CP violation with disappearance experiments only, by combining JUNO for electron neutrinos and DUNE or Hyper-Kamiokande for muon neutrinos. While the maximum sensitivity to discover CP is quite modest (1.6σ with 6 years of JUNO and 13 years of DUNE), some values of δ may be disfavored by $> 3\sigma$ depending on the true value of δ.

Neutrino oscillation experiments will be entering the precision era in the next decade with the advent of high statistics experiments like DUNE, HK, and JUNO. Correctly estimating the confidence intervals from data for the oscillation parameters requires very large Monte Carlo data sets involving calculating the oscillation probabilities in matter many, many times. In this paper, we leverage past work to present a new, fast, precise technique for calculating neutrino oscillation probabilities in matter optimized for long-baseline neutrino oscillations in the Earth's crust including both accelerator and reactor experiments. For ease of use by theorists and experimentalists, we provide fast c++ and fortran codes.

CP-Violation with Neutrino Disappearance and NuFast

Peter B. Denton

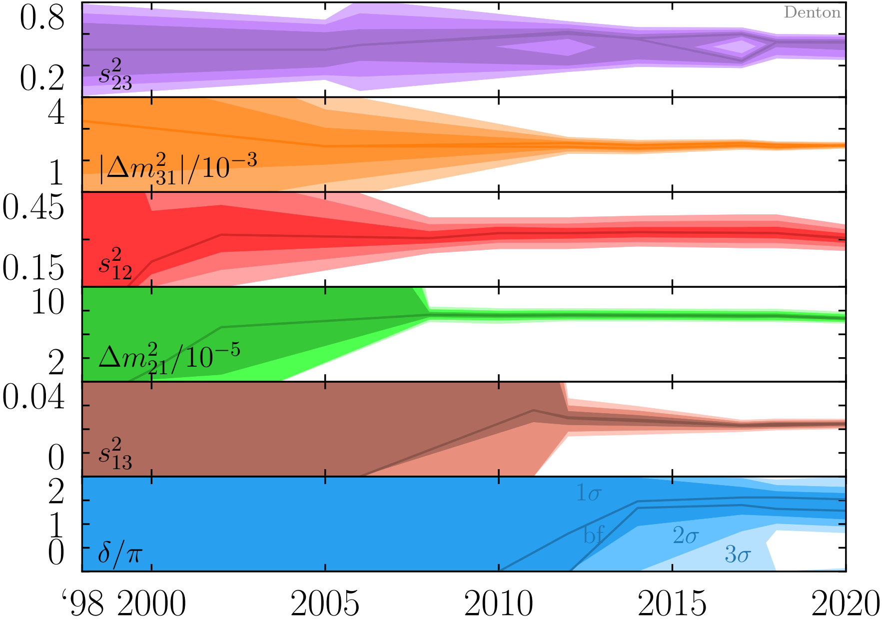
CETUP*

July 8, 2024

2309.03262 PRL (in press)
2405.02400 with S. Parke

Brookhaven National Laboratory

Brookhaven NDI
Neutrino Discovery Initiative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$|\Delta m^2_{31}|/10^{-3}$
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s^2_{12}
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$\Delta m^2_{21}/10^{-5}$
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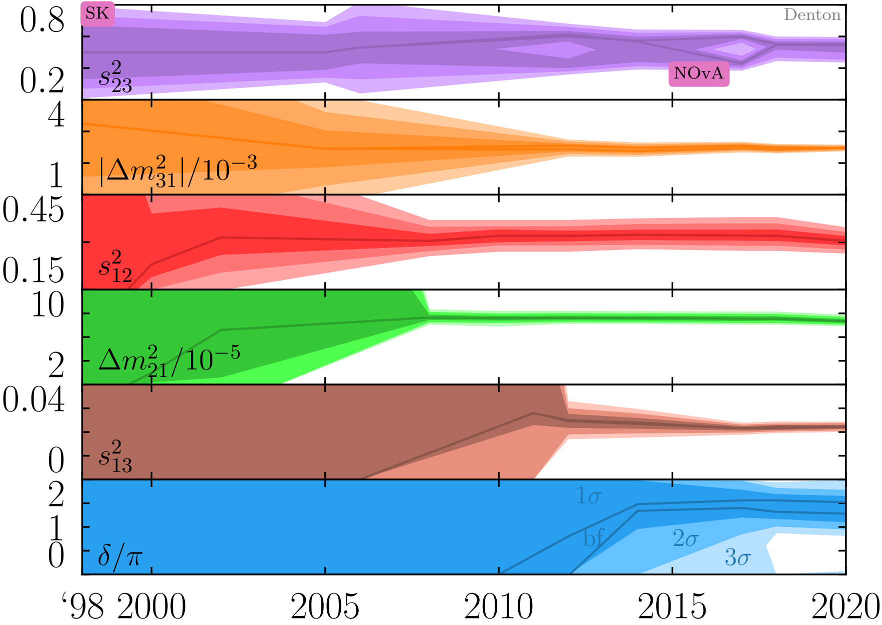

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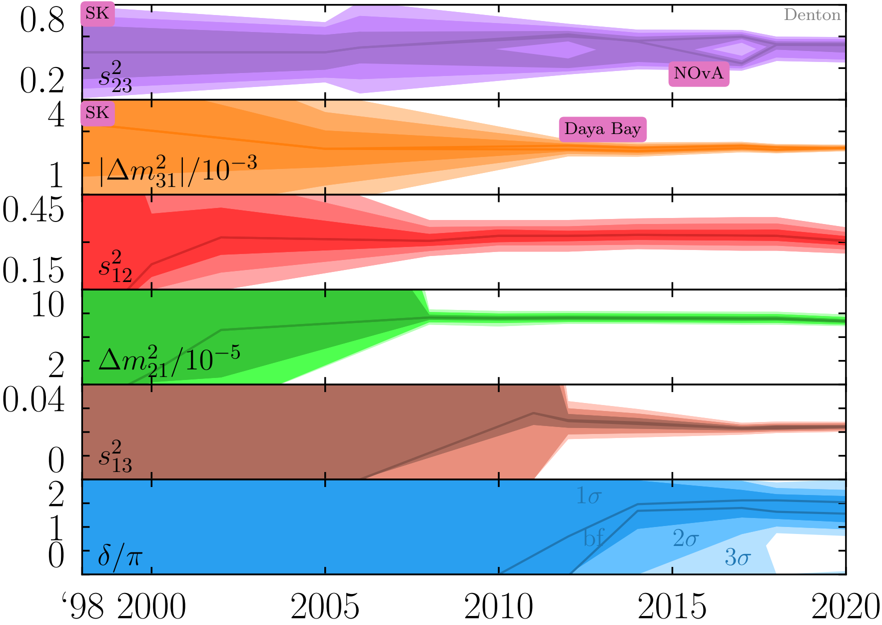

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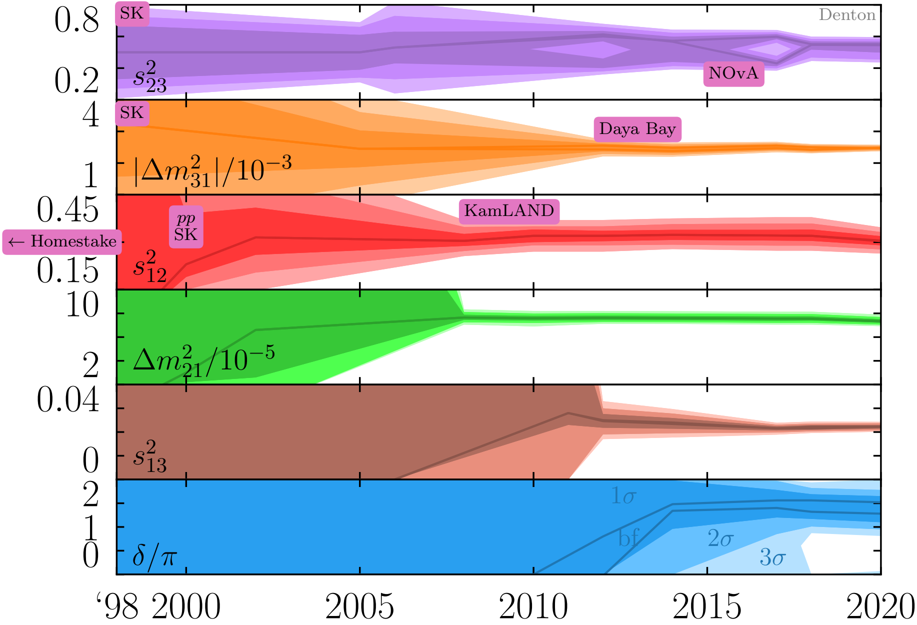

