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INFN
Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati

Asymptotically free E_6 GUT and the generation of neutrino mass

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2024-07-02

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CETUP 2024*
Lead, South Dakota

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Outline

- Motivation: GUTs and E_6
- About the E_6 group
- Model building considerations in asymptotcally free E_6 GUT
  - Symmetry breaking
  - Yukawa sector, including neutrinos

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I will present **ongoing work**. . .

Past (published) work on an E_6 GUT model:

$\rightarrow$ see 2305.16398 and 2403.20278

$\rightarrow$ talk by Borut Bajc (week 5 of CETUP* 2024)

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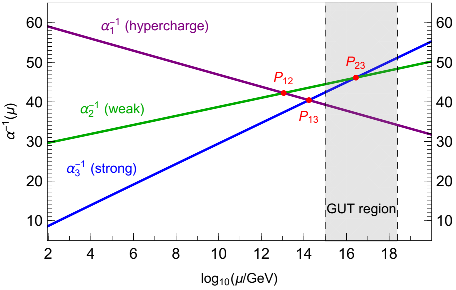
Motivation — Unification of gauge couplings?

- RG running of gauge couplings in the Standard Model (SM)
- Unification at M_{GUT}? GUT — Grand Unified Theory



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Window of opportunity: grey band

$\gtrsim$ $10^{15}\,\mathrm{GeV}$
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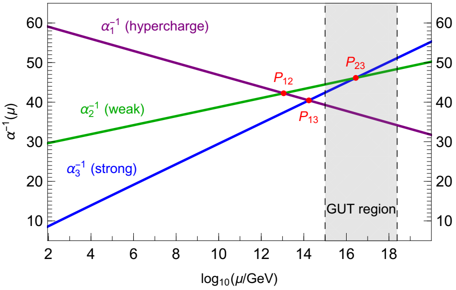

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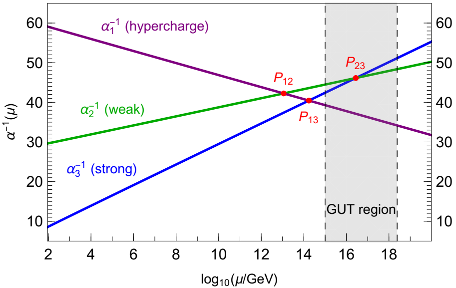

Indeed, SM approximately unifies there!

Motivation — where does E_6 come from?

- Requirements for a unified gauge group G:
  (a) G is simple
  (b) $G \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
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- Classification of simple finite-dimensional Lie algebras:

root system	name	comment	$\mathbb{C}$-irreps?
A_n	$\mathrm{SU}(n+1)$	rotations in $\mathbb{C}^{n+1}$	all n
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C_n	$\mathrm{Sp}(2n)$	rotations in $\mathbb{H}^n$	/
D_n	$\mathrm{SO}(2n)$	rotations in $\mathbb{R}^{2n}$	odd n
$\mathrm{E}_6, \mathrm{E}_7, \mathrm{E}_8, \mathrm{F}_4, \mathrm{G}_2$		exceptional	E_6

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  (2) $\mathrm{SO}(4n+2)$, $n \geq 2$
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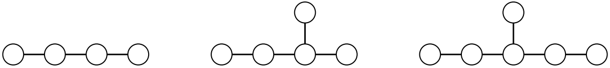
- Minimal choices: $G_{\mathrm{SM}} \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6$

About E_6 — comparison with other GUT groups



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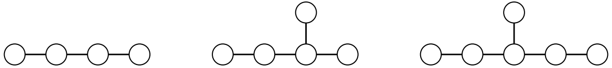

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	$\mathrm{SU}(5)$	$\mathrm{SO}(10)$	E_6
rank	4	5	6
dimension	24	45	78
fund. irrep	5	10	27

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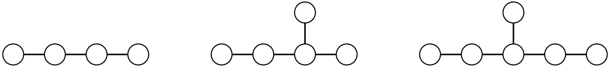

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- Some E_6 maximal subgroups:
  - $\mathrm{E}_6 \supset \mathrm{SO}(10) \times \mathrm{U}(1)$

$$78 = 45(0) \oplus 1(0) \oplus 16(-3) \oplus \overline{16}(+3) \tag{1}$$

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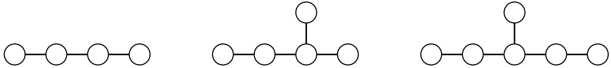

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  - $\mathrm{E}_6 \supset \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$

$$78 = (8,1,1) \oplus (1,8,1) \oplus (1,1,8) \oplus (3,\bar{3},\bar{3}) \oplus (\bar{3},3,3) \tag{2}$$

