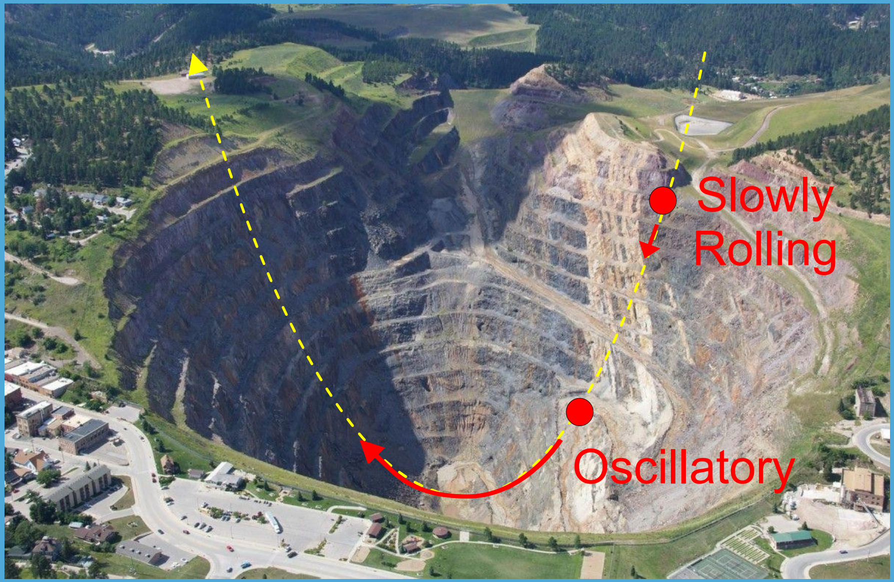
Cosmological Stasis and its Realization from Dynamical Scalars



Brooks Thomas

LAFAYETTE COLLEGE

Work supported in part by NSF

Based on work done in collaboration with:

Keith Dienes, Fei Huang, Lucien Heurtier, and Tim Tait [arXiv:2406.06830]

Keith Dienes, Fei Huang, Lucien Heurtier, Doojin Kim and Tim Tait [arXiv:2111.04753]

Keith Dienes, Fei Huang, Lucien Heurtier, Doojin Kim and Tim Tait [arXiv:2212.01369]

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CETUP* Workshop, Lead, South Dakota, June 27th, 2024

Cosmological Stasis

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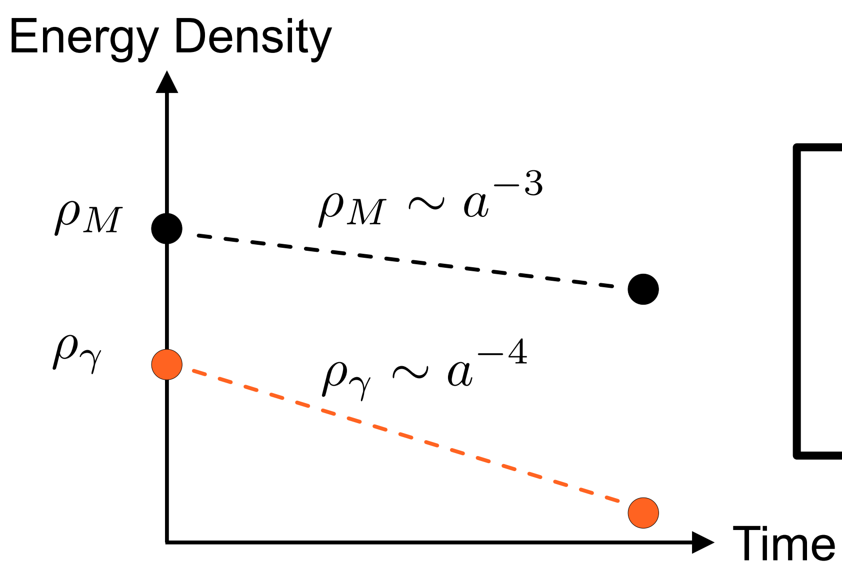

Boltzmann Equations

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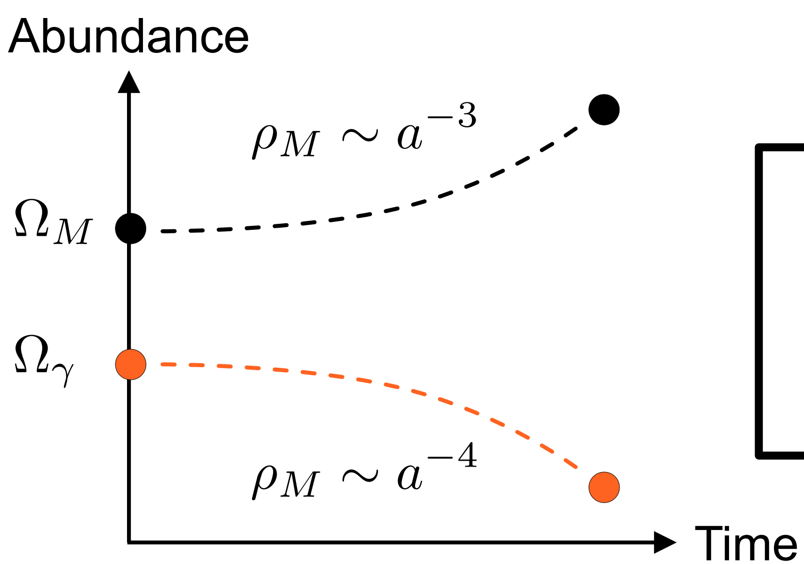

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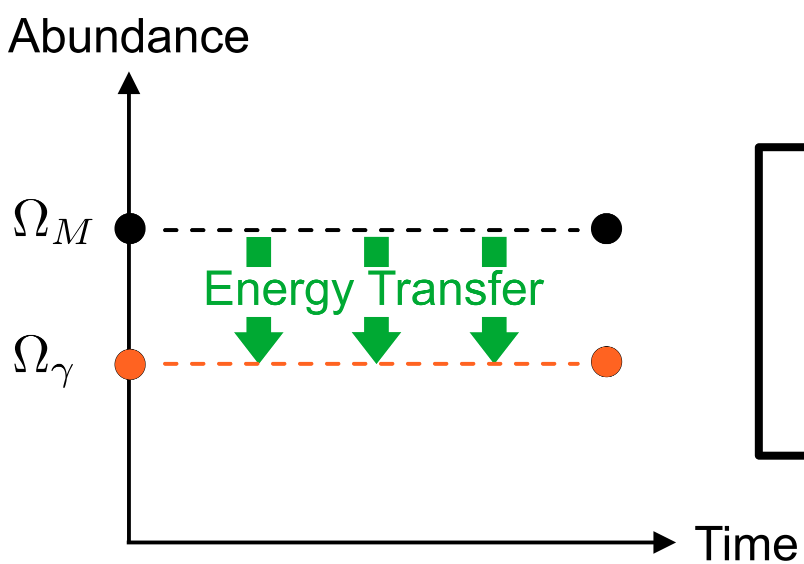

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"Pump" from component with lower to higher w.

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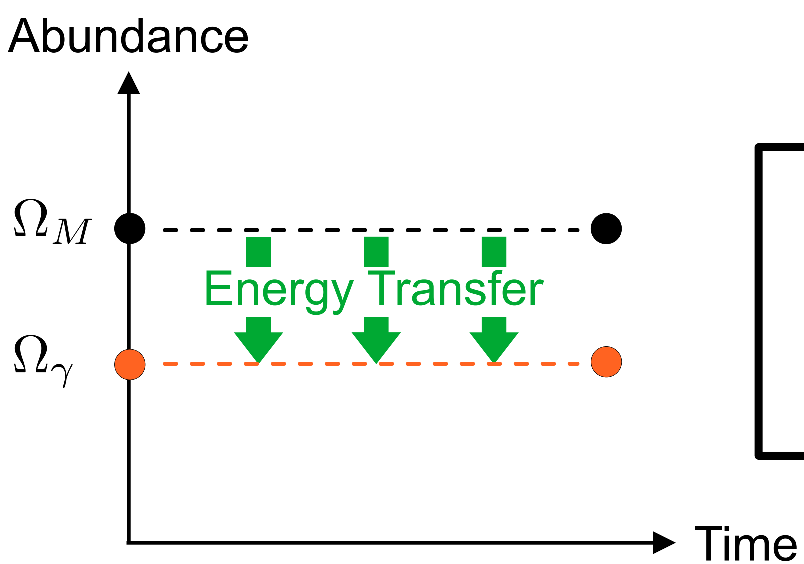

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$\propto t^{-1}$, so $P(t)$ should be as well.

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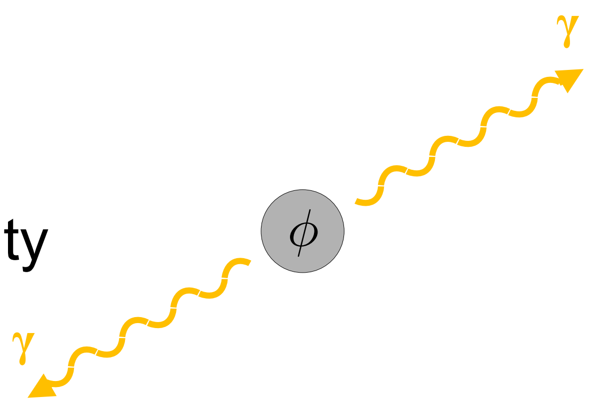

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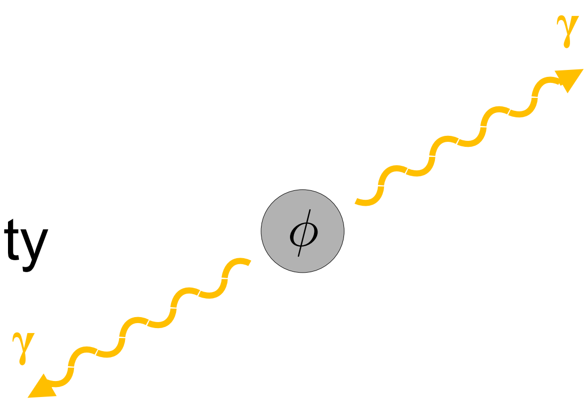

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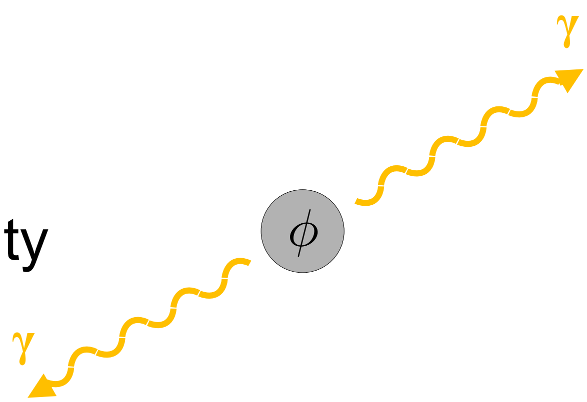

- The exponential decay of a single matter species, which occurs over a relatively short time period, is insufficient for achieving stasis.
- However, a ***tower of matter states*** ϕ_ℓ, where $\ell = 0, 1, 2, \ldots, N - 1$, whose decay widths Γ_ℓ and initial abundances $\Omega_\ell^{(0)}$ scale across the tower as a function of their mass m_ℓ can indeed give rise to a pump that compensate for the effect of cosmic expansion over a extended period.

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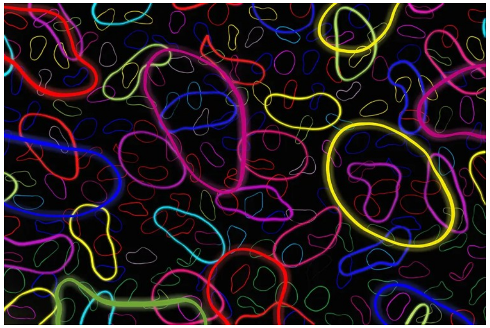
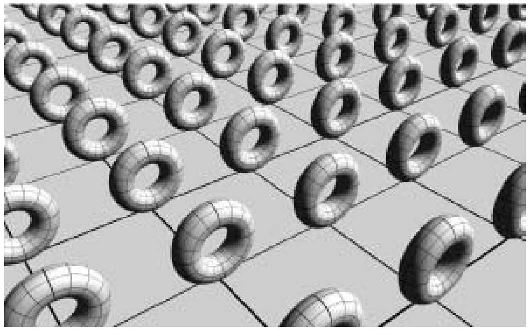
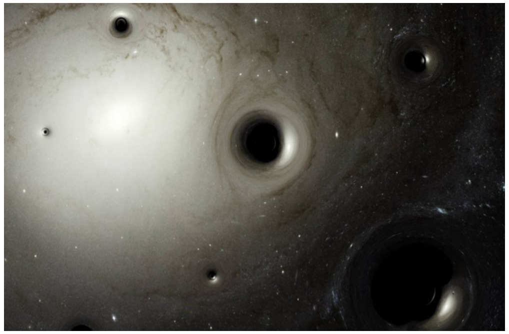

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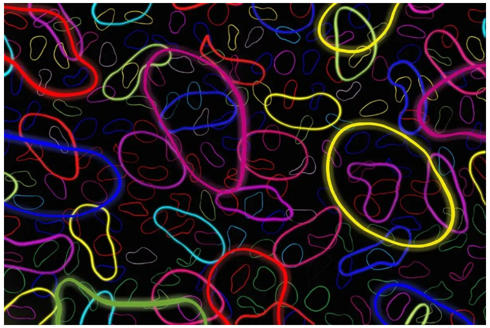
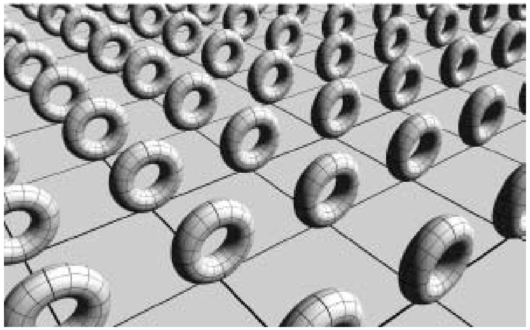
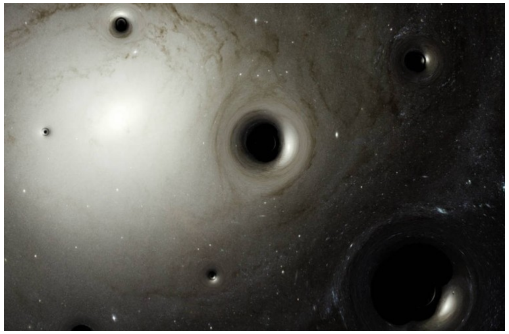

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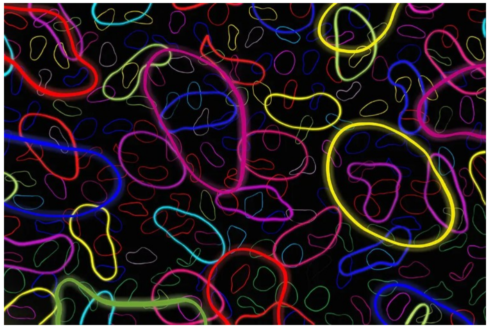
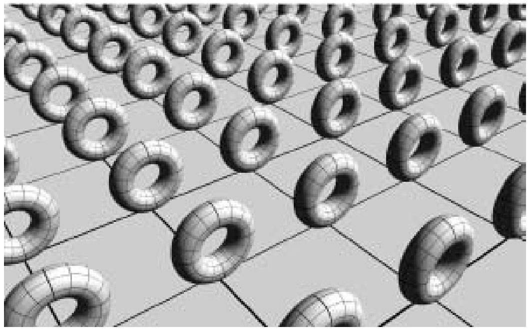
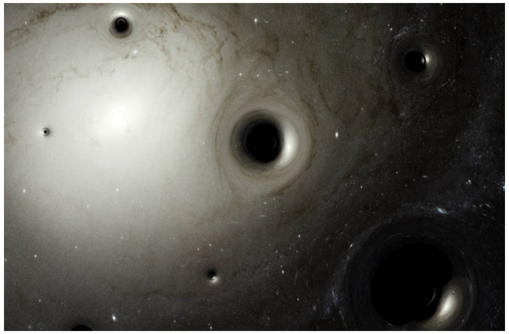
- The modified cosmological histories associated with stasis can affect the evolution of **scalar and tensor perturbations**.