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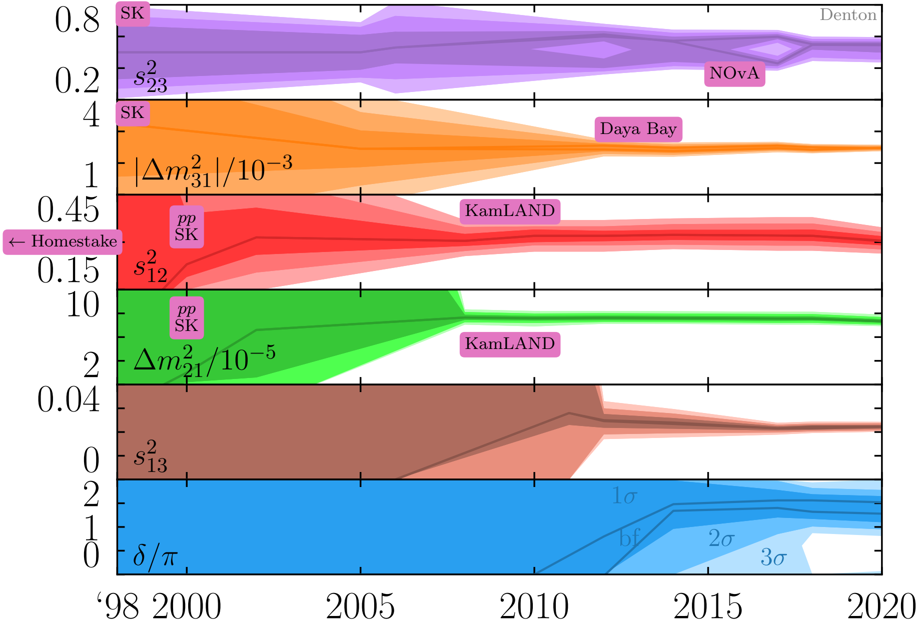
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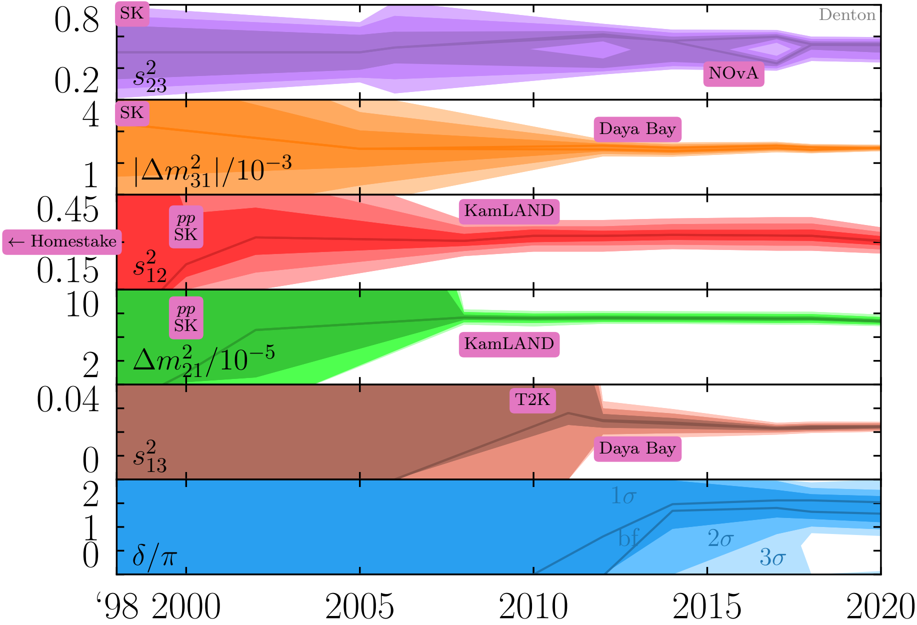
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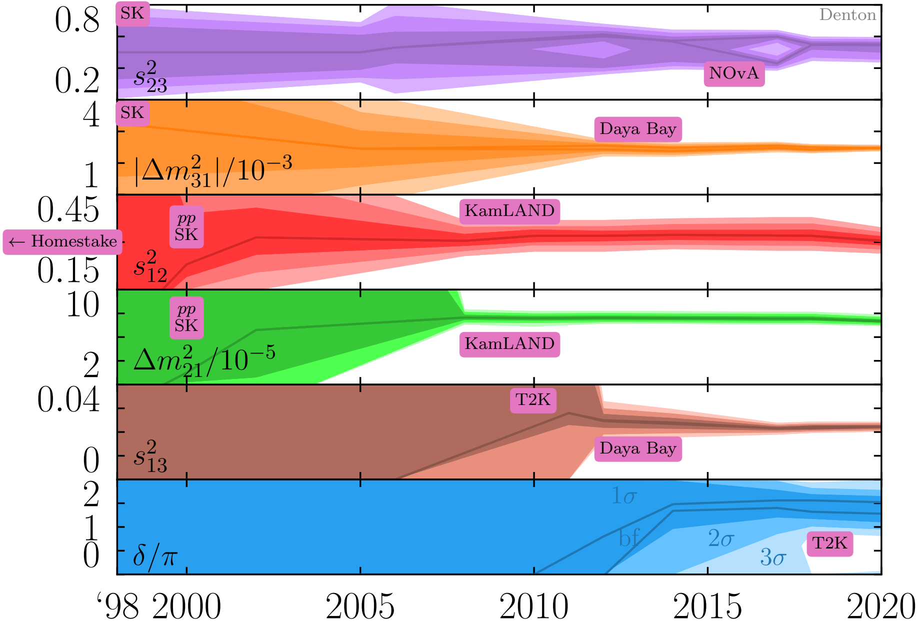

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Four known unknown in particle physics: all neutrinos

Atmospheric mass ordering

θ_{23} octant

Complex phase

Absolute mass scale

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Atmospheric mass ordering

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Outline

1. Why CPV is interesting
2. Other non-standard probes of CPV
3. Relationship between appearance, disappearance, CP, T, CPT
4. Three ways to see why there is CPV information in disappearance
   4.1 Parameter counting
   4.2 Direct analytic calculation
   4.3 Numerical test
5. Role of the matter effect
6. Sensitivities
7. `NuFast`

Why is CPV interesting?

δ and CP violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

δ and CP violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

1. Strong interaction: no observed EDM $\Rightarrow$ CP (nearly) **conserved**

$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. 1509.04411

2. Quark mass matrix: non-zero but **small** CP violation

$$\frac{|J_{\rm CKM}|}{J_{\max}} = 3\times10^{-4}$$

CKMfitter 1501.05013

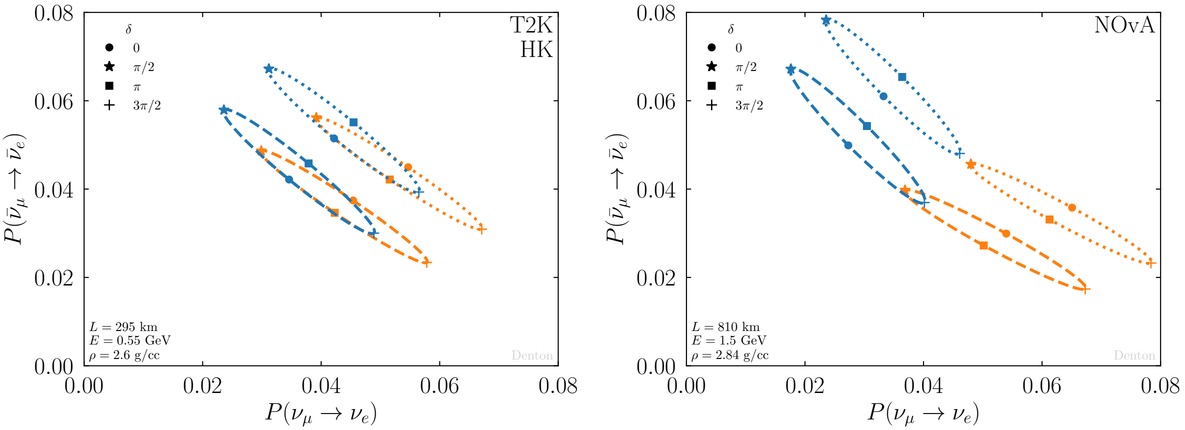
3. Lepton mass matrix: **?**

$$\frac{|J_{\rm PMNS}|}{J_{\max}} < 0.34$$

PBD, J. Gehrlein, R. Pestes 2008.01110

$J_{\max} = \frac{1}{6\sqrt{3}} \approx 0.096$

δ: what is it really?