About E_6 — fermions in the fundamental representation 1

- Decomposition of the fundamental representation
  - $\mathrm{E}_6 \supset \mathrm{SO}(10) \times \mathrm{U}(1)$

$$27 = 16(+1) \oplus 10(-2) \oplus 1(+4) \tag{3}$$

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- Reminder: $\mathrm{SO}(10) \supset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

$$16 = Q \oplus u^c \oplus d^c \oplus L \oplus e^c \oplus \nu^c, \tag{5}$$

$$10 = d' \oplus d'^c \oplus L' \oplus L'^c, \tag{6}$$

$$1 = n, \tag{7}$$

where

$$Q \sim (3,2,+\tfrac{1}{6}),\quad u^c \sim (\bar{3},1,-\tfrac{2}{3}),\quad d^c, d'^c \sim (\bar{3},1,+\tfrac{1}{3}),\quad d' \sim (3,1,-\tfrac{1}{3}), \tag{8}$$
$$L, L' \sim (1,2,-\tfrac{1}{2}),\quad L'^c \sim (1,2,+\tfrac{1}{2}),\quad e^c \sim (1,1,+1),\quad \nu^c, n \sim (1,1,0).$$

About E_6 — fermions in the fundamental representation 2

- Trinification decomposition of irrep 27

$$(3,3,1) \sim \begin{pmatrix} u_1 & d_1 & d'_1 \\ u_2 & d_2 & d'_2 \\ u_3 & d_3 & d'_3 \end{pmatrix}, \quad (1,\bar{3},3) \sim \begin{pmatrix} \nu'^c & e' & e \\ e'^c & \nu' & \nu \\ e^c & \nu^c & n \end{pmatrix}, \quad (\bar{3},1,\bar{3}) \sim \begin{pmatrix} u^c_1 & u^c_2 & u^c_3 \\ d^c_1 & d^c_2 & d^c_3 \\ d'^c_1 & d'^c_2 & d'^c_3 \end{pmatrix}.$$

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  - All SM fermions of one generation inside
  - 2 right-handed neutrinos per generation
    $\rightarrow \mathrm{E}_6$ GUT is a theory of neutrino mass — similar to $\mathrm{SO}(10)$

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  - vector-like exotics in every generation: lepton doublet, down-type q
    $\to$ these remain heavy, chiral part survives to EW scale
  - The group E_6 is free of chiral anomalies

About E_6 — how to build a Yukawa sector?

- Simplest way to build a Yukawa sector: $3 \times 27_F + $ scalars

$$27 \otimes 27 = \overline{27}_s \oplus 351'_s \oplus 351_a \tag{9}$$

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$$\mathcal{L} \supset {16_F}^2 \cdot 10 + {16_F}^2 \cdot \overline{126} + {16_F}^2 \cdot 120 \tag{11}$$

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- This E_6 Yukawa sector:
  (a) Neutrino masses: seesaw type I+II (requires $351'$)
  (b) Is realistic already with $27 \oplus 351'$
  (known from $\mathrm{SO}(10)$ fits with 2 symmetric Yukawa matrices)

E_6 model building — requirements and possibilities

- Requirements from scalar sector:
  (1) breaks $\mathrm{E}_6 \to \cdots \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
  (in non-SUSY GUT: expect intermediate stages)
  (2) admits a realistic Yukawa sector
  (need a home for the SM Higgs)

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- Irreducible representations of E_6:

$$\mathbf{1}, \quad \mathbf{27}, \quad \mathbf{78}, \quad \mathbf{351}, \quad \mathbf{351'}, \quad \mathbf{650}, \quad \mathbf{1728}, \quad \mathbf{2430}, \quad \ldots \tag{12}$$

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  - $\to$ $\mathbf{27} \oplus \mathbf{351}'$ for Yukawa sector [2403.20278]

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  - → can break through trinification $\mathrm{SU}(3)^3$ due to 650 [2305.16398]
  - → $27 \oplus 351'$ for Yukawa sector [2403.20278]
  - → will be presented at CETUP* in week 5 by Borut Bajc

E_6 model building — constraints

- RGE for gauge coupling $\alpha = g^2/4\pi$:

$$\frac{d}{dt}\alpha^{-1} = -\frac{1}{2\pi}\left(a + \frac{1}{4\pi} b\,\alpha + \mathcal{O}(\alpha^2)\right) \tag{13}$$

- For theories with $3 \times 27_F$:

$$a = -38 + 1\,N_{27} + 2\,N_{78} + 25\,N_{351} + 28\,N_{351'} + 25\,N_{650} + 160\,N_{1728} + 135\,N_{2430} \tag{14}$$