A Concrete Realization

[Dienes, Huang, Heurtier, Kim, Tait, BT '21]

- Let's consider a tower of N such states states with...

Masses	Decay Widths	Initial Abundances
$m_\ell = m_0 + (\Delta m)\ell^\delta$	$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$	$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$

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- Towers of states with mass spectra of this form arise naturally in many extensions of the Standard Model.
    - KK excitations of a 5D scalar: $\begin{cases} mR \ll 1 \longrightarrow \delta \sim 1 \\ mR \gg 1 \longrightarrow \delta \sim 2 \end{cases}$
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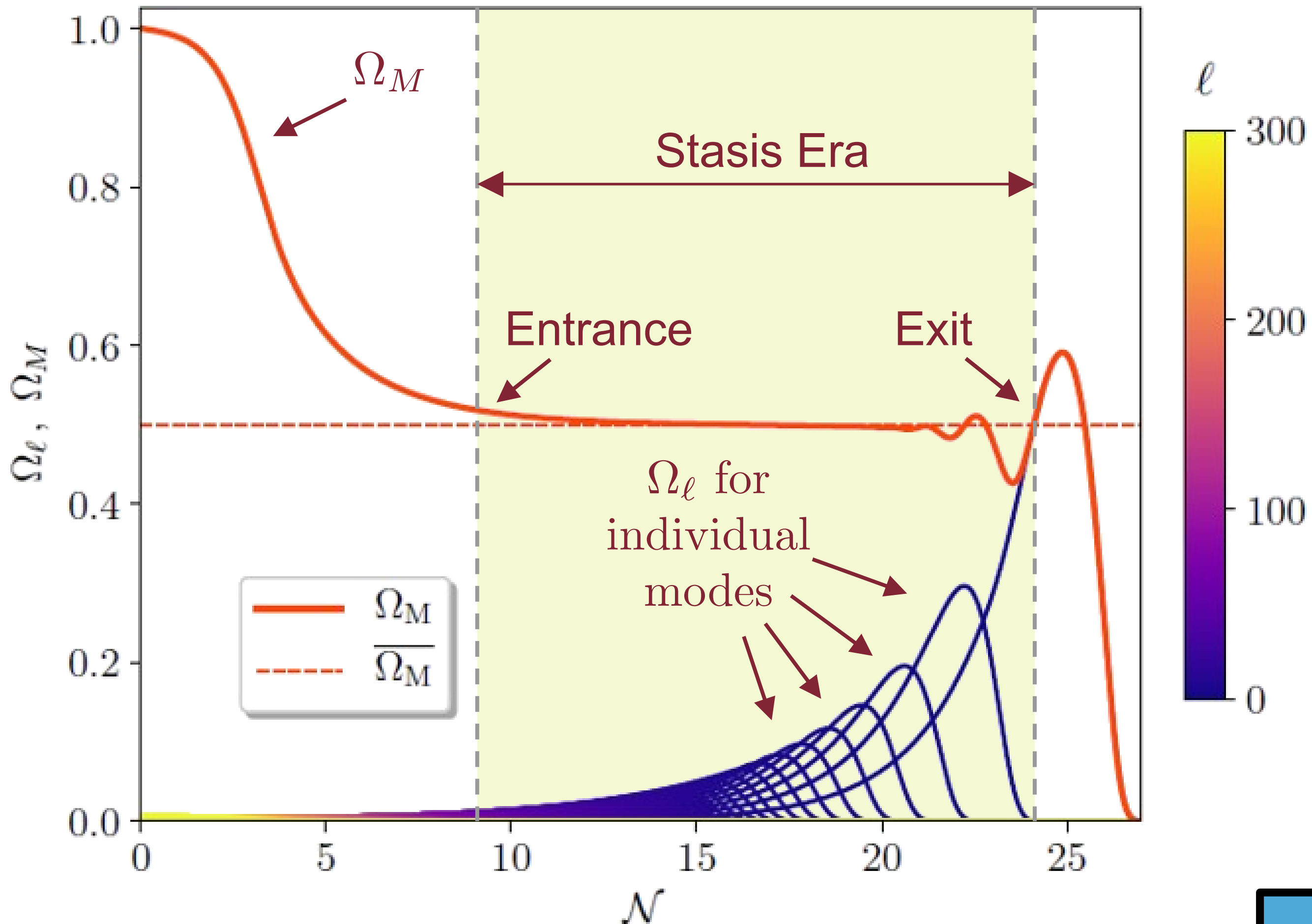
- Scaling of initial abundances depends on how they're generated:
  - Misalignment production $\longrightarrow$ $\alpha < 0$
  - Thermal freeze-out $\longrightarrow$ $\alpha < 0 \text{ or } \alpha > 0$
  - Universal inflaton decay $\longrightarrow$ $\alpha \sim 1$
  - ...

The Emergence of Stasis

- In BSM scenarios of this sort, stasis emerges generically, with minimal additional assumptions.



Parameter Choices

$$\alpha = 1$$
$$\gamma = 7$$
$$\delta = 1$$
$$N = 300$$
$$\frac{m_0}{\Delta m} = 1$$
$$\frac{\Gamma_{N-1}}{H^{(0)}} = 0.01$$

- The matter and radiation abundances during stasis turn out to depend on the model parameters α, γ, and δ.

Stasis Abundances

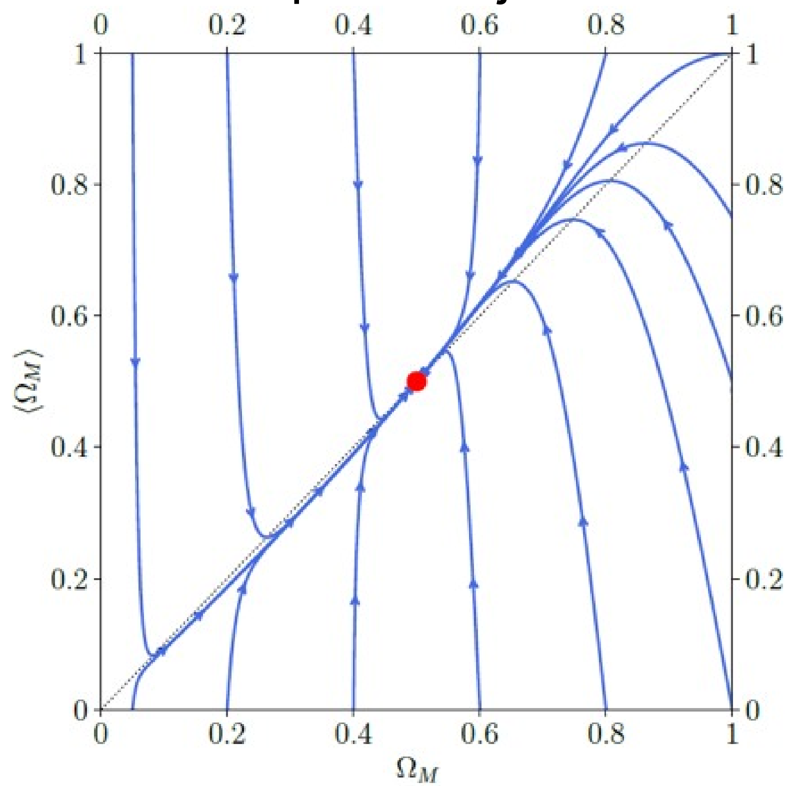
$$\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)}$$

$$\overline{\Omega}_\gamma = 1 - \overline{\Omega}_M$$

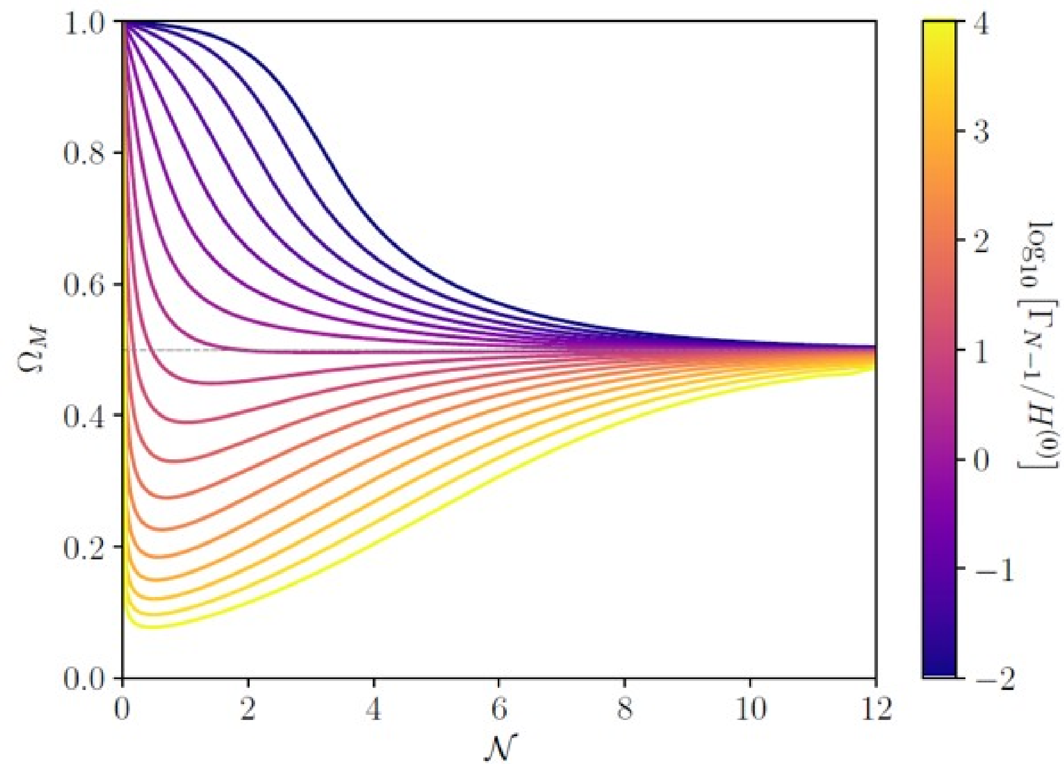
Stasis as a Global Attractor

- Perhaps even more importantly, achieving cosmological stasis does not require a fine-tuning of the initial conditions for Ω_M and H – or, alternatively, for Ω_m and its time-average $\langle \Omega_M \rangle$ – or for the ratio $\Gamma_{N-1}/H^{(0)}$.
- In fact, stasis is a ***global attractor*** in the sense that Ω_M and Ω_γ will ***evolve toward their stasis values*** regardless of what these initial conditions are.

State-Space Trajectories



Ω_M vs. t For Different $\Gamma_{N-1}/H^{(0)}$



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- **Inflationary oservables**: Such a modification of the cosmological timeline can alter predictions for CMB observables such as the tensor-to-scalar ratio r and spectral index n_s that characterize the primordial perturbation spectrum. **[Dienes, Huang, Heurtier, Kim, Tait, BT '22]**

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- **Gravitational waves**: Such modifications of the cosmological timeline also alter the contribution to the graviational-wave background generated by other sources.

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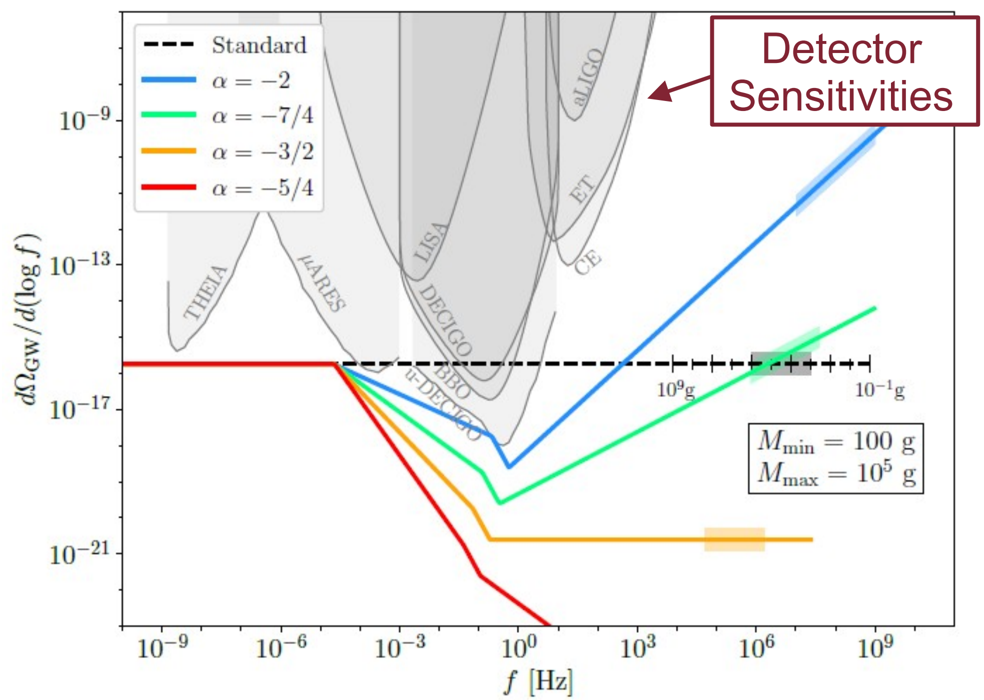
GW Spectra in Primordial-Black-Hole-Induced Stasis



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- **Gravitational waves**: Such modifications of the cosmological timeline also alter the contribution to the graviational-wave background generated by other sources. [Dienes, Huang, Heurtier, Kim, Tait, BT '22]
- **Density perturbations**: Such perturbations evolve differently than they do in the standard cosmology, since $w_{\text{eff}} \neq 1/3$ during stasis. Possible implications for small-scale structure. [Dienes, Huang, Heurtier, Hoover, Paulsen, Tait, BT '24]