δ: what is it not?

$$\delta \not\Rightarrow \text{Baryogenesis/Leptogenesis}$$

The amount of leptogenesis is a function of:

1. δ
2. the heavy mass scale
3. α, β (Majorana phases)
4. CP phases in the RH neutrinos
5. …

C. Hagedorn, et al. 1711.02866

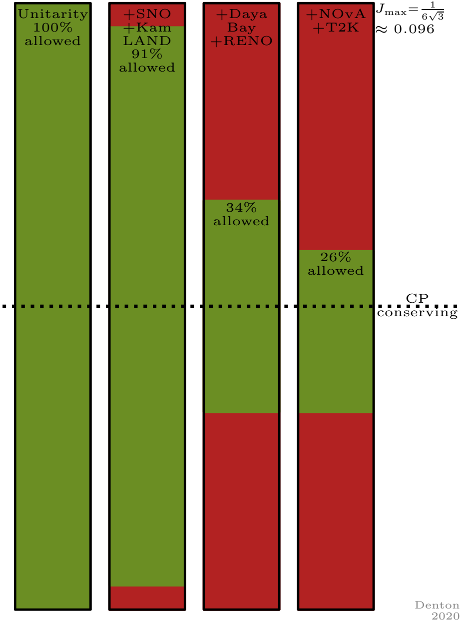
K. Moffat, et al. 1809.08251

$$\text{Measuring } \delta = 0, \pi \quad \not\Rightarrow \quad \text{no leptogenesis}$$
$$\text{Measuring } \delta \neq 0, \pi \quad \not\Rightarrow \quad \text{leptogenesis}$$

δ, J: current status

Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1}\frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin\delta| = 1$ allowed



When δ and when J?

If the goal is **CP violation** the Jarlskog invariant should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12}, θ_{13}, θ_{23}, and J, I can't determine the sign of $\cos\delta$ which is physical

e.g. $P(\nu_\mu \to \nu_\mu)$ depends on $\cos\delta$

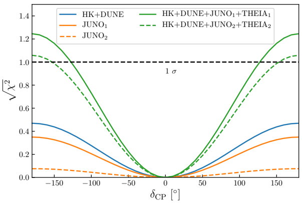
Other non-standard CPV probes

1. Some information in solar due to loops in elastic scattering

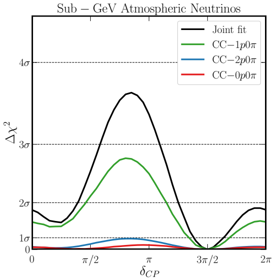
V. Brdar, X-J. Xu 2306.03160
K. Kelly, et al. 2407.03174

2. Sub-GeV atmospherics

K. Kelly, et al. 1904.02751
See also e.g. A. Suliga, J. Beacom 2306.11090



Solar (no systematics)



Atmospherics at DUNE

Appearance, disappearance, and CP

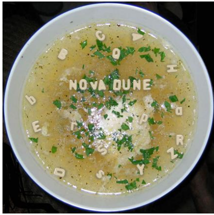
Appearance vs. Disappearance

Oscillation experiments can do
appearance or disappearance experiments:

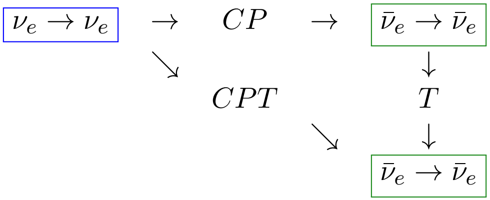
Disappearance
K2K, MINOS, T2K, NOνA
KamLAND, Daya Bay, RENO, Double CHOOZ
(Sort of) SNO, Borexino, SK-solar
JUNO, DUNE, HK

Appearance
T2K, NOνA
OPERA
Atm ν_τ hints @ SK & IceCube
DUNE, HK

Neither appearance nor disappearance
SK-atm, IceCube



CP, T: Disappearance

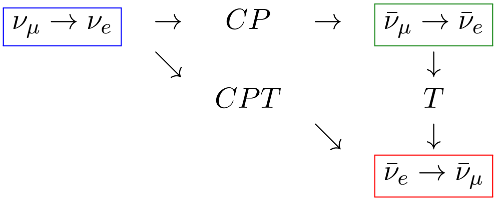


Disappearance measurements are even eigenstates of CP

$$CP[P(\nu_e \to \nu_e)] = P(\bar{\nu}_e \to \bar{\nu}_e) \overset{CPT}{=} P(\nu_e \to \nu_e)$$

Assume that CPT is a good symmetry

CP, T: Appearance



Appearance measurements are not eigenstates of CP

Appearance and Disappearance, CP even and CP odd terms

Disappearance:

$$\begin{aligned}P(\nu_\alpha \to \nu_\alpha) = 1 &- 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} \\ &- 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} \\ &- 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32} \\ = P_{\alpha\alpha}^{CP+}\end{aligned}$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L/4E$$

Appearance and Disappearance, CP even and CP odd terms

Disappearance:

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Appearance:

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = &- 4\Re[U_{\alpha 1}U^*_{\beta 1}U^*_{\alpha 2}U_{\beta 2}] \sin^2\Delta_{21} \\ &- 4\Re[U_{\alpha 1}U^*_{\beta 1}U^*_{\alpha 3}U_{\beta 3}] \sin^2\Delta_{31} \\ &- 4\Re[U_{\alpha 3}U^*_{\beta 3}U^*_{\alpha 2}U_{\beta 2}] \sin^2\Delta_{32} \\ &\pm 8J_{CP} \sin\Delta_{21}\sin\Delta_{31}\sin\Delta_{32} \\ =&\ P^{CP+}_{\alpha\beta} + P^{CP-}_{\alpha\beta} \end{aligned}$$

$\Delta_{ij} \equiv \Delta m^2_{ij} L/4E$

Sign depends on α, β

Conventional Wisdom

1. Appearance is sensitive to CPV [True]

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2. Disappearance has no CPV sensitivity [False]

Conventional Wisdom

1. Appearance is sensitive to CPV [True]
2. Disappearance has no CPV sensitivity [False]
3. Any δ dependence in disappearance is in ν_μ not ν_e [Confusing/False]

$$\begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}-s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12}-s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Correct Statements

- Appearance is the best way to measure δ and CPV

  ...given known oscillation parameters, systematics, and realistic experiments

  - Probes mostly $\sin\delta$ not $\cos\delta$
  - Don't need both ν and $\bar{\nu}$ (but systematics)
- **Disappearance can measure δ**
  - CPV can be discovered with only disappearance measurements
  - Probes mostly $\cos\delta$ not $\sin\delta$
  - Requires measurements of two flavors
  - "Works through unitarity" (as do nearly all oscillation measurements)

Parameter Counting

1. Four parameters in the PMNS matrix

Majorana phases are irrelevant in oscillations

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2. Disappearance experiments of one flavor can measure up to three amplitudes
   - Electron neutrino row:
     - KamLAND measured one
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$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4\sum_{i>j} C_{ij}^\alpha \sin^2\Delta_{ij}$$

$$C_{ij}^\alpha = |U_{\alpha i}|^2 |U_{\alpha j}|^2$$

$$|U_{\alpha i}| = \left(\frac{C_{ij}^\alpha C_{ik}^\alpha}{C_{jk}^\alpha}\right)^{1/4}$$

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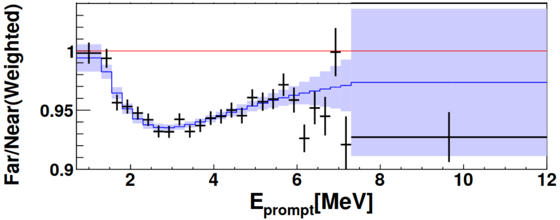

Daya Bay 1809.02261

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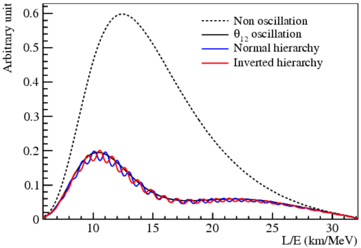
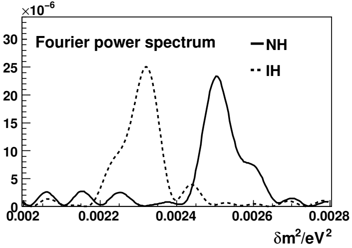


JUNO 1507.05613



L. Zhan, et al. 0807.3203

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4. Given good measurements of the ν_e and ν_μ disappearance, 4 independent parameters will be measured
   - Any row can be “simple” (e.g. $c_{12}c_{13}$, $s_{12}c_{13}$, ...) $\Rightarrow$ no one row is ever enough
   - That is, CPV is physical and cannot depend on parameterization

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   - That is, CPV is physical and cannot depend on parameterization
5. This is sufficient to constrain $\cos\delta$ and three mixing angles
6. If we determine $\cos\delta \neq \pm 1 \quad \Rightarrow \quad$ CP is violated!