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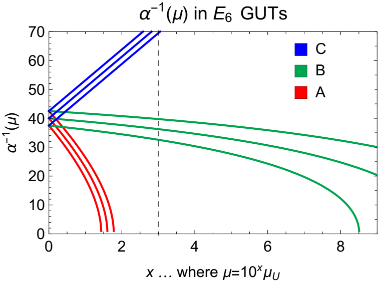
A	$a = +16$	$b = +11956$
B	$a = -9$	$b = +5956$
C	$a = -29$	$b = -52$



E_6 model building — constraints

- RGE for gauge coupling $\alpha = g^2/4\pi$:

$$\frac{d}{dt}\alpha^{-1} = -\frac{1}{2\pi}\left(a + \frac{1}{4\pi} b\,\alpha + \mathcal{O}(\alpha^2)\right) \tag{13}$$

- For theories with $3 \times 27_F$:

$$a = -38 + 1\,N_{27} + 2\,N_{78} + 25\,N_{351} + 28\,N_{351'} + 25\,N_{650} + 160\,N_{1728} + 135\,N_{2430} \tag{14}$$

	fermions	scalars
A	3×27	$650 \oplus 27 \oplus 351'$
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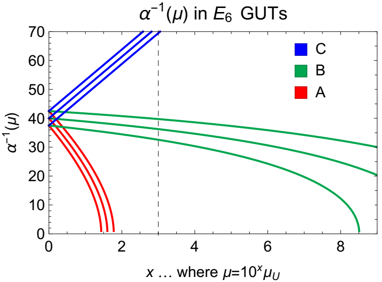

- For asymptotic freedom: constrained to use of irreps 27 and 78

Asymptotically free E_6 — symmetry breaking

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    $\to$ technically this could be flipped $\mathrm{SO}(10)$
    $\to$ further subtleties: global $\mathrm{SO}(78)$ symmetry of potential

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  $\to$ SM singlets in $\mathbf{27}$ are also $\mathrm{SU}(5)$ singlets

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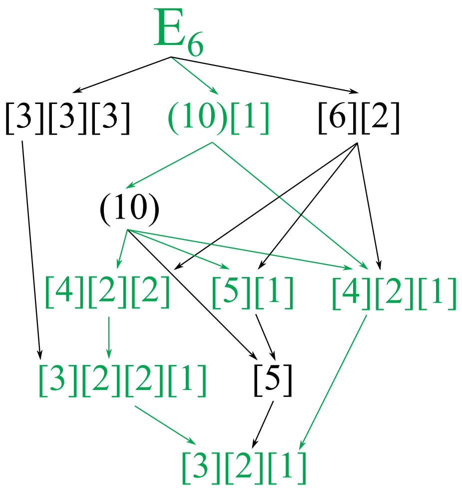
- $n \times 27$ (without 78): $\mathrm{SU}(5)$
  $\to$ SM singlets in 27 are also $\mathrm{SU}(5)$ singlets
- Viable [preliminary result]: $27 \oplus 78$
  $\to$ one solution is to $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$

Asymptotically free E_6 — breaking patterns



- Imperfect/incomplete diagram:
  - $\rightarrow$ shows some subgroups
  - $\rightarrow$ ignores embedding details
- Notation: $[n] := \mathrm{SU}(n)$
  $(n) := \mathrm{SO}(n)$
- Without 78:
  breaking gets stuck in [5]
- For $78 \oplus 27$:
  plausible stages in green
- Explicit solution found:
  [3][2][2][1] (left-right model)

  Others? To be worked out...

Asymptotically free E_6 — Yukawa sector considerations

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$$\frac{y_i}{\Lambda} \, 27_F \, 27_F \cdot 78 \cdot 27_i \tag{17}$$

$\rightarrow$ asymptotic freedom: $\Lambda = M_{Pl} \;\Rightarrow\; \frac{\langle 78 \rangle}{\Lambda} \lesssim 10^{-2.5}$

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- Problems: $\rightarrow$ $n = 1$: $M_U \propto M_D \propto Y_1$, hence no CKM
  $\rightarrow$ $n \geq 1$: $M_D \propto M_E^T$ (not realistic)

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$$\mathcal{L}_Y = Y_i\, 27_F\, 27_F\, 27_i + \overline{Y}_i\, \overline{27}_F\, \overline{27}_F\, 27_i^* + (m + \eta\, 78)\, 27_F\, \overline{27}_F \quad (18)$$

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  $\rightarrow$ for suitable $\langle 78\rangle$: different for D and E sectors

  (left-right solution: $D \neq E$; $m, \langle 78\rangle$ at GUT scale)

Yukawa sector — quarks and charged leptons (UDE)