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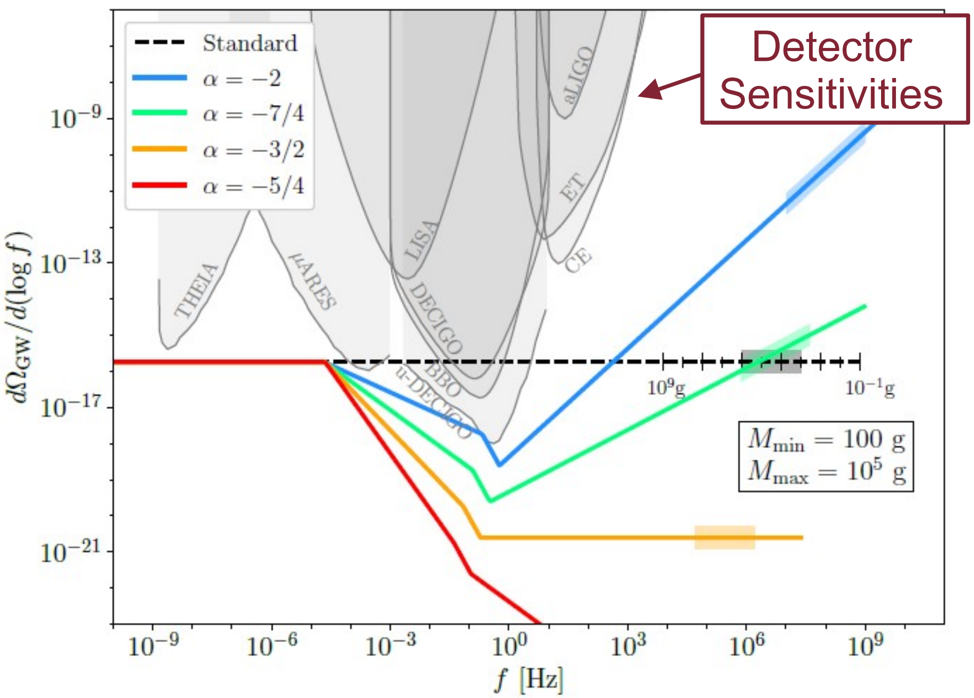

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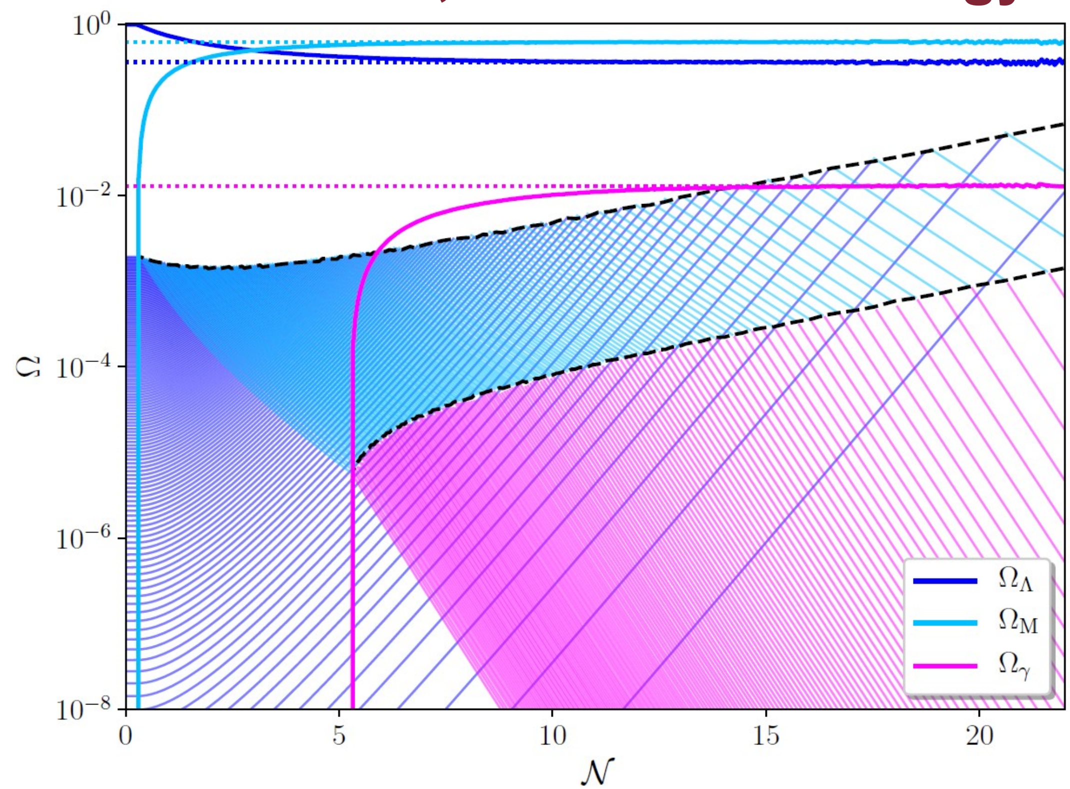
- A stasis can also arise involving **more than two components** – for example, vacuum energy, matter, and radiation – can also arise in BSM contexts. Such stases also turn out to be global attractors. **[Dienes, Huang, Heurtier, Tait, BT '23]**

Lucien Heurtier

Fei Huang

Triple Stasis Involving Matter, Radiation, and Vacuum Energy



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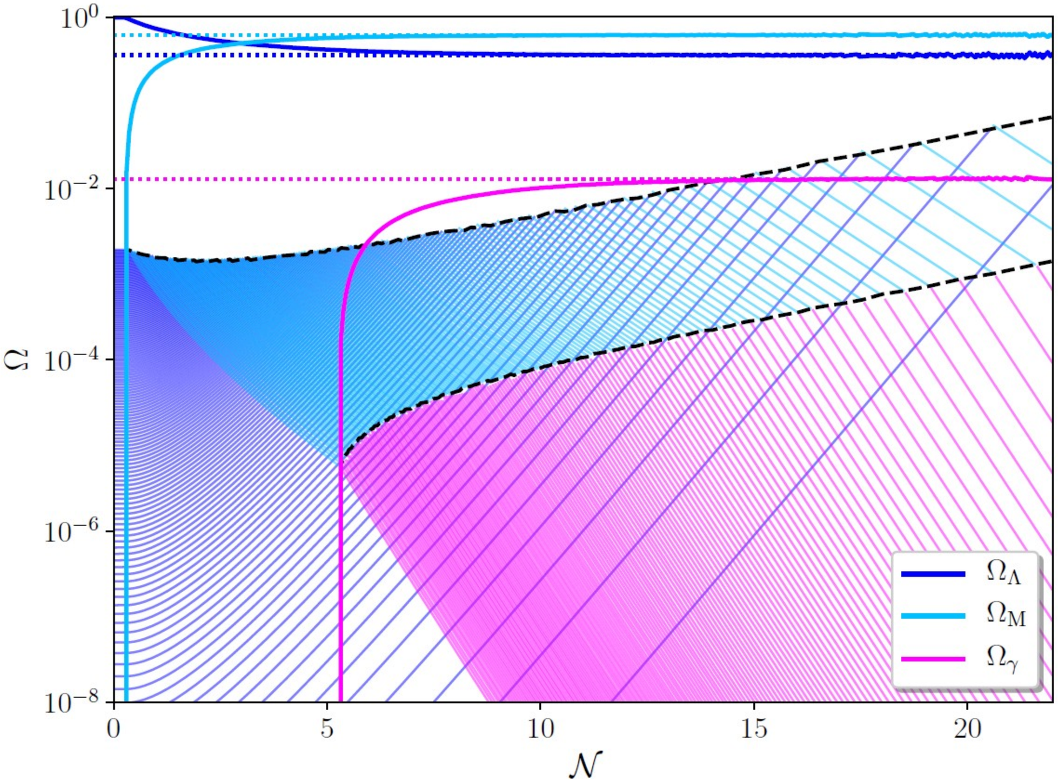

- A set of **axion-like scalar fields** whose background values transition from overdamped to underdamped oscillation can give rise to stasis… and *that's* what I'll be focusing on for the remainder of this talk!

Stasis from Dynamical Scalar Fields

[Dienes, Huang, Heurtier, Tait, BT '24]

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- Such a stasis, as we'll see, would be characterized by an effective equation-of-state parameter **between that of vacuum energy** ($w_\Lambda = -1$) **and matter** ($w_M = 0$).
- Moreover, stases involving dynamical scalars give rise to some **phenomena not seen in other realizations of stasis** which could potentially useful for addressing fundamenal questions in cosmology.

Warm-Up: A Single Scalar

- To set the stage, let's recall how the homogeneous zero-mode of a **single scalar field** ϕ of mass m with a quadratic potential $V(\phi)$ evolves in a flat FRW universe.

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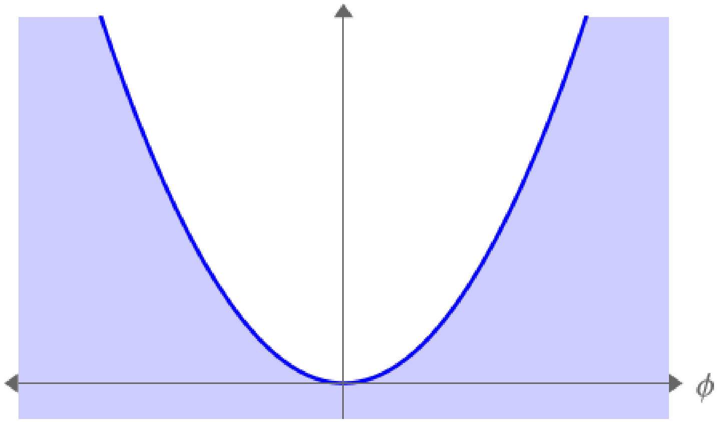
$$V(\phi) = \frac{1}{2}m\phi^2$$



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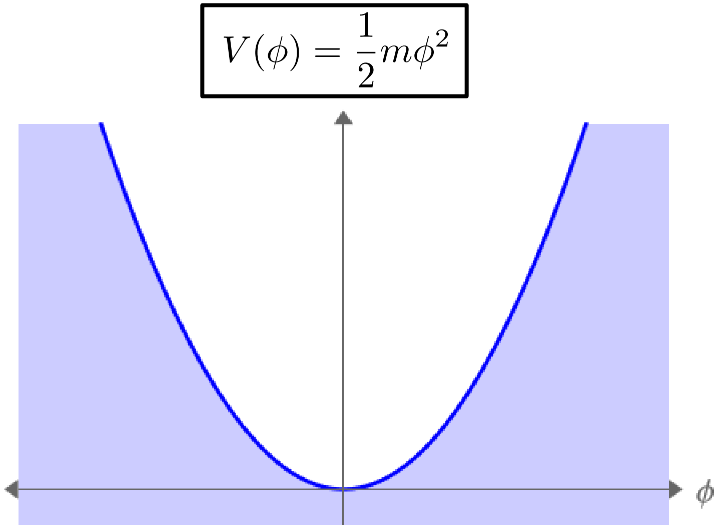

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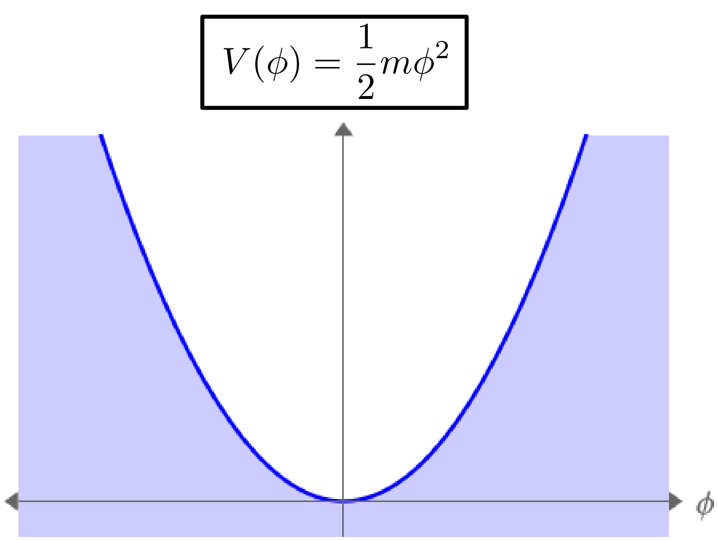

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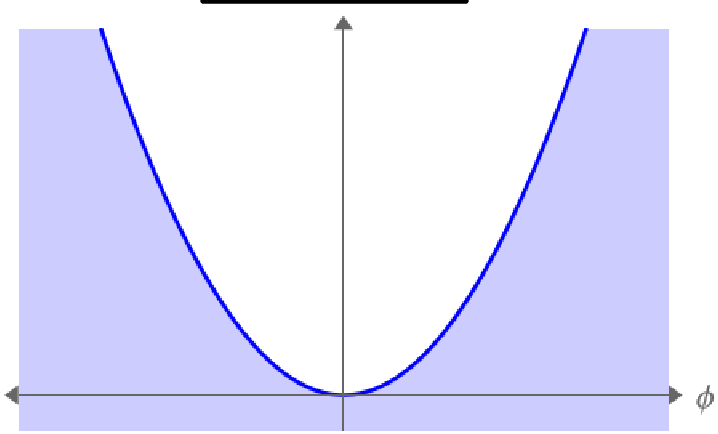

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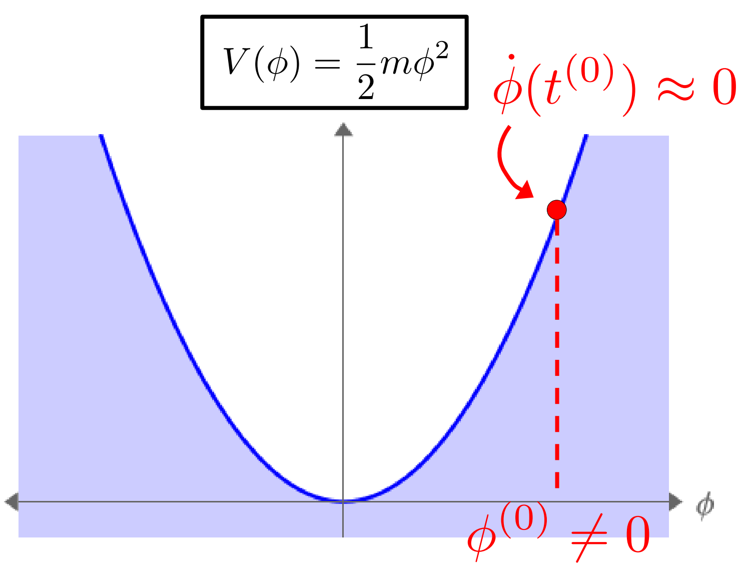

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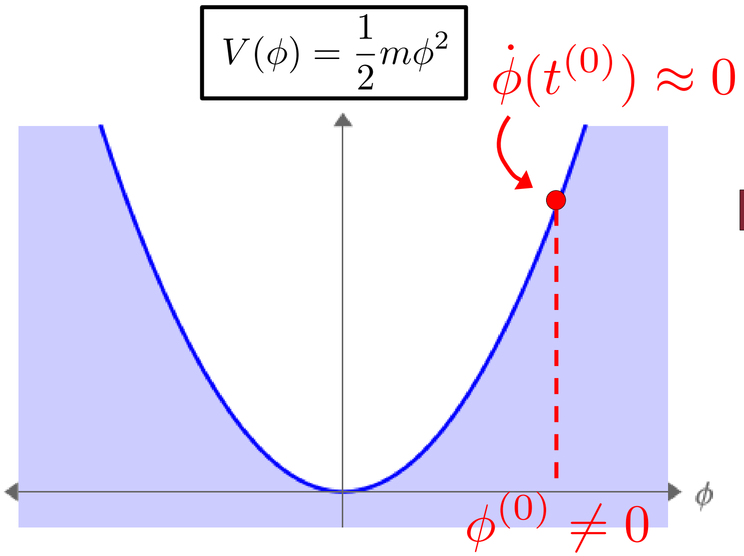