Direct Analytic Calculation

Disappearance experiments measure various $|U_{\alpha i}|^2$ terms

Suppose 4 are measured: $|U_{e2}|^2$, $|U_{e3}|^2$, $|U_{\mu 2}|^2$, $|U_{\mu 3}|^2$

Actually this gives all 9 magnitudes by unitarity

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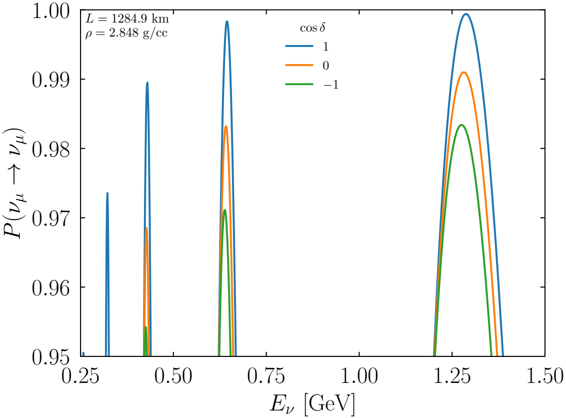
Actually this gives all 9 magnitudes by unitarity

$$J_{CP}^2 = |U_{e2}|^2|U_{\mu 2}|^2|U_{e3}|^2|U_{\mu 3}|^2 - \frac{1}{4}\left(1 - |U_{e2}|^2 - |U_{\mu 2}|^2 - |U_{e3}|^2 - |U_{\mu 3}|^2 + |U_{e2}|^2|U_{\mu 3}|^2 + |U_{e3}|^2|U_{\mu 2}|^2\right)^2$$

Can show that if any one $|U_{\alpha i}|^2 = 0 \Rightarrow J = 0$

Disappearance can tell us if CP is violated,
but not if nature prefers ν's or $\bar{\nu}$'s

Where is $|U_{\mu 2}|^2$?



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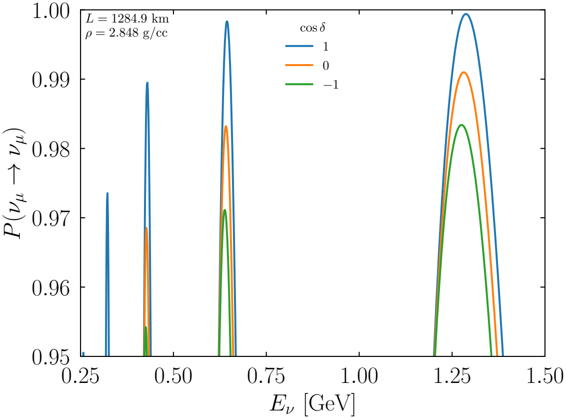

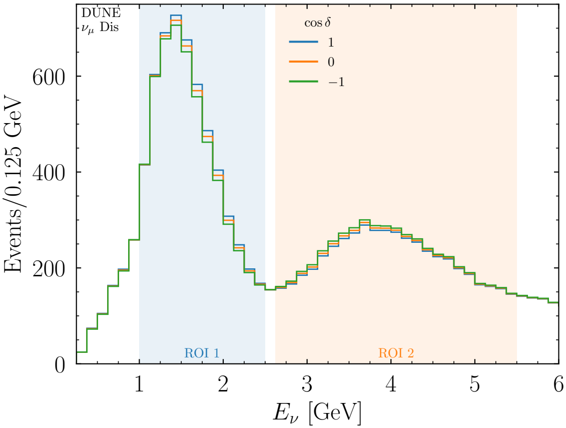

6.5 yrs ν_μ rates

$\cos\delta$	ROI 1	ROI 2
1	5506	5038
0	5418	5115
-1	5334	5193

Approximate size of $|U_{\mu2}|^2$ signal: (21) sector

- There is no δ information in $|U_{\mu1}|^2+|U_{\mu2}|^2$ (sum of Δ_{31} and Δ_{32} terms)

Approximate size of $|U_{\mu 2}|^2$ signal: (21) sector

- There is no δ information in $|U_{\mu 1}|^2 + |U_{\mu 2}|^2$ (sum of Δ_{31} and Δ_{32} terms)
- There is δ information in $|U_{\mu 1}|^2 |U_{\mu 2}|^2$

Approximate size of $|U_{\mu2}|^2$ signal: (21) sector

- There is no δ information in $|U_{\mu1}|^2 + |U_{\mu2}|^2$ (sum of Δ_{31} and Δ_{32} terms)
- There is δ information in $|U_{\mu1}|^2|U_{\mu2}|^2$
- This comes from the Δm^2_{21} term

DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

Approximate size of $|U_{\mu2}|^2$ signal: (21) sector

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- This comes from the Δm^2_{21} term

DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

- This term is

$$\approx -4c_{23}^2\left(s_{12}^2c_{12}^2+s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

$$\approx -2\quad(0.21\quad+\qquad 0.03\cos\delta)\left(\frac{\pi}{33}\right)^2$$

$\Delta m^2_{21}/|\Delta m^2_{31}|\approx 33$

Approximate size of $|U_{\mu 2}|^2$ signal: (21) sector

- There is no δ information in $|U_{\mu 1}|^2+|U_{\mu 2}|^2$ (sum of Δ_{31} and Δ_{32} terms)
- There is δ information in $|U_{\mu 1}|^2|U_{\mu 2}|^2$
- This comes from the Δm^2_{21} term

DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

- This term is

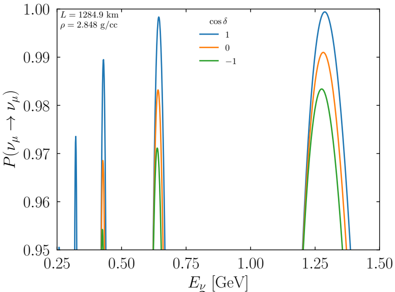
$$\approx -4c_{23}^2\left(s_{12}^2c_{12}^2+s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

$$\approx -2\quad(0.21\quad+\quad 0.03\cos\delta)\left(\frac{\pi}{33}\right)^2$$

$\Delta m^2_{21}/|\Delta m^2_{31}|\approx 33$

- So the probability is large for $\cos\delta=-1$?
- So the effect is $\sim -0.0005\cos\delta$?

Approximate size of $|U_{\mu 2}|^2$ signal: (21) sector



DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

- There is no δ information in $|U_{\mu 1}|^2 + |U_{\mu 2}|^2$
- There is δ information in $|U_{\mu 1}|^2 |U_{\mu 2}|^2$
- This comes from the Δm^2_{21} term

- This term is

$$\approx -4c_{23}^2\left(s_{12}^2 c_{12}^2 + s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

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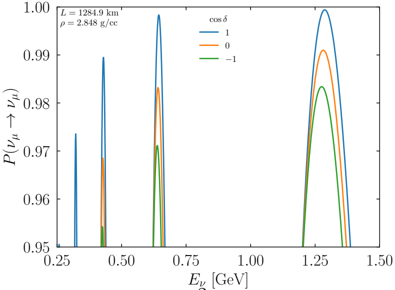

DUNE and HK can measure Δm^2_{21} somewhat
PBD, J. Gehrlein 2302.08513

- There is no δ information in $|U_{\mu 1}|^2 + |U_{\mu 2}|^2$
- There is δ information in $|U_{\mu 1}|^2 |U_{\mu 2}|^2$
- This comes from the Δm^2_{21} term
- This term is

$$\approx -4c_{23}^2 \left(s_{12}^2 c_{12}^2 + s_{23} c_{23} s_{13} \sin 2\theta_{12} \cos 2\theta_{12} \cos\delta\right) \sin^2 \Delta_{21}$$

$$\approx -2 \quad (0.21 \quad + \quad 0.03 \cos\delta) \left(\frac{\pi}{33}\right)^2$$

$\Delta m^2_{21} / |\Delta m^2_{31}| \approx 33$

- So the probability is large for $\cos\delta = -1$? — Sign is wrong
- So the effect is $\sim -0.0005 \cos\delta$? — Magnitude is ~ 16 too small

Matter effects matter: (21) sector

- Let's start at

$$\approx -4c_{23}^2\left(s_{12}^2c_{12}^2+s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

Matter effects matter: (21) sector

- Let's start again at

$$\approx -4c_{23}^2\left(s_{12}^2c_{12}^2 + s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

- Solar splitting modified by

$$\Delta m_{21}^2 \to \Delta m_{21}^2 \mathcal{S}_\odot$$

$$\mathcal{S}_\odot \approx \sqrt{(\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2)^2 + \sin^2 2\theta_{12}} \approx 3.4$$

at $E = 1.3$ GeV
PBD, S. Parke 1902.07185

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at $E = 1.3$ GeV

PBD, S. Parke 1902.07185

- Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \to \frac{\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

$a \propto \rho E$

Matter effects matter: (21) sector

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PBD, S. Parke 1902.07185

- Mixing angle is modified

$$\cos 2\theta_{12} = 0.37 \to \frac{\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

- So the sign is swapped

$a \propto \rho E$

$$\sin 2\theta_{12}\cos 2\theta_{12} = 0.35 \to -0.26$$

Matter effects matter: (21) sector

- Let's start again at

$$\approx -4c_{23}^2\left(s_{12}^2c_{12}^2+s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

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$$\sin 2\theta_{12}\cos 2\theta_{12} = 0.35 \to -0.26$$