- Explicit results for quarks and charged leptons (LR solution)

$$\mathcal{L} \supset \begin{pmatrix} u \\ \bar{u}^c \end{pmatrix}^T \begin{pmatrix} -Y_i v_{3i} & m + \frac{b_3 \eta}{6} \\ m - \frac{a_3 \eta}{6} & -\overline{Y} v_{3i}^* \end{pmatrix} \begin{pmatrix} u^c \\ \bar{u} \end{pmatrix} \tag{19}$$

$$+ \begin{pmatrix} d^c \\ d'^c \\ \bar{d} \\ \bar{d}' \end{pmatrix}^T \begin{pmatrix} Y_i v_{1i}^* & Y_i V_{1i} & m - \frac{a_3 \eta}{6} & 0 \\ -Y_i v_{2i}^* & -Y_i V_{2i} & 0 & m + \frac{a_3 \eta}{3} \\ m + \frac{b_3 \eta}{6} & 0 & \overline{Y}_i v_{1i} & -\overline{Y}_i v_{2i} \\ 0 & m - \frac{b_3 \eta}{3} & \overline{Y}_i V_{1i}^* & -\overline{Y}_i V_{2i}^* \end{pmatrix} \begin{pmatrix} d \\ d' \\ \bar{d}^c \\ \bar{d}'^c \end{pmatrix} \tag{20}$$

$$+ \begin{pmatrix} e \\ e' \\ \bar{e}^c \\ \bar{e}'^c \end{pmatrix}^T \begin{pmatrix} -Y_i {v_{1i}}^* & Y_i V_{1i} & m - \frac{2a_3 + b_3}{6} \eta & 0 \\ Y_i {v_{2i}}^* & -Y_i V_{2i} & 0 & m + \frac{a_3 - b_3}{6} \eta \\ m + \frac{a_3 + 2b_3}{6} \eta & 0 & -\overline{Y}_i v_{1i} & \overline{Y}_i v_{2i} \\ 0 & m + \frac{a_3 - b_3}{6} \eta & \overline{Y}_i V_{1i}^* & -\overline{Y}_i V_{2i}^* \end{pmatrix} \begin{pmatrix} e^c \\ e'^c \\ \bar{e} \\ \bar{e}' \end{pmatrix} \tag{21}$$

- Colors: fermions in $4 \times 27_F$, fermions in $\overline{27}_F$, $\langle 78 \rangle$, $\langle 27 \rangle$, $\langle 27 \rangle_{EW}$
- LR solution: : $V_{2i}|_{i=1}$, a_3 and b_3 independent; $M_I \sim V_{1i} \ll V_{2i}$
- Works for $n \times 27$, $n \geq 2$.

Yukawa sector —neutrinos

- Neutrino sector even more complicated: states to be considered are

$$\{\nu, \nu', \nu'^c, \nu^c, n, \bar{\nu}, \bar{\nu}', \bar{\nu}'^c, \bar{\nu}^c, \bar{n}\} \tag{22}$$

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Yukawa sector — loop contributions to neutrinos

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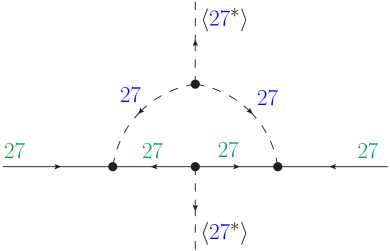

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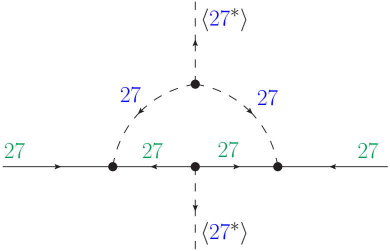

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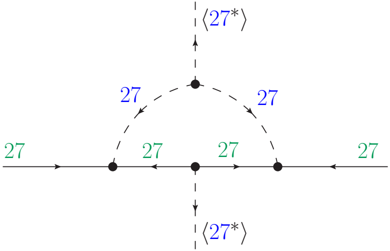

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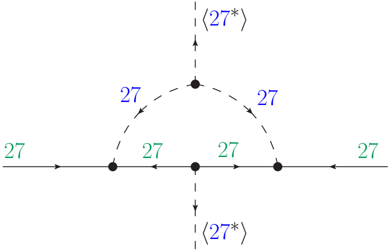

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  - → needs vector-like fermions 10_F

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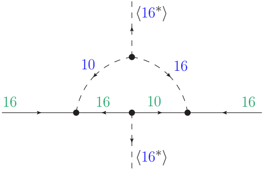

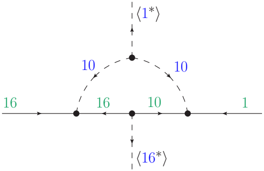

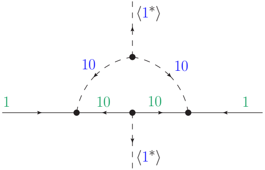

Conclusions

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