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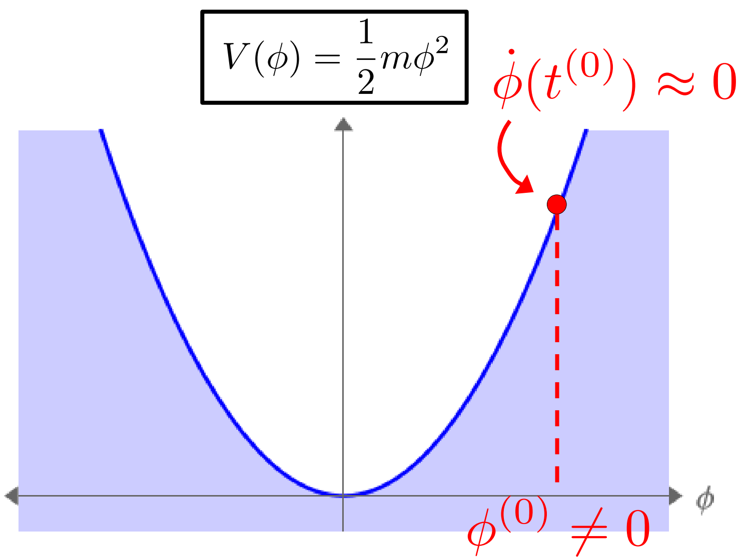

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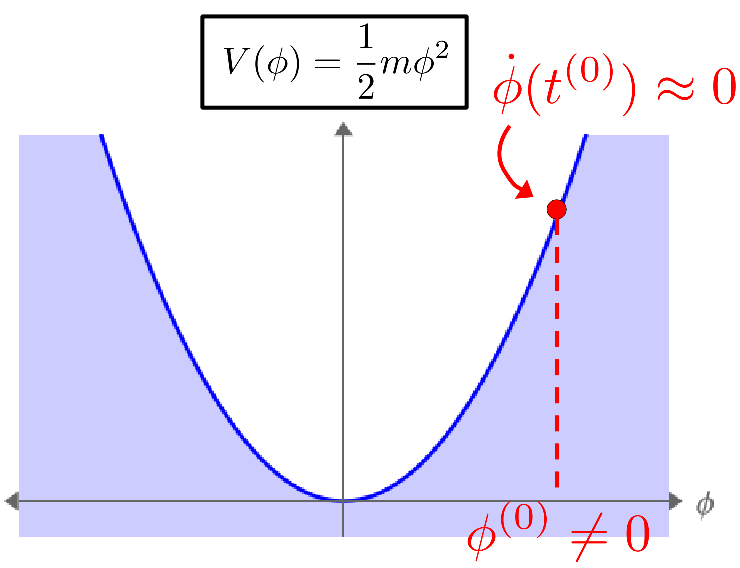
$$H \approx \frac{\kappa}{3t}, \quad \text{where} \quad \kappa \equiv \frac{2}{1+w}$$



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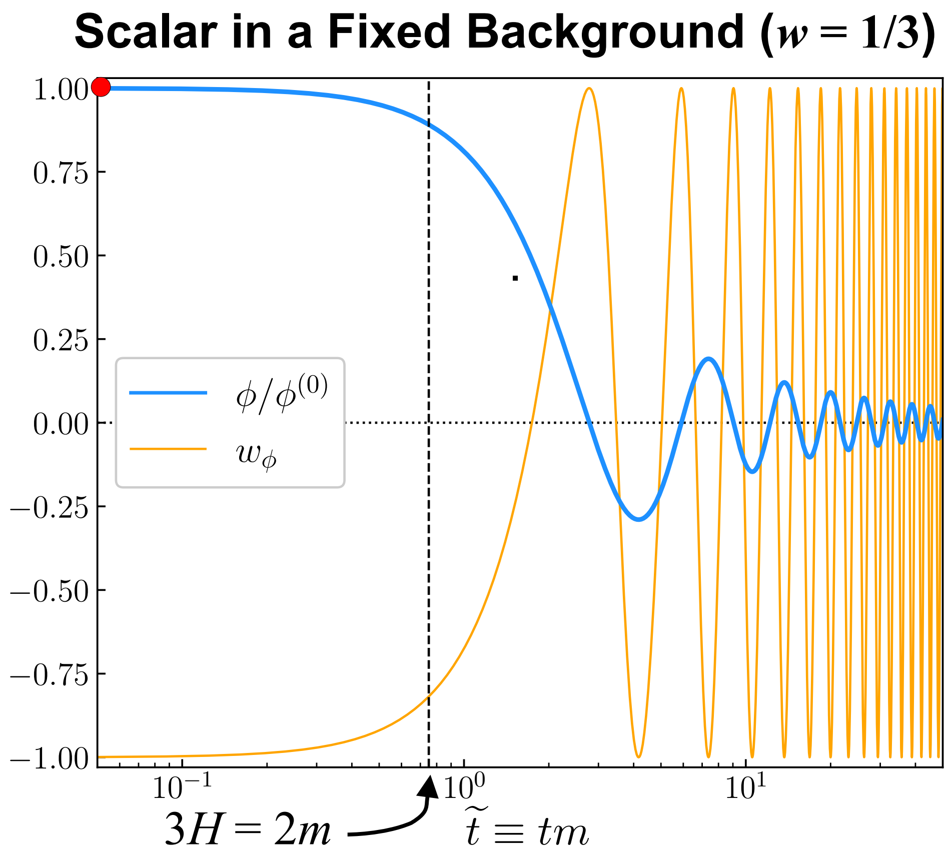

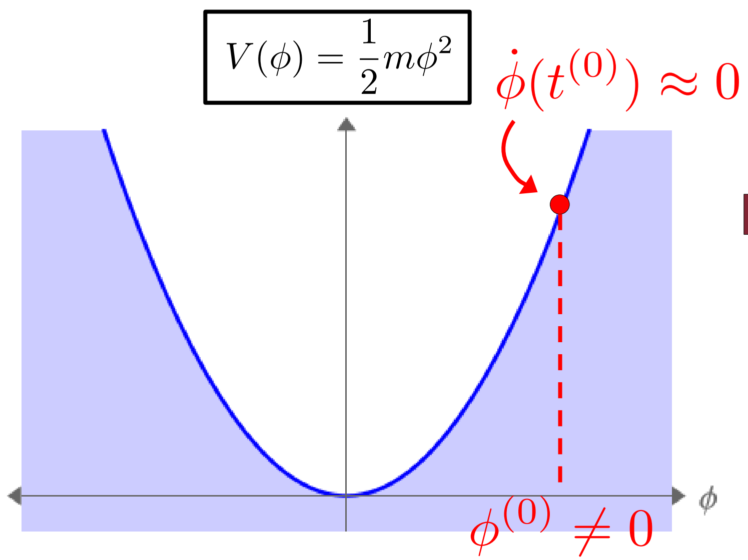

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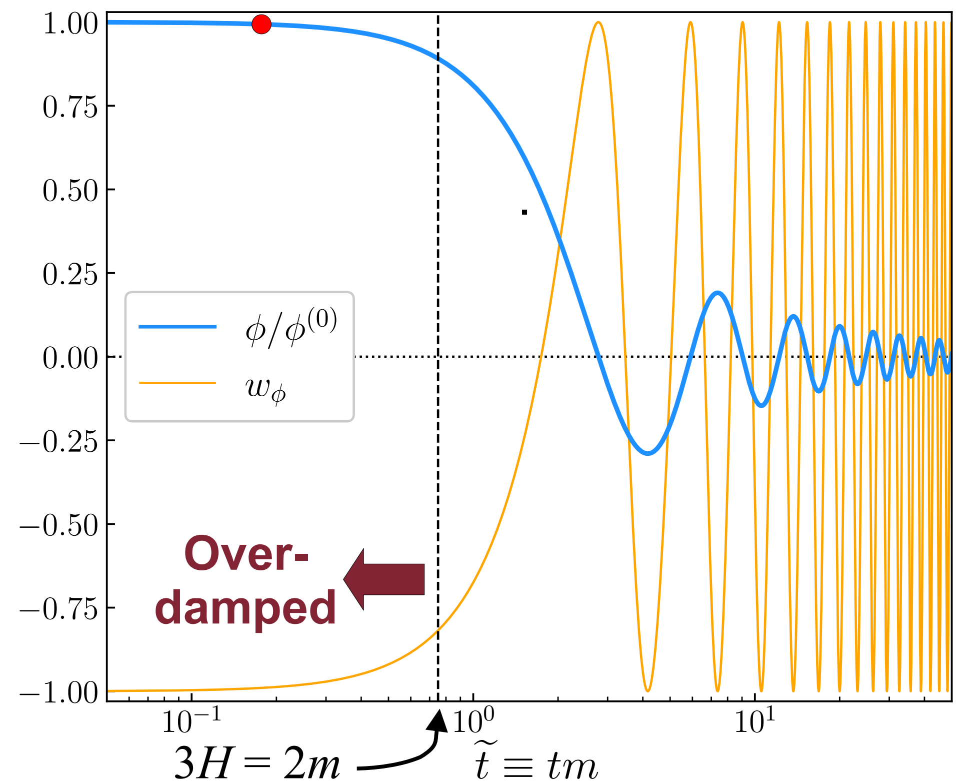

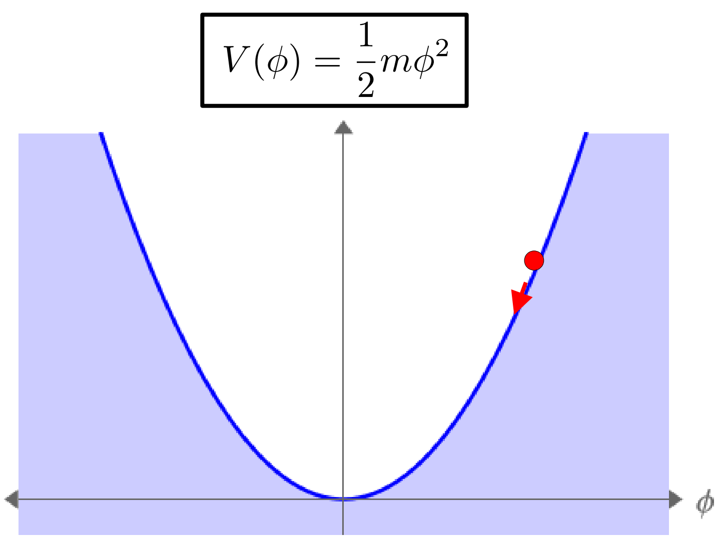

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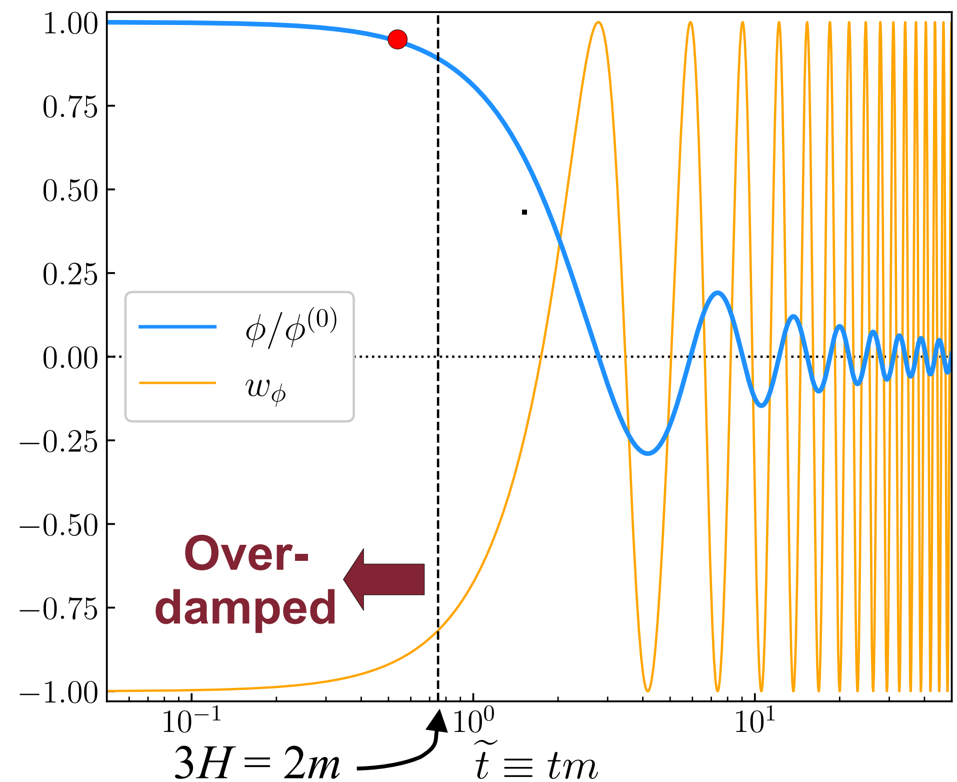

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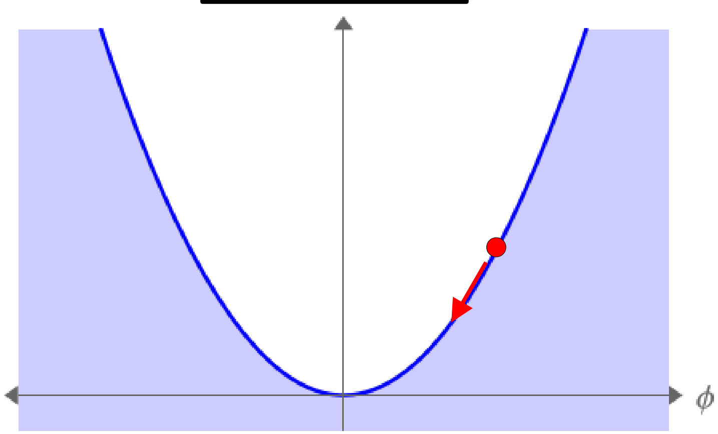