- Also s_{13} increases in matter $\sim 15\%$: total effect is $0.004\cos\delta$

Matter effects matter: (21) sector

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$$\approx -4c_{23}^2\left(s_{12}^2c_{12}^2 + s_{23}c_{23}s_{13}\sin 2\theta_{12}\cos 2\theta_{12}\cos\delta\right)\sin^2\Delta_{21}$$

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at $E = 1.3$ GeV
PBD, S. Parke 1902.07185

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$$\cos 2\theta_{12} = 0.37 \to \frac{\cos 2\theta_{12} - c_{13}^2 a/\Delta m_{21}^2}{\mathcal{S}_\odot} \approx -0.96 < 0$$

- So the sign is swapped

$a \propto \rho E$

$$\sin 2\theta_{12}\cos 2\theta_{12} = 0.35 \to -0.26$$

- Also s_{13} increases in matter $\sim 15\%$: total effect is $0.004\cos\delta$
- This gets us **half** of the effect, and the correct sign

Matter effects matter: (32) sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at the maximum is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

Matter effects matter: (32) sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at the maximum is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at max

Matter effects matter: (32) sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at the maximum is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at max
- Leading order in s_{13}:

$$
\begin{aligned}
&\approx -4 s^2_{23}\left(c^2_{12} c^2_{23} - 2 s_{13} s_{12} c_{12} s_{23} c_{23} \cos\delta\right) \sin^2 \Delta_{32} \\
&\approx -2 \quad (0.0094 \qquad - 0.023 \cos\delta) 0.1 \qquad \text{(matter)}
\end{aligned}
$$

Matter effects matter: (32) sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at the maximum is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at max
- Leading order in s_{13}:

$$\approx -4 s^2_{23}(c^2_{12}c^2_{23} - 2 s_{13} s_{12} c_{12} s_{23} c_{23} \cos\delta) \sin^2 \Delta_{32}$$
$$\approx -2 \quad (0.0094 \qquad - 0.023 \cos\delta) 0.1 \qquad \text{(matter)}$$

- Adds in another $\approx 0.004 \cos\delta$ effect

Matter effects matter: (32) sector

- $\frac{\Delta m^2_{\mu\mu} L}{4E}$ in matter at the maximum is $\sim \pi$

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283
PBD, S. Parke 2401.10326

- The Δm^2_{32} component is a bit off π at max
- Leading order in s_{13}:

$$\begin{aligned} &\approx -4 s^2_{23} (c^2_{12} c^2_{23} - 2 s_{13} s_{12} c_{12} s_{23} c_{23} \cos\delta) \sin^2 \Delta_{32} \\ &\approx -2 \quad (0.0094 \qquad - 0.023 \cos\delta) 0.1 \qquad \text{(matter)} \end{aligned}$$

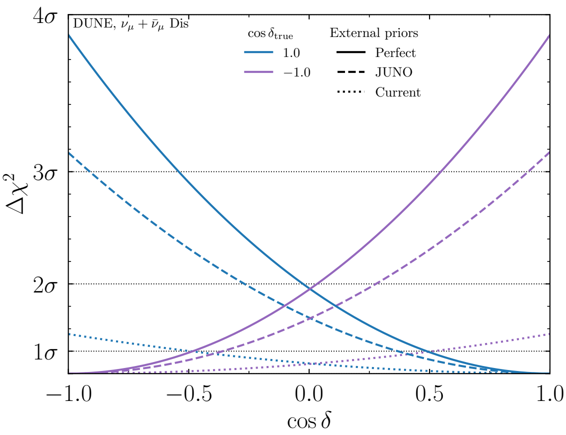
- Adds in another $\approx 0.004 \cos\delta$ effect
- Total is $\approx 0.008 \cos\delta$ which agrees with exact calculation

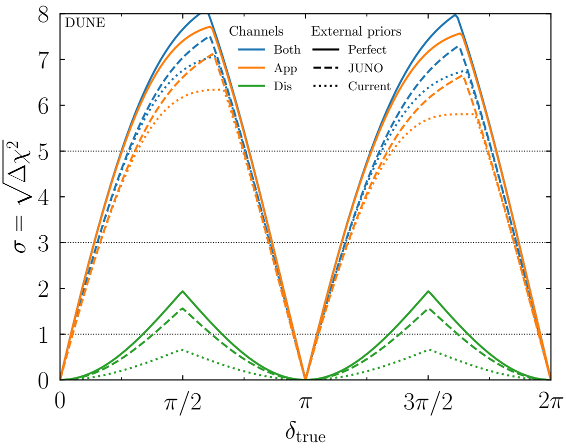
Numerical Studies

Inputs are *only*:

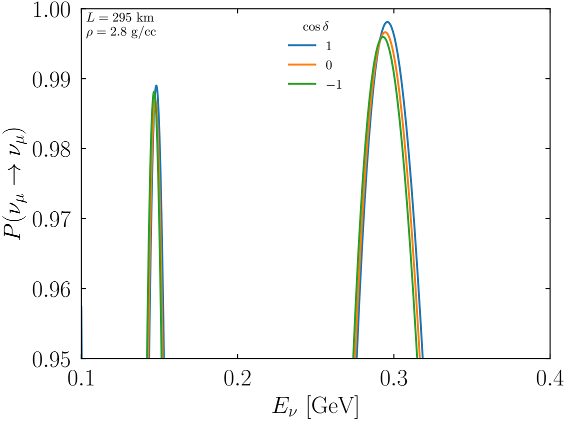
- Daya Bay data for θ_{13} 1809.02261
- KamLAND data for θ_{12} and Δm^2_{21} 1303.4667
- JUNO 6 yrs precision sensitivity on θ_{12}, Δm^2_{21}, Δm^2_{31} 2204.13249
- DUNE 6.5+6.5 yrs disappearance channels sensitivity only 2103.04797

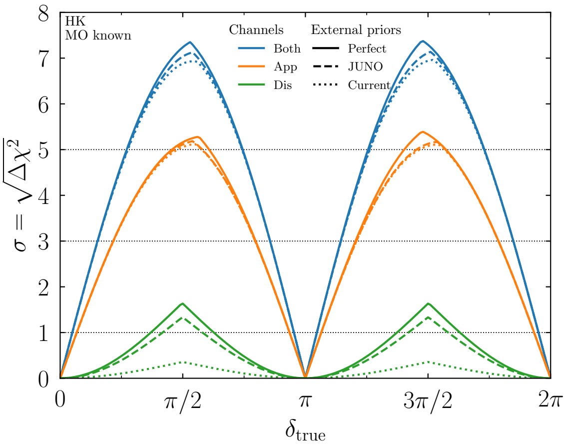
JUNO and DUNE disappearance Sensitivities





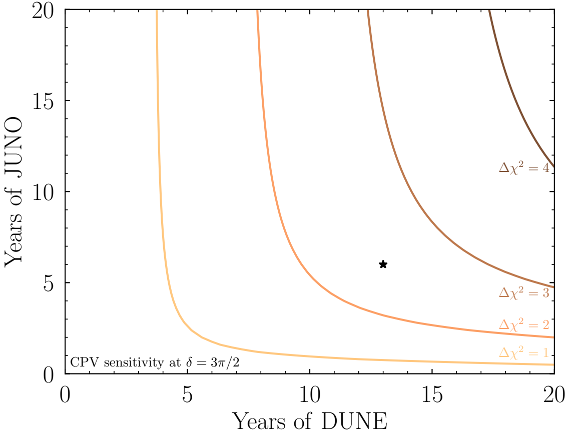
JUNO and HK disappearance Sensitivities





Varying Runtime/Power

Significance to disfavor $|\cos\delta| = 1$ at $\cos\delta = 0$



Improvement requires **both** experiments!

NuFast

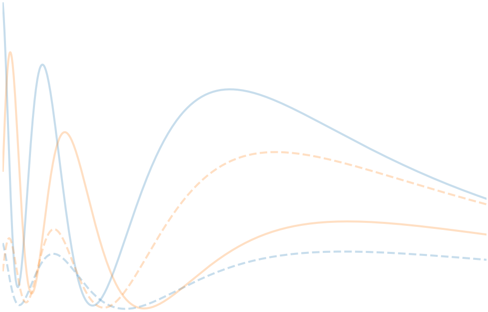
A fast code for long-baseline
neutrino oscillation probabilities in matter

2405.02400 with S. Parke

The problem

$$\Delta m^2_{21},\, \Delta m^2_{31}$$
$$s^2_{23},\, s^2_{13},\, s^2_{12}$$
$$\delta$$

$\Rightarrow$
in matter



Many approaches

Solve the Schrödinger equation

$$i\frac{d}{dt}|\nu\rangle = H(t)|\nu\rangle$$

If $H(t) = H$ (constant density)

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = \left[e^{-iHL}\right]_{\beta\alpha} \qquad P = |\mathcal{A}|^2$$

Exponential requires computing eigenvalues and eigenvectors of H

Many approaches

Modify vacuum probabilities

- Get the eigenvalues by solving the cubic

Cardano 1545
V. Barger, et al. PRD 22 (1980) 2718

- Get the eigenvectors

H. Zaglauer, K. Schwarzer Z.Phys. C40 (1988) 273
K. Kimura, A. Takamura, H. Yokomakura hep-ph/0205295
PBD, S. Parke, X. Zhang 1907.02534
A. Abdulahi, S. Parke 2212.12565

Other approaches?
Are approximations useful?
Optimal hybrids?
What is the goal?

Fermilab computing experts bolster NOvA evidence, 1 million cores consumed

July 3, 2018 | Marcia Teckenbrock

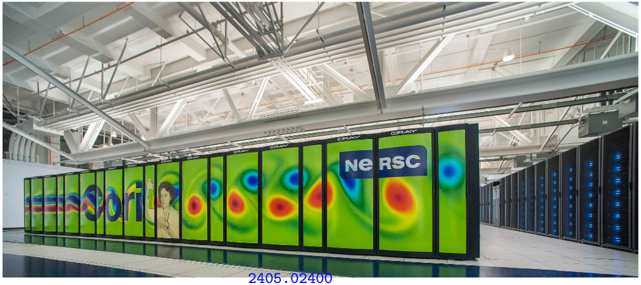
Share Tweet Email

Array

How do you arrive at the physical laws of the universe when you're given experimental data on a renegade particle that interacts so rarely with matter, it can cruise through light-years of lead? You call on the power of advanced computing.

The NOvA neutrino experiment, in collaboration with the Department of Energy's Scientific Discovery through Advanced Computing (SciDAC-4) program and the HEPCloud program at DOE's Fermi National Accelerator Laboratory, was able to perform the largest-scale analysis ever to support the recent evidence of antineutrino oscillation, a phenomenon that may hold clues to how our universe evolved.

Using Cori, the newest supercomputer at the National Energy Research Scientific Computing Center (NERSC), located at Lawrence Berkeley National Laboratory, NOvA used over 1 million computing cores, or CPUs, between May 14 and 15 and over a short timeframe one week later. This is the largest number of CPUs ever used concurrently over this duration — about 54 hours — for a single high-energy physics experiment. This unprecedented amount of computing enabled scientists to carry out some of the most complicated techniques used in neutrino physics, allowing them to dig deeper into the seldom seen interactions of neutrinos. This Cori allocation was more than 400 times the amount of Fermilab computing allocated to the NOvA experiment and 50 times the total computing capacity at Fermilab allocated for all of its rare-physics experiments. A continuation of the analysis was performed on NERSC's Cori and Edison supercomputers one week later. In total, nearly 35 million core-hours were consumed by NOvA in the 54-hour period. Executing the same analysis on a single desktop computer would take 4,000 years.



FNAL Newsroom

Monte-Carlo estimates of statistical significances

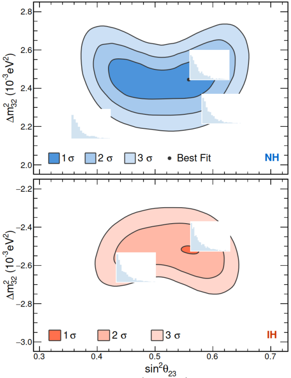
Wilks' theorem is often wrong

At each point in parameter space, simulate the experiment many times

"many" means $\gg 1/p$ for a desired p-value

This is sometimes called Feldman-Cousins

G. Feldman, R. Cousins physics/9711021
This isn't actually what was novel in the FC paper



Study found most of the time was spent computing probabilities

NOvA/T2K are $\sim 3\sigma$ experiments, but DUNE/HK will be $\gtrsim 5\sigma$ experiments!

DUNE sensitivities require computing the probabilities "a zillion times"

Start at the end

What is needed for experiments?

1. All 9 channels ($\nu_\alpha \to \nu_\beta$)
   - DUNE will certainly do ν_τ appearance

     See e.g. P. Machado, H. Schulz, J. Turner 2007.00015
   - $\nu_\tau \to \nu_\beta$ channels are not needed, but come from free from unitarity
   - JUNO only needs $\nu_e \to \nu_e$
2. Different energies, baselines, and densities
3. ν and $\bar{\nu}$
4. NO and IO
5. Oscillation parameters are mostly known
   - Don't need to consider e.g. $\Delta m^2_{21} > |\Delta m^2_{31}|$ or $\theta_{23} \sim 10°$

How to achieve speed

1. Avoid costly operations
   - `sin`, `cos` (and inverse functions) are very slow
   - `sqrt` is quite slow, but not as bad as trigs
   - Division is slower than multiplication ($0.2x$ may be faster than $x/5$)
2. Reduce repeated calculations
   - Compute $\frac{L}{4E}$ in the correct units once
   - Compute each of the three $\sin\frac{\Delta m^2_{ij}L}{4E}$ once

All of these are compiler dependent

Optimal structure of the probability

1. Amplitude requires four trig functions of kinematic variables $(\Delta m^2_{ij} L/4E)$ ✗
2. Writing the probabilities out requires three trig functions ✓
3. Disappearance structure is straightforward:

$$P_{\alpha\alpha} = 1 - 4\sum_{i>j} |V_{\alpha i}|^2 |V_{\alpha j}|^2 \sin^2 \frac{\Delta\lambda_{ij} L}{4E}$$

H in matter has eigenvalues λ_i and eigenvectors $V_{\alpha i}$

Optimal structure of the probability

4. Appearance structure:

$$P_{\mu e}^{CP+}=2\sum_{i>j}(|V_{\tau k}|^2-|V_{\mu i}|^2|V_{ej}|^2-|V_{\mu j}|^2|V_{ei}|^2)\sin^2\frac{\Delta\lambda_{ij}L}{4E}$$

Fun fact:

$$2\Re(V_{\alpha i}V_{\beta j}^*V_{\alpha j}^*V_{\beta i})$$
$$=|V_{\alpha k}|^2|V_{\beta k}|^2-|V_{\alpha i}|^2|V_{\beta i}|^2-|V_{\alpha j}|^2|V_{\beta j}|^2$$
$$=|V_{\gamma k}|^2-|V_{\alpha i}|^2|V_{\beta j}|^2-|V_{\alpha j}|^2|V_{\beta i}|^2$$

$$P_{\mu e}^{CP-}=-8J\frac{\Delta m_{21}^2\Delta m_{31}^2\Delta m_{32}^2}{\Delta\lambda_{21}\Delta\lambda_{31}\Delta\lambda_{32}}\sin\frac{\Delta\lambda_{21}L}{4E}\sin\frac{\Delta\lambda_{31}L}{4E}\sin\frac{\Delta\lambda_{32}L}{4E}$$

Leverages NHS identity:
V. Naumov IJMP 1992
P. Harrison, W. Scott hep-ph/9912435

Note that CP even/odd is actually T even/odd due to matter

Account for matter

1. Need the eigenvalues λ_i
2. For eigenvectors, naively need $\Re(V_{\alpha i}V^*_{\beta j}V^*_{\alpha j}V_{\beta i})$
3. Given our form, need only the $|V_{\alpha i}|^2$ and J
   - Don't need any phase information of the eigenvectors!

Leverages PBD, S. Parke, X. Zhang 1907.02534

4. Can compute the $|V_{\alpha i}|^2$ from the λ_i and submatrix eigenvalues (requires only a square root) using Eigenvector-Eigenvalue Identity

$$|V_{\alpha i}|^2 = \frac{\prod_{k=1}^{n-1}(\lambda_i - \xi_k^\alpha)}{\prod_{k=1;k\neq i}^{n}(\lambda_i - \lambda_k)}$$

See e.g. PBD, S. Parke, T. Tao, X. Zhang 1908.03795
Can actually avoid the $\sqrt{\ }$ in practice

Eigenvalues are hard

The eigenvalues in matter λ_i depend on S:

$$S = \cos\left\{\frac{1}{3}\cos^{-1}\left[\frac{2A^3 - 9AB + 27C}{2\left(A^2 - 3B\right)^{3/2}}\right]\right\}$$

where

$$A = \sum \lambda_i = \Delta m^2_{21} + \Delta m^2_{31} + a$$

$$B = \sum_{i>j} \lambda_i \lambda_j = \Delta m^2_{21}\Delta m^2_{31} + a[\Delta m^2_{21}(1 - |U_{e2}|^2) + \Delta m^2_{31}(1 - |U_{e3}|^2)]$$

$$C = \prod \lambda_i = a\Delta m^2_{21}\Delta m^2_{31}|U_{e1}|^2$$

Approximate eigenvalues

1. Instead, approximate one eigenvalue
   - λ_3 is best because is never parametrically small and easy to approximate
2. From DMP:

$$\lambda_3 \approx \Delta m_{31}^2 + \frac{1}{2}\Delta m_{ee}^2\left(x - 1 + \sqrt{(1-x)^2 + 4xs_{13}^2}\right)$$

$$x \equiv \frac{a}{\Delta m_{ee}^2} \qquad \Delta m_{ee}^2 \equiv \Delta m_{31}^2 - s_{12}^2\Delta m_{21}^2$$