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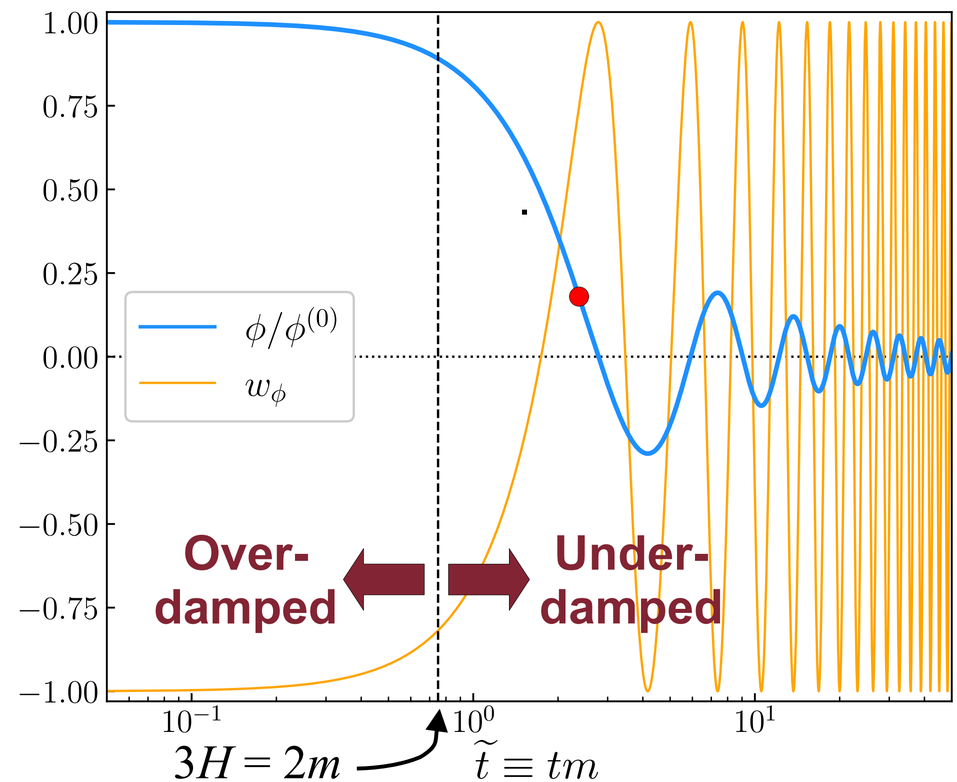

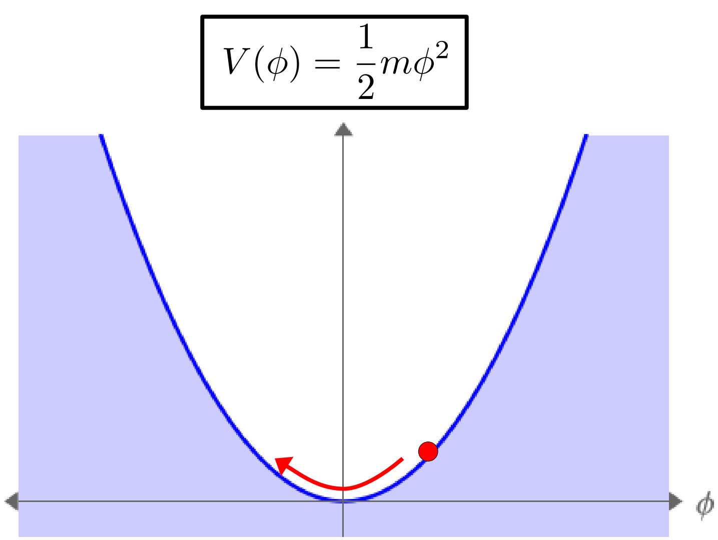

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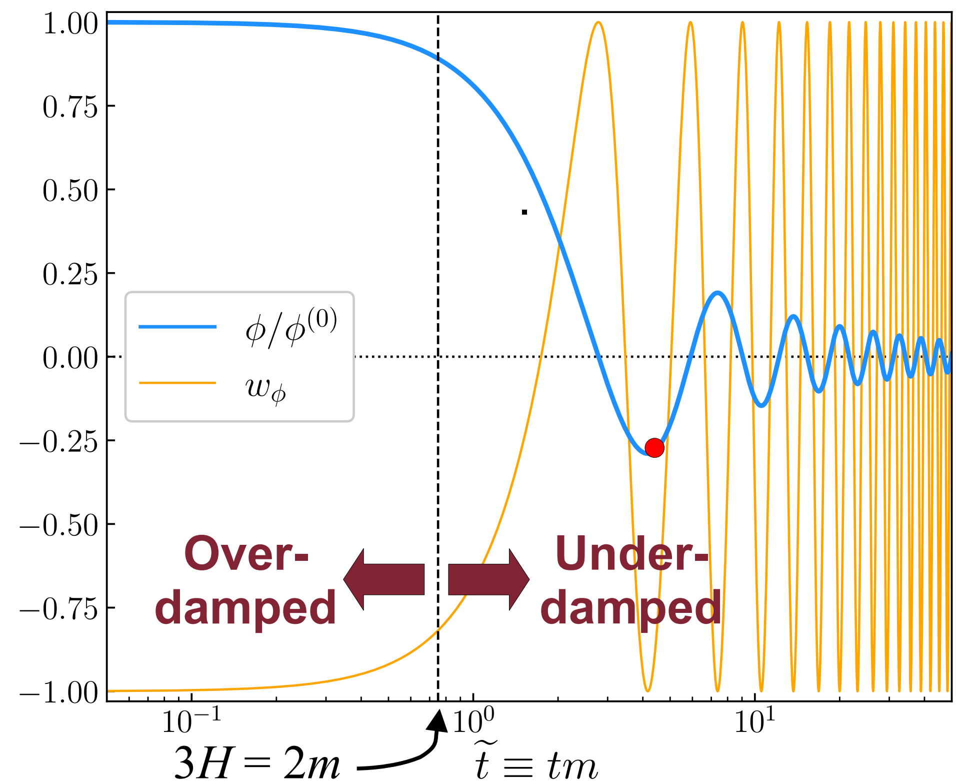

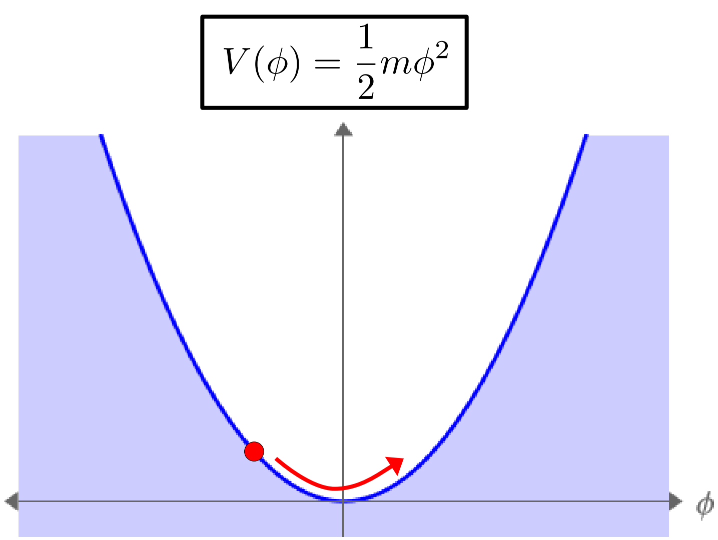

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➡ **Behaves like massive matter**

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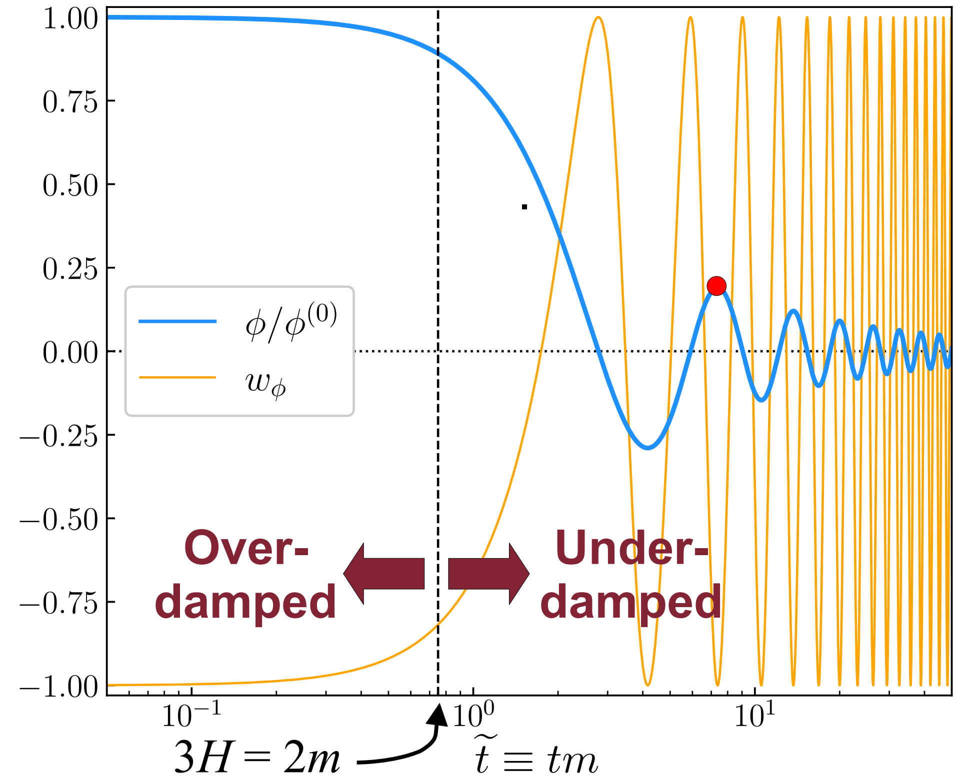

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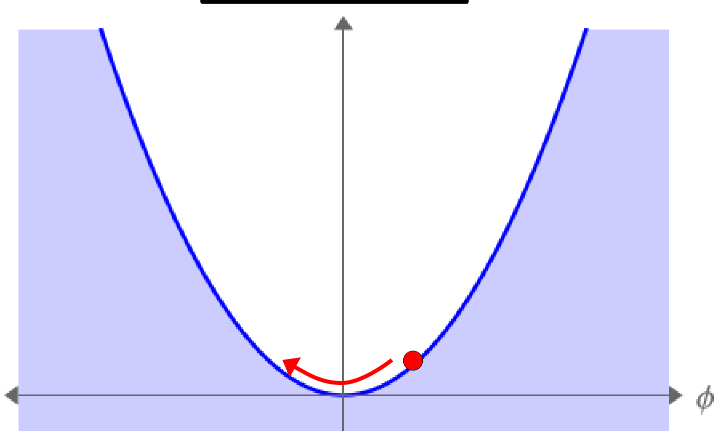

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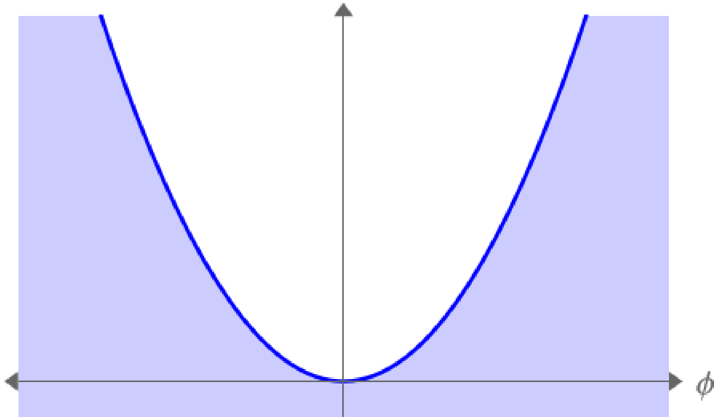

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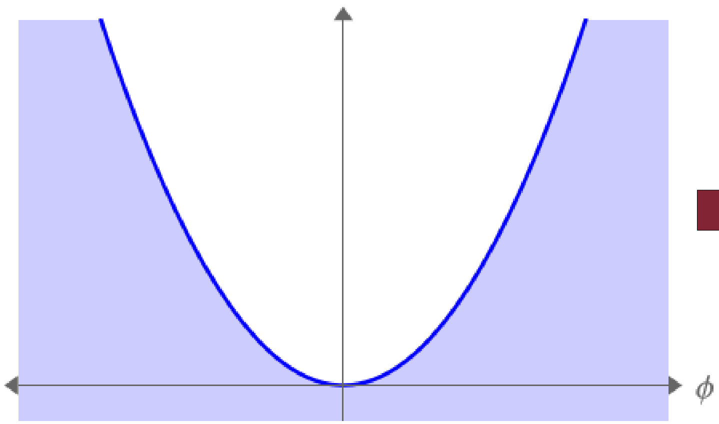

For Each Field

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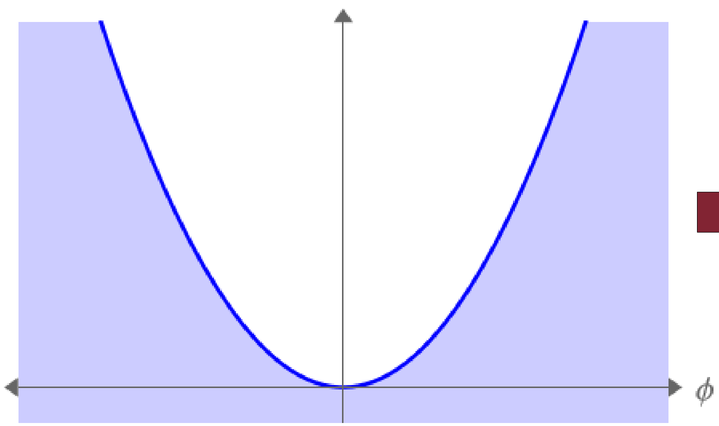

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- We'll also assume (for the moment) that there's **no background energy component**: the collective energy density of the ϕ_ℓ dominates the universe.

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N "copies" of this

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Let's see what the cosmology of such a tower of scalar-field zero modes looks like!

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- All that now remains is to specify the initial conditions for our scalars.

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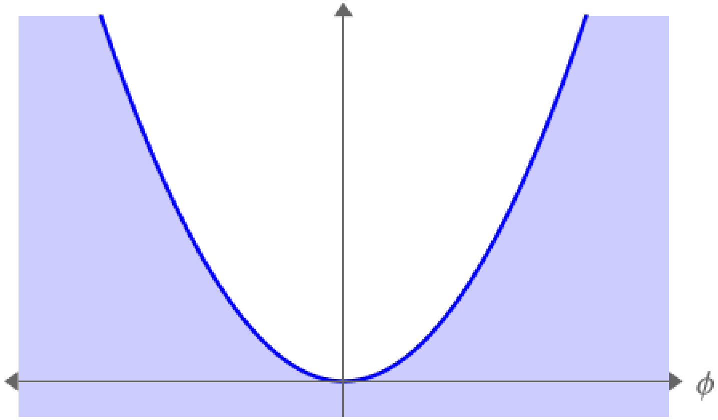

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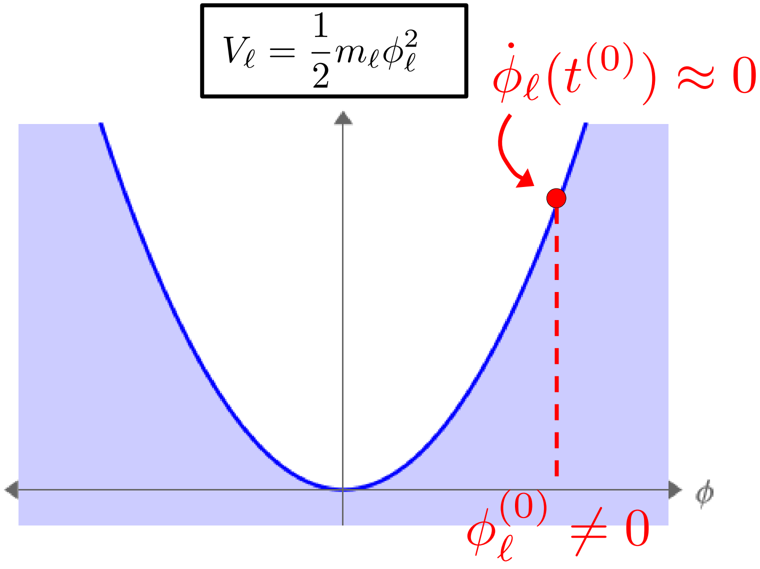

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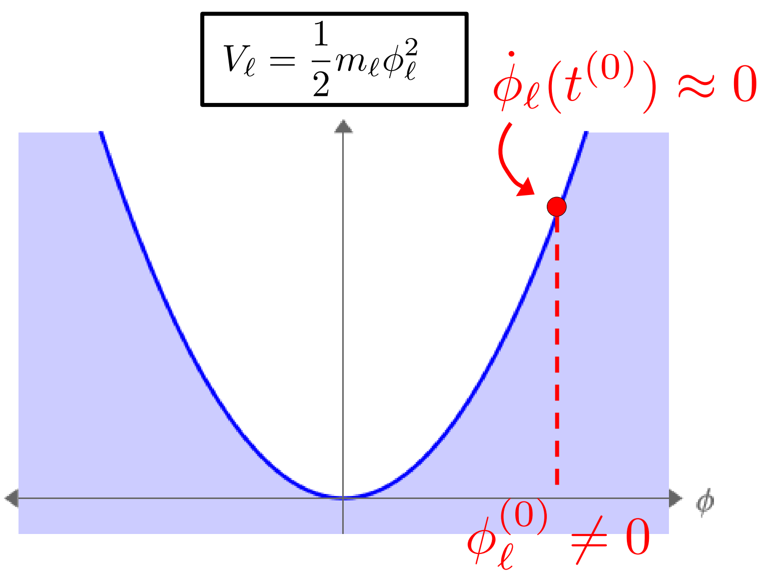

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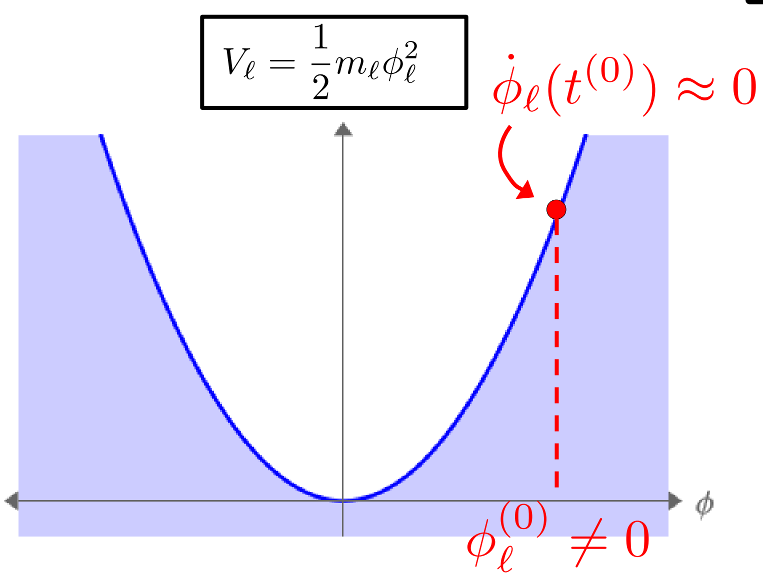

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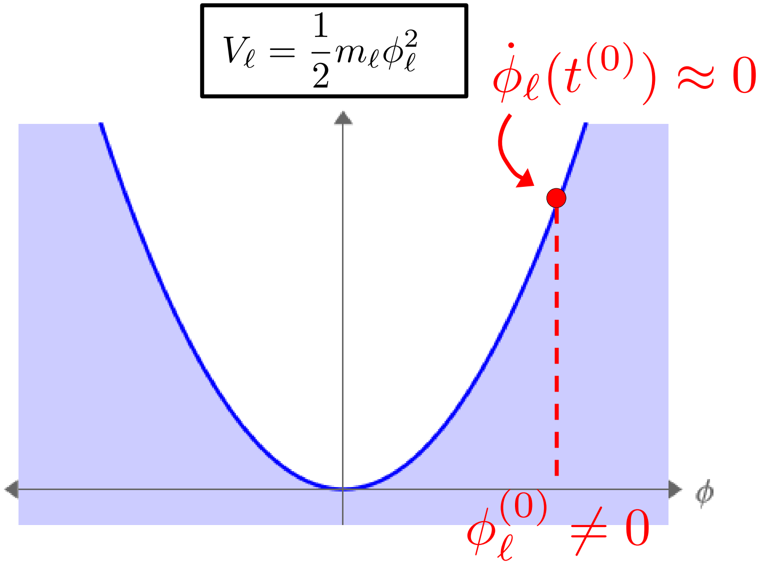

- For a given mass spectrum, the overall scale of the abundances can be parameterized by the ratio $\phi_0^{(0)}/M_P$, or, equivalently, by the ratio $H^{(0)}/m_{N\text{-}1}$.

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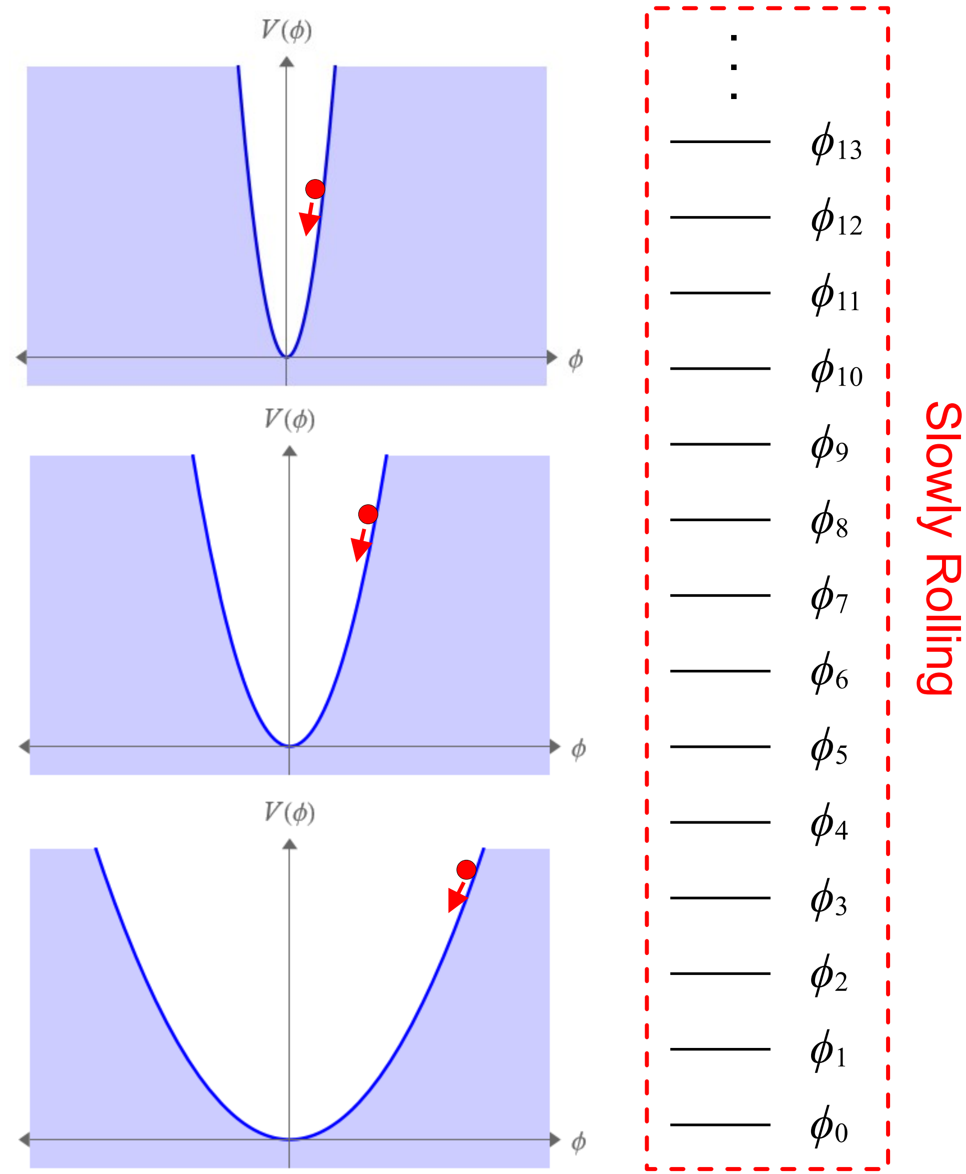

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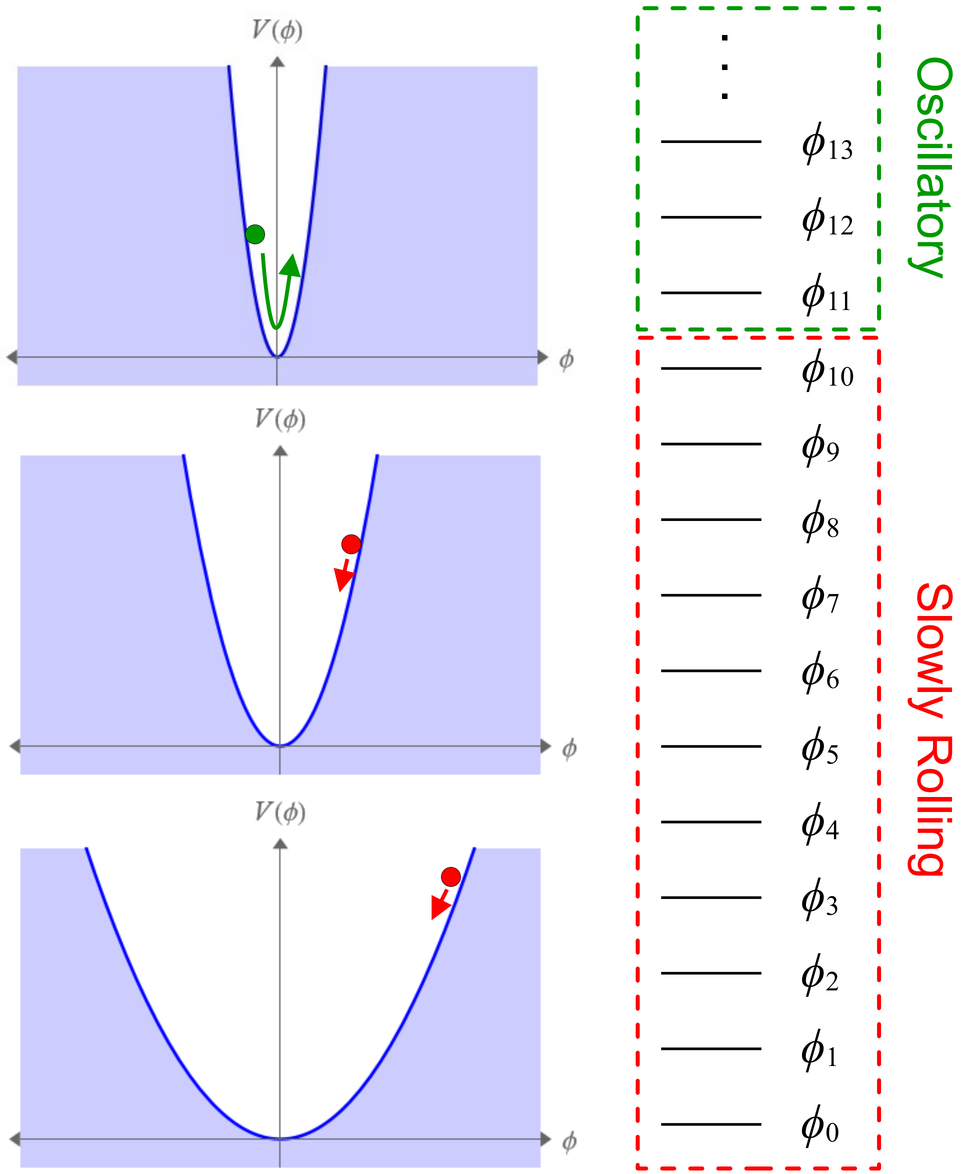

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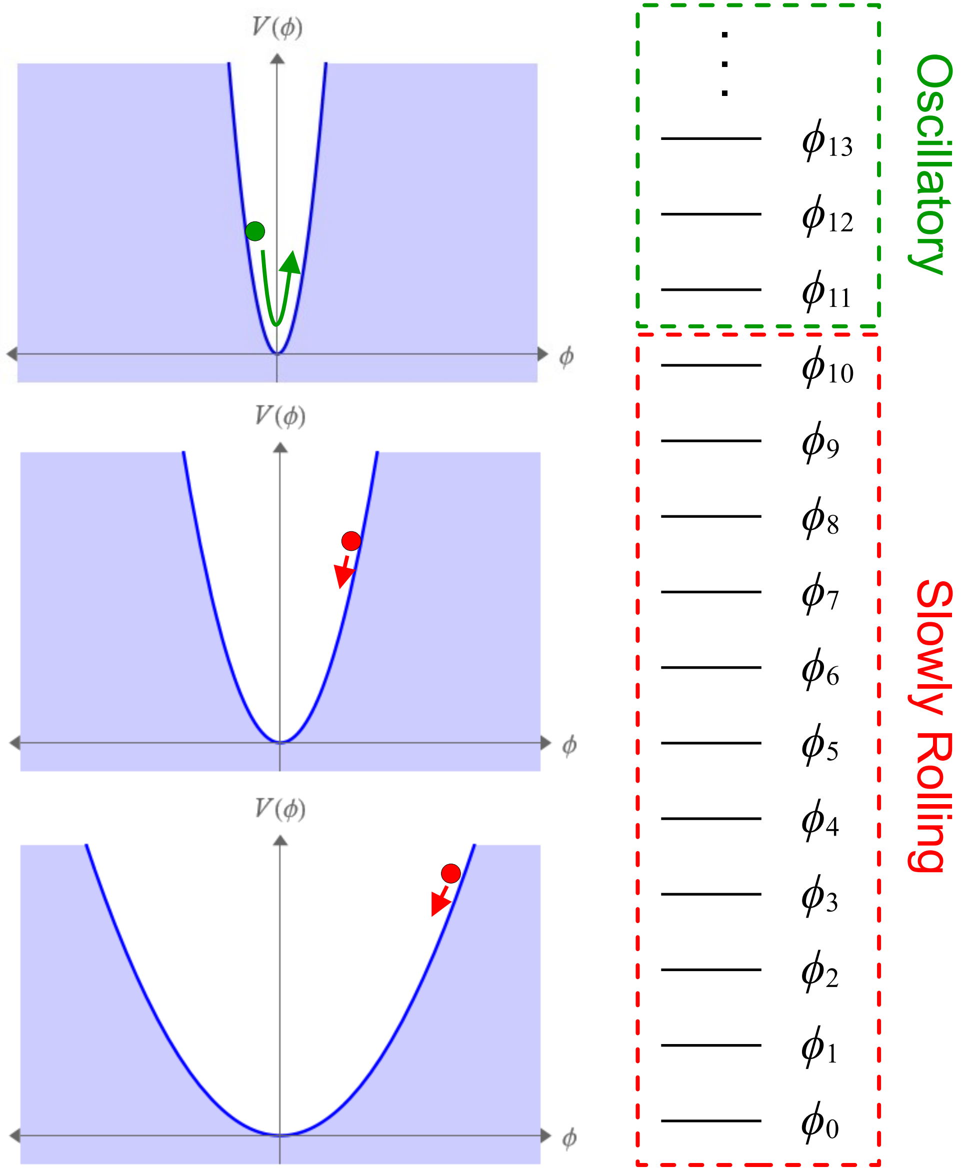

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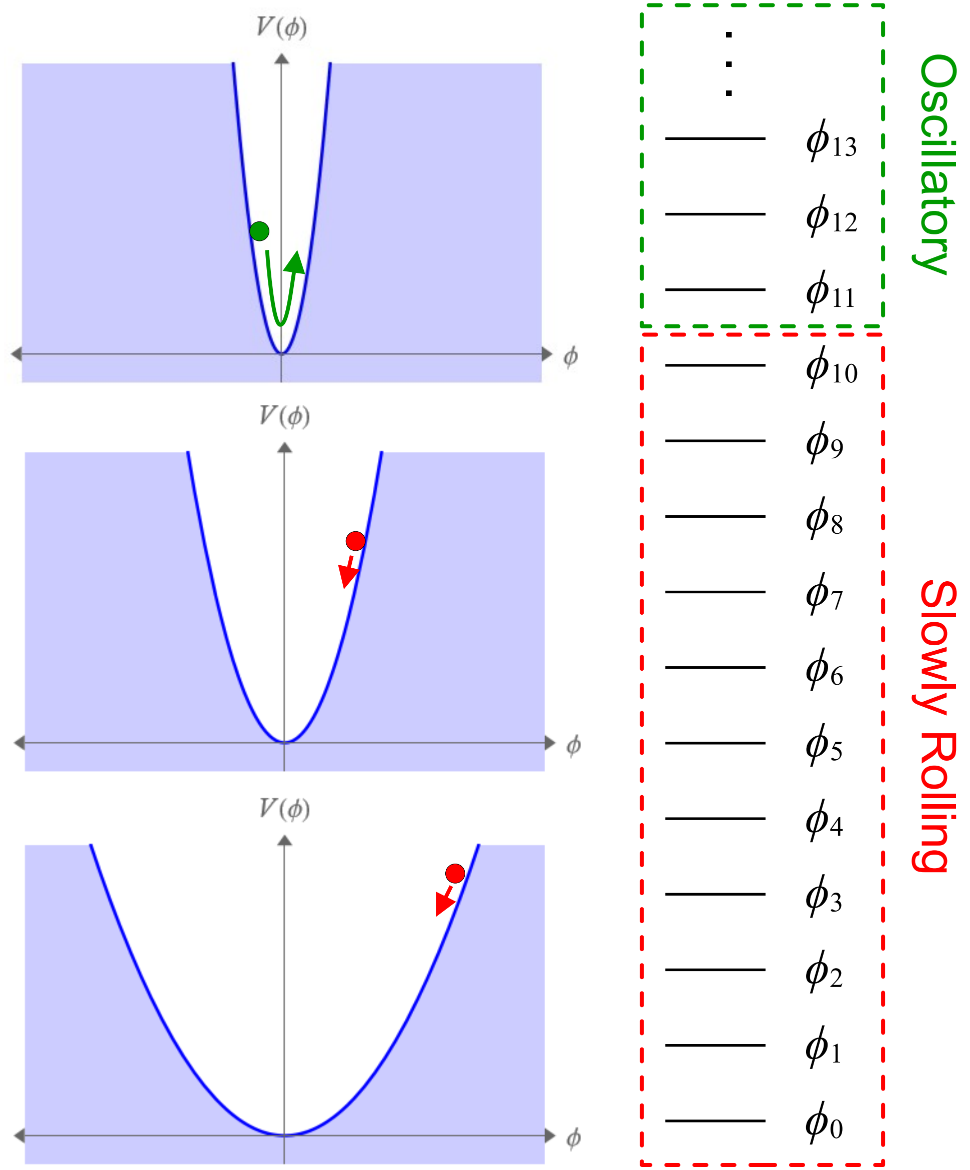
- Thus, we can divide the tower into **two regions**, which we treat as different energy components:



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- Each ϕ_ℓ transitions to the underdamped phase when when $3H(t) = 2m_\ell$.
- As time goes on, increasingly lighter fields begin oscillating
- At any given time t, there is a critical value ℓ_c of ℓ below which the ϕ_ℓ remain overdamped.
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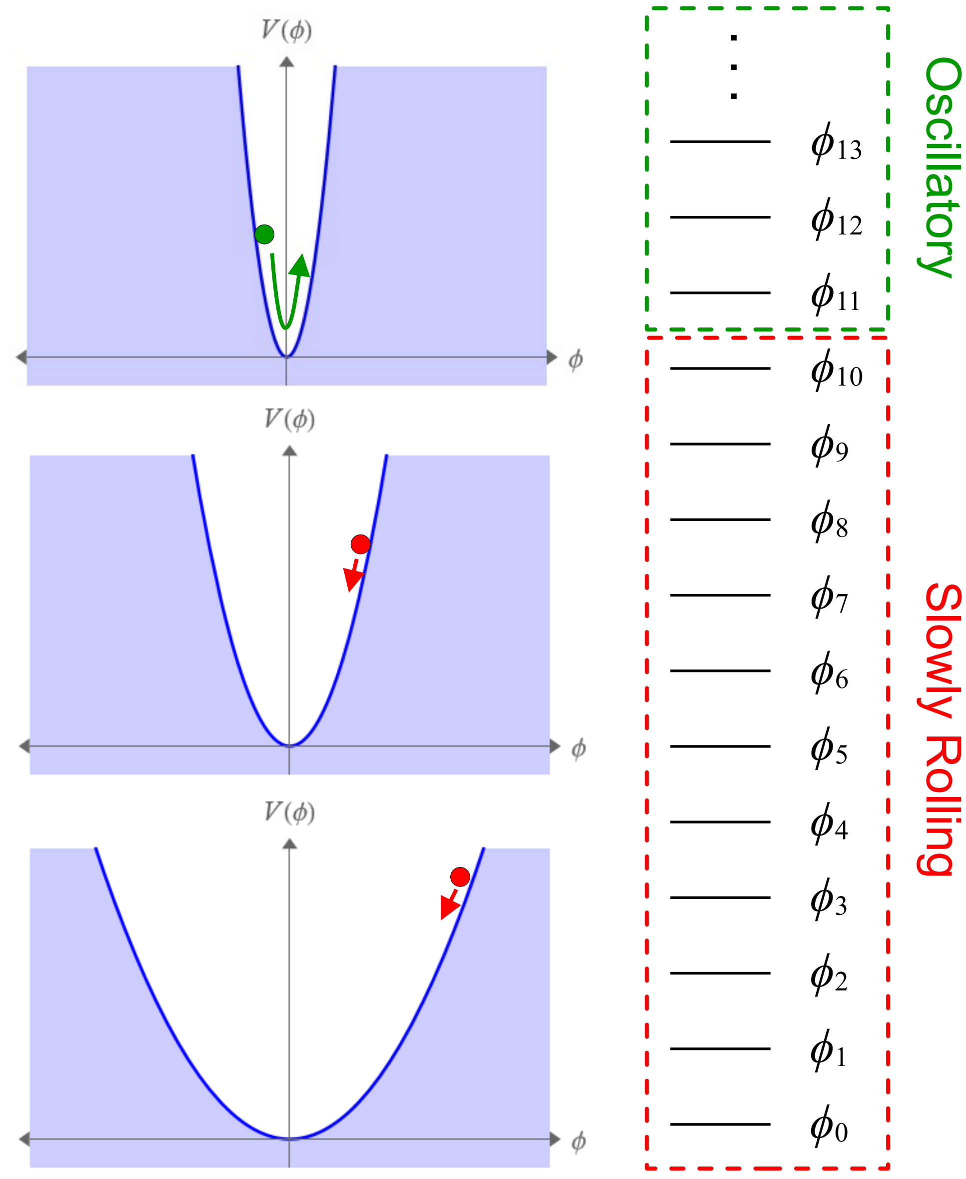

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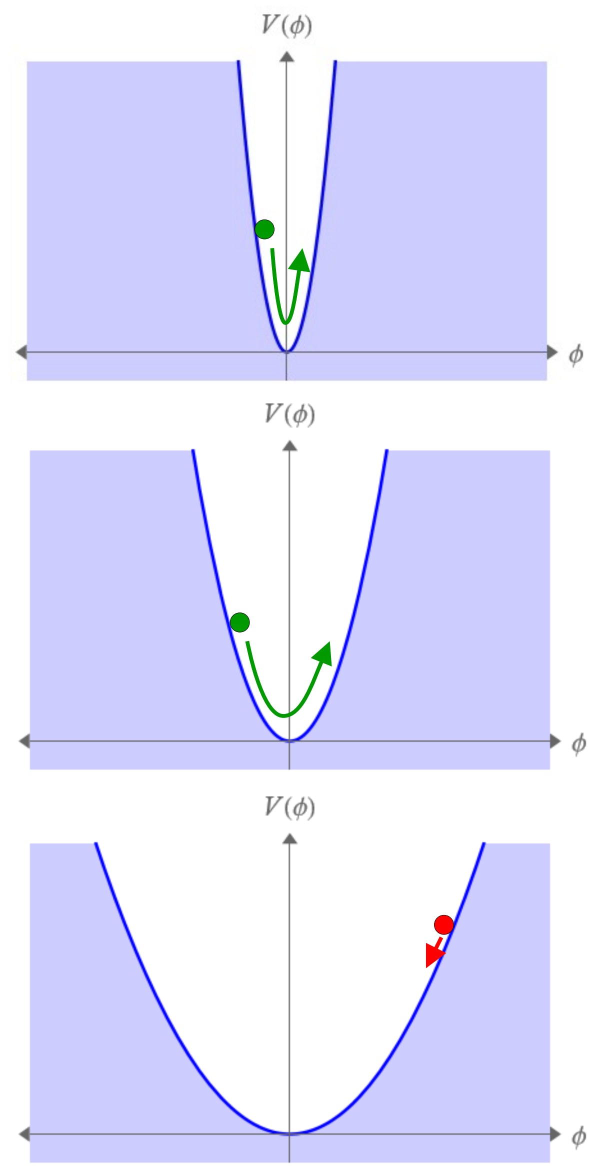

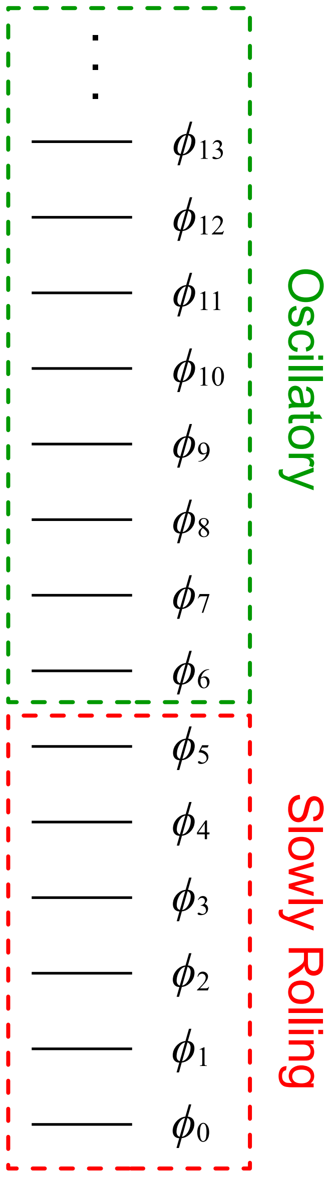

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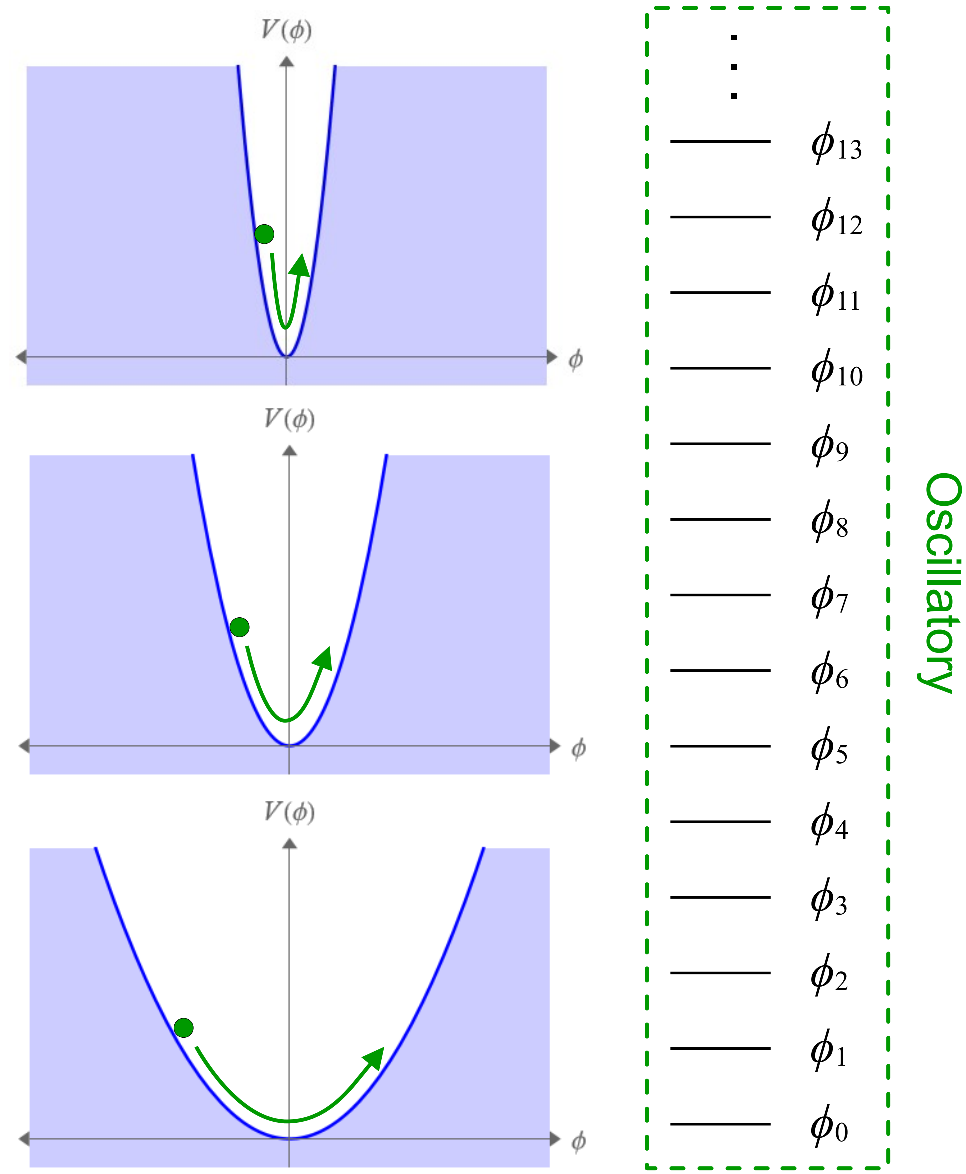

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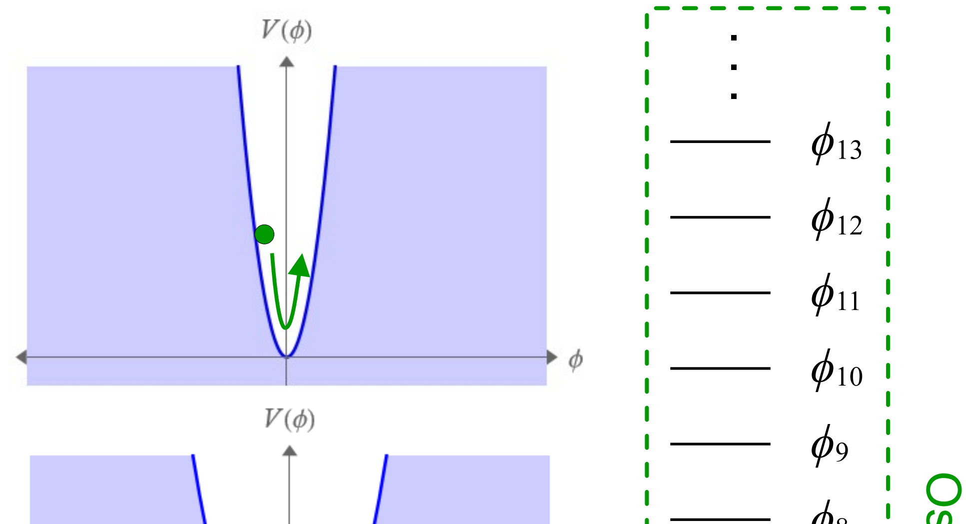

The Question:

Can we achieve a stasis between these slow-roll and oscillatory cosmological energy components, which act like vacuum energy and matter, respectively?

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Towers which satisfy this relation give rise to stasis. For any δ, this corresponds to $\phi_\ell^{(0)} \sim \ell^{-1/2}$

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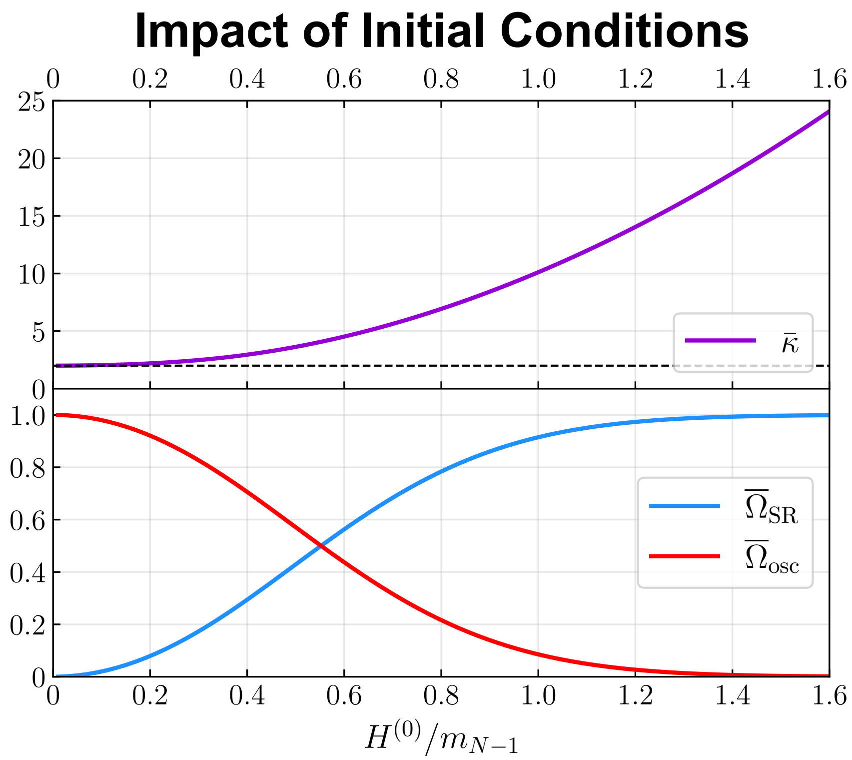

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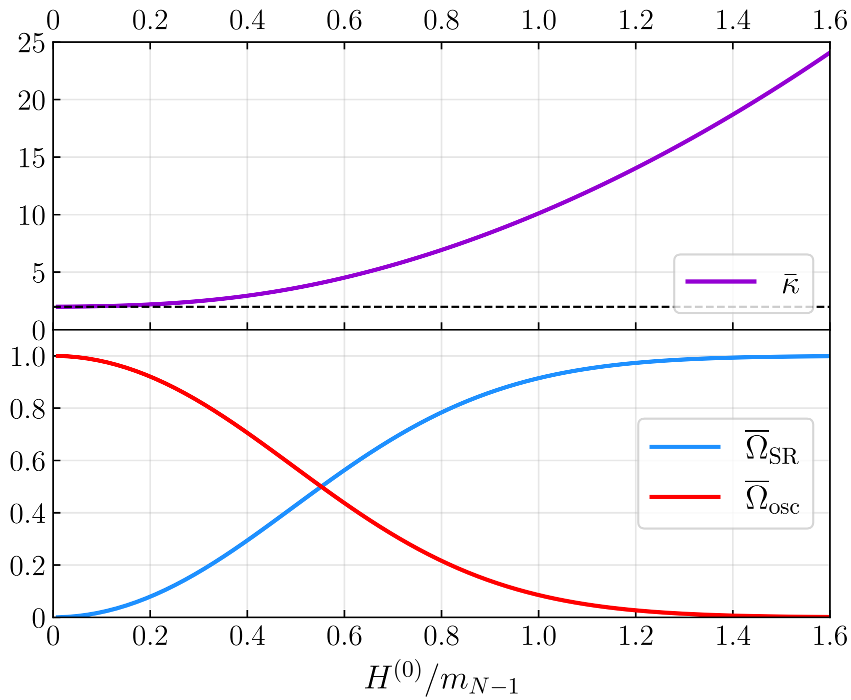

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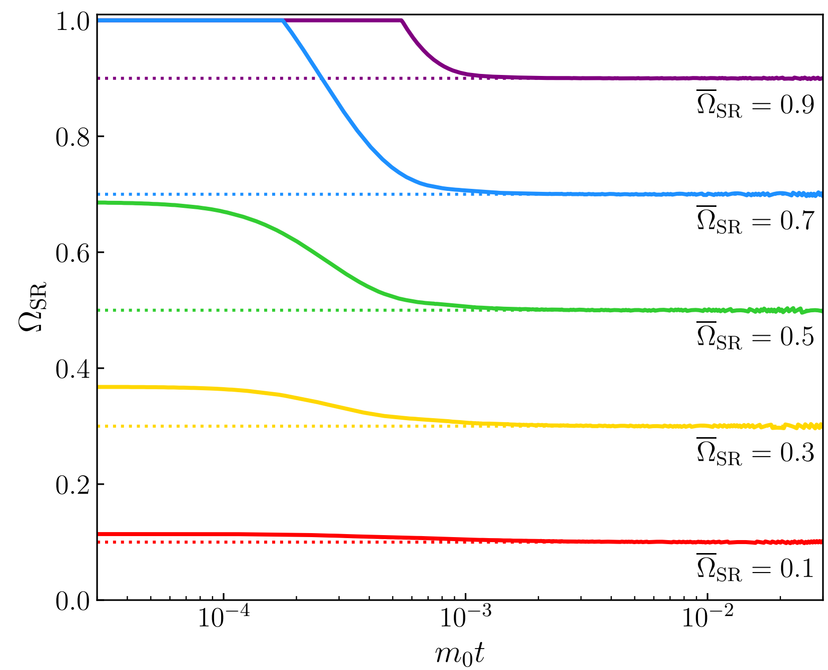

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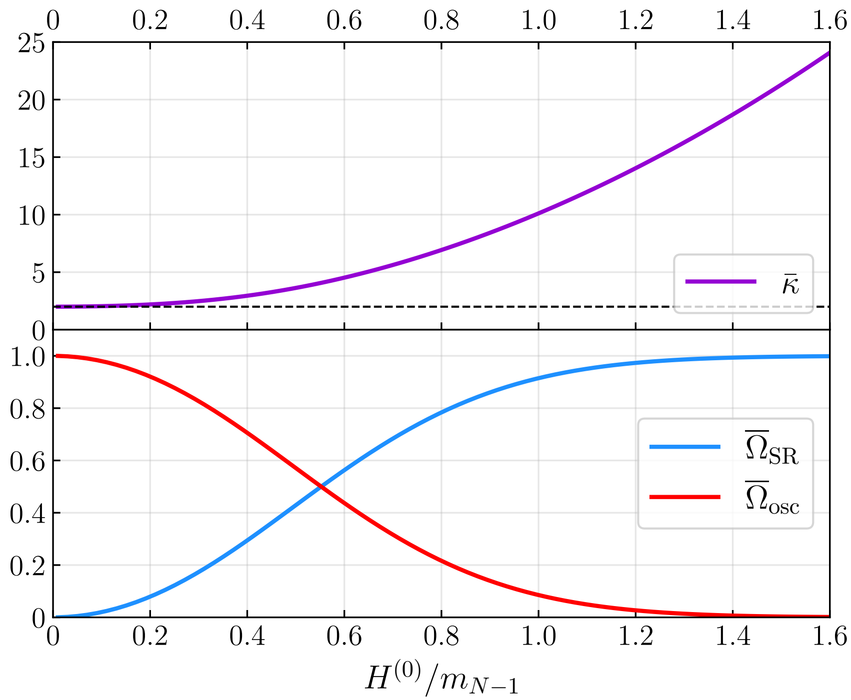

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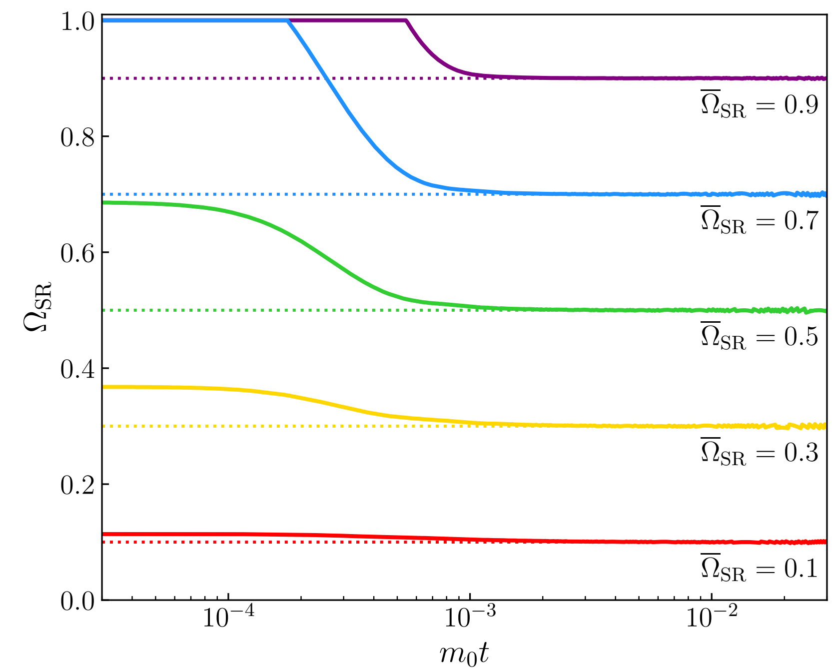

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Duration of Stasis

$$\mathcal{N}_s \approx \frac{\overline{\kappa}}{3}\left[\delta \log N + \log\left(\frac{\Delta m}{m_0}\right) + \log\left(\frac{3H^{(0)}}{2m_{N-1}}\right)\right]$$

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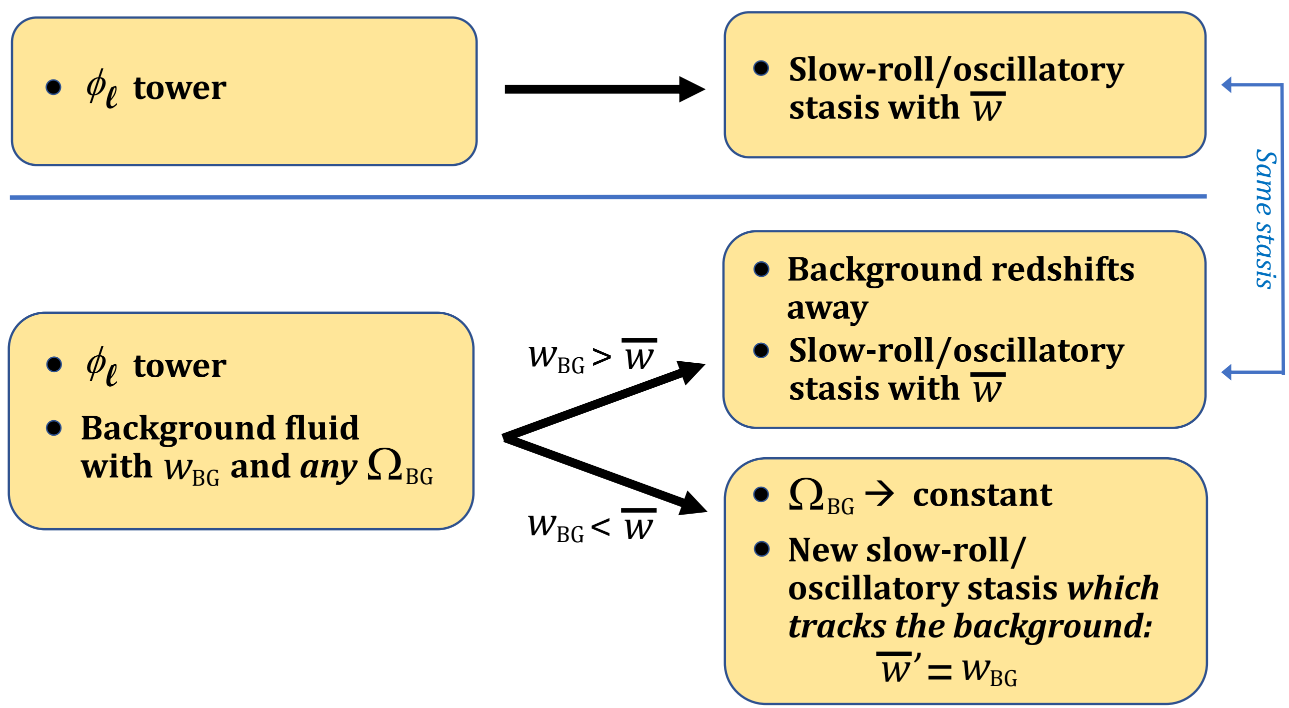