H. Minakata, S. Parke 1505.01826
PBD, H. Minakata, S. Parke 1604.08167
H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

3. Get other two eigenvalues by picking two of A, B, C conditions

Requires one more $\sqrt{\ \ }$

The approximation

- This is the only approximation used in the entire approach
- In vacuum the approximation returns to the correct value

Many approximations in the literature are not correct in vacuum limit
See G. Barenboim, PBD, S. Parke, C. Ternes 1902.00517

- In fact can iteratively improve λ_3 with rapid convergence via Newton's method:

$$\lambda_3 \to \lambda_3 - \frac{X(\lambda_3)}{X'(\lambda_3)}$$

$$X(\lambda) = \lambda^3 - A\lambda^2 + B\lambda - C = 0$$

  - Precision improvement starts at 10^{-5} for the first step
  - The improvement is quadratic thereafter
- One line of code, just loop as many times as desired

All 9 channels

Given P_{ee}, $P_{\mu\mu}$, $P_{\mu e}^{CP+}$, and $P_{\mu e}^{CP-}$:

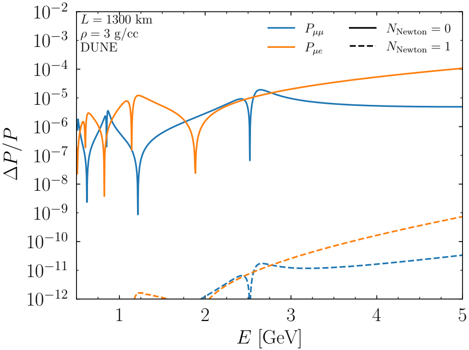
	$P_{\alpha e}$	$P_{\alpha\mu}$	$P_{\alpha\tau}$
$P_{e\beta}$	P_{ee}	$P_{\mu e}^{CP+} - P_{\mu e}^{CP-}$	$1 - P_{ee} - P_{\mu e}^{CP+} + P_{\mu e}^{CP-}$
$P_{\mu\beta}$	$P_{\mu e}^{CP+} + P_{\mu e}^{CP-}$	$P_{\mu\mu}$	$1 - P_{\mu\mu} - P_{\mu e}^{CP+} - P_{\mu e}^{CP-}$
$P_{\tau\beta}$	$1 - P_{ee} - P_{\mu e}^{CP+} - P_{\mu e}^{CP-}$	$1 - P_{\mu\mu} - P_{\mu e}^{CP+} + P_{\mu e}^{CP-}$	$-1 + P_{ee} + P_{\mu\mu} + 2P_{\mu e}^{CP+}$

Total approach

1. Inputs: 6 oscillation parameters, experimental details (L, E, ρ, Y_e)
2. Calculate λ_3 approximately
   - Iteratively improve with Newton's method, if desired
3. Calculate the $|V_{\alpha i}|^2$'s with the Eigenvector-Eigenvalue Identity
4. Calculate the sines of the kinematic terms
5. Calculate the CP odd term with the NHS identity
6. Calculate key probabilities: P_{ee}, $P_{\mu\mu}$, and $P_{\mu e}$
7. Calculate remaining probabilities

Precision

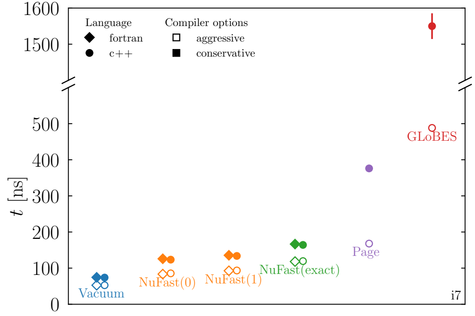
Is this approximation okay?
DUNE requires $\lesssim 1\%$ level precision



Slightly better for HK
$\sim 10^{-10}$ for JUNO
See backups

Speed

Is this algorithm fast?
How does it compare?



- "Conservative" = default, `-O0`
- "Aggressive" = `-Ofast` and `-ffast-math`
- Some variation expected due to architecture

See also J. Page 2309.06900
and P. Huber, et al. hep-ph/0701187

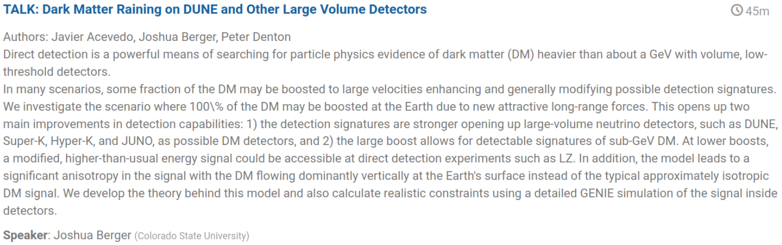
NuFast: the code

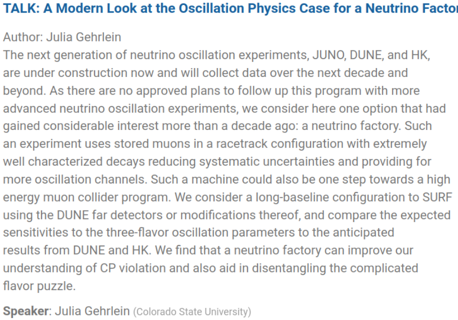
- Code is on github: github.com/PeterDenton/NuFast
- Implementations in c++ and f90
- Easy to use and there are comments!

```
// NuFast.cpp
// Calculate the probabilities:
Probability_Matter_LBL(s12sq, s13sq, s23sq, delta,
    Dmsq21, Dmsq31, L, E, rho, Ye, N_Newton,
    &probs_returned);
// Print out the probabilities to terminal
for (int alpha = 0; alpha < 3; alpha++)
{
   for (int beta = 0; beta < 3; beta++)
   {
      printf("%d %d %g\n", alpha, beta,
          probs_returned[alpha][beta]);
   } // beta, 3
} // alpha, 3
```

- $\bar{\nu}$: $E < 0$; IO: $\Delta m^2_{31} < 0$
- Folder called `Benchmarks` to make the plots and speed tests in the paper

Other talks at CETUP to Look Out For





Discussion and Conclusions

- **Disappearance can discover CPV**
- Requires two good measurements: JUNO and DUNE/HK
- Can rule out some values of δ at $> 3\sigma$
- Works in vacuum or matter; matter slightly minimizes HK's effect
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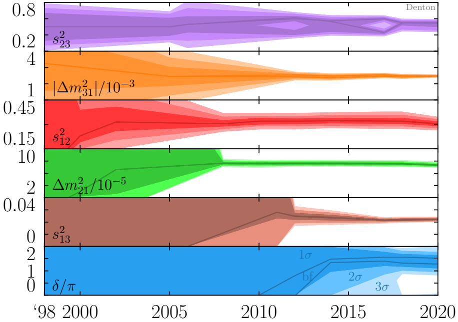
- `NuFast` for fast oscillation probabilities
- Atmospherics and nighttime solar in the works!

Backups

References



SK hep-ex/9807003

M. Gonzalez-Garcia, et al. hep-ph/0009350

M. Maltoni, et al. hep-ph/0207227

SK hep-ex/0501064

SK hep-ex/0604011

T. Schwetz, M. Tortola, J. Valle 0808.2016

M. Gonzalez-Garcia, M. Maltoni, J. Salvado 1001.4524

T2K 1106.2822

D. Forero, M. Tortola, J. Valle 1205.4018

D. Forero, M. Tortola, J. Valle 1405.7540

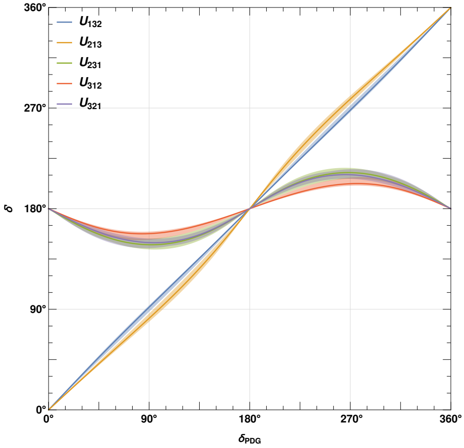
P. de Salas, et al. 1708.01186

F. Capozzi et al. 2003.08511

Complex phase in different parameterizations

- Can relate the complex phase in one parameterization to that in another
- U_{132} and U_{213} similar to U_{123}
- δ constrained to $\sim [150°, 210°]$ in U_{231}, U_{312}, U_{321}
- Bands indicate 3σ uncertainty on θ_{12}, θ_{13}, θ_{23}
- "50% of possible values of δ"
  $\Rightarrow$ parameterization dependent

DUNE TDR II 2002.03005



Quark mixing

From the PDG, V_{CKM} in the V_{123} parameterization is

$$\theta_{12} = 13.09^\circ \quad \theta_{13} = 0.2068^\circ \quad \theta_{23} = 2.323^\circ \quad \delta_{\text{PDG}} = 68.53^\circ$$

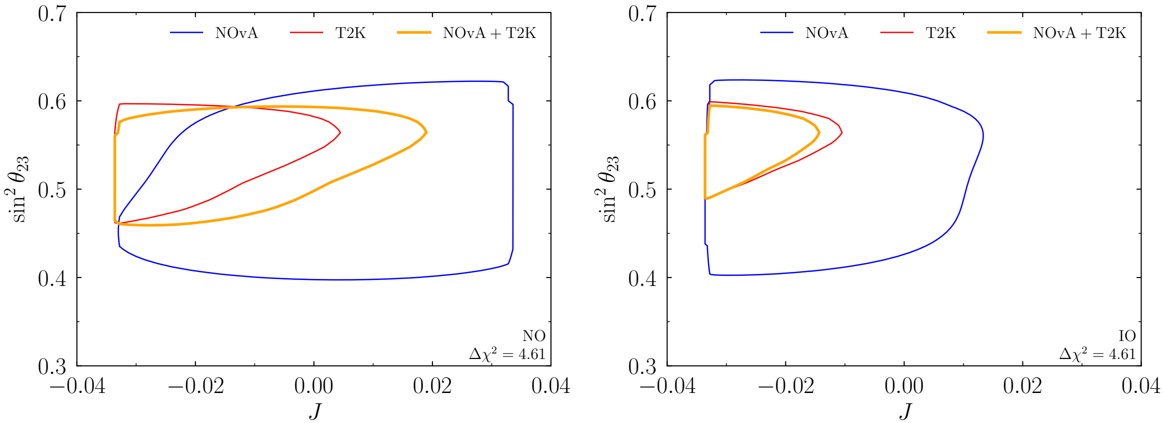
Looks like "large" CPV:

$$\sin\delta_{\text{PDG}} = 0.93 \sim 1$$

yet $J_{\text{CKM}}/J_{\max} = 3 \times 10^{-4}$.

Switch to V_{212} parameterization, $\Rightarrow \delta' = 1^\circ$ and $\sin\delta' = 0.02$.

Standard oscillation parameters



Can see that the combination doesn't like the NO while it does like the IO

IO preferred over NO at $\Delta\chi^2 = 2.3$

CP violation in oscillations

In vacuum at first maximum:

$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m^2_{21}}{\Delta m^2_{32}}$$

$$J \equiv s_{12} c_{12} s_{13} c^2_{13} s_{23} c_{23} \sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

- Extracting δ from data requires every other oscillation parameter
- J requires only Δm^2_{21} (up to matter effects)

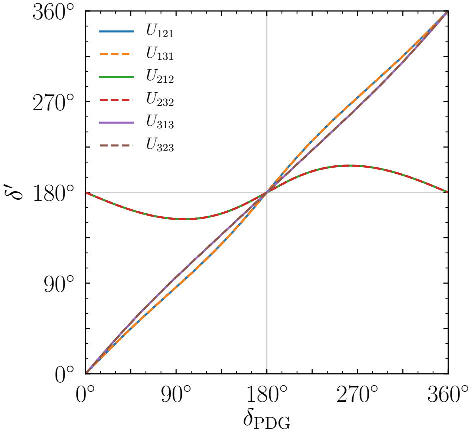
Matter effects are easily accounted for

$$\hat{J} \simeq \frac{J}{\sqrt{(c_{212} - c^2_{13} a/\Delta m^2_{21})^2 + s^2_{212}} \sqrt{(c_{213} - a/\Delta m^2_{ee})^2 + s^2_{213}}}$$

PBD, S. Parke 1902.07185

PBD, H. Minakata, S. Parke 1604.08167

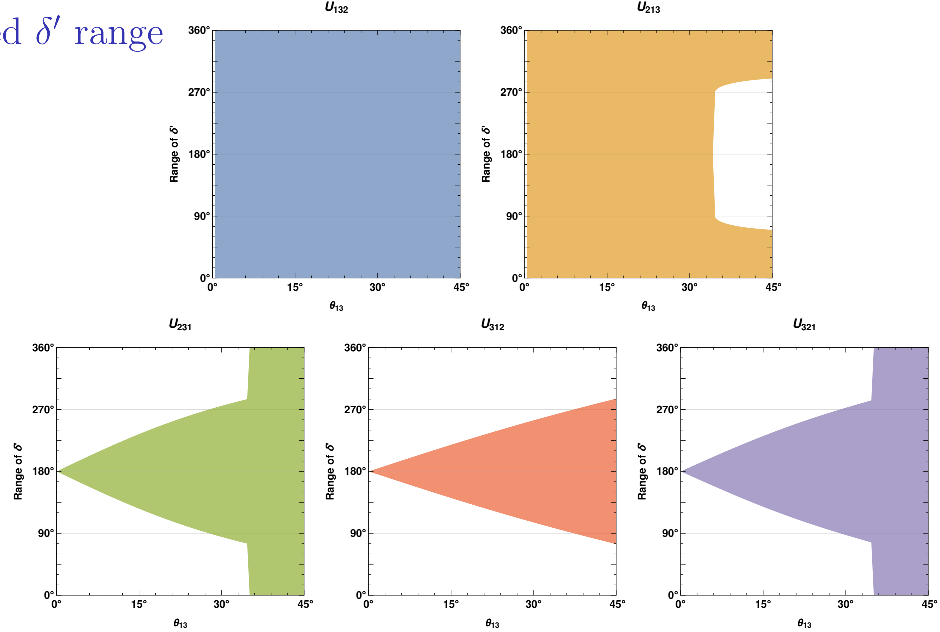
Repeated rotations



	U_{121}	U_{131}	U_{212}	U_{232}	U_{313}	U_{323}
$\lvert U_{e2}\rvert$	✓	✓	✓	✓	✗	✗
$\lvert U_{e3}\rvert$	✓	✓	✗	✗	✓	✓
$\lvert U_{\mu3}\rvert$	✗	✗	✓	✓	✓	✓

Note that $e^{i\delta}$ must be on first or third rotation

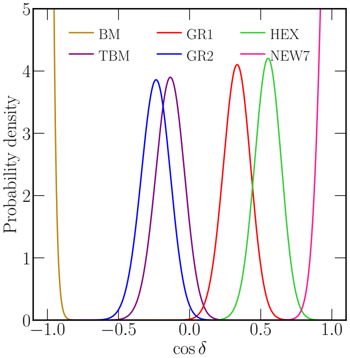
Allowed δ' range



The importance of $\cos\delta$

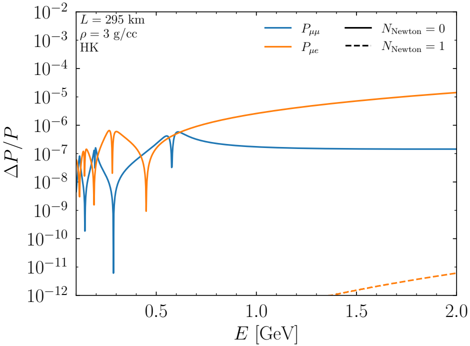
- If only $\sin\delta$ is measured $\Rightarrow$ sign degeneracy: $\cos\delta = \pm\sqrt{1-\sin^2\delta}$
- Most flavor models predict $\cos\delta$

J. Gehrlein, et al. 2203.06219

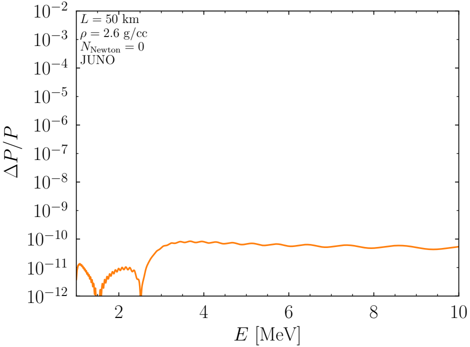


L. Everett, et al. 1912.10139

Precision

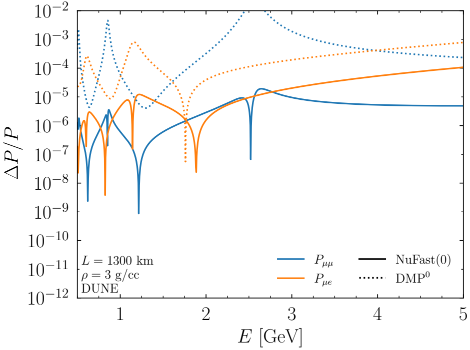


$L = 295$ km
$\rho = 3$ g/cc
HK
$P_{\mu\mu}$
$P_{\mu e}$
$N_{\rm Newton} = 0$
$N_{\rm Newton} = 1$
$\Delta P/P$
E [GeV]



$L = 50$ km
$\rho = 2.6$ g/cc
$N_{\rm Newton} = 0$
JUNO
$\Delta P/P$
E [MeV]

Comparison to DMP



DMP: PBD, H. Minakata, S. Parke 1604.08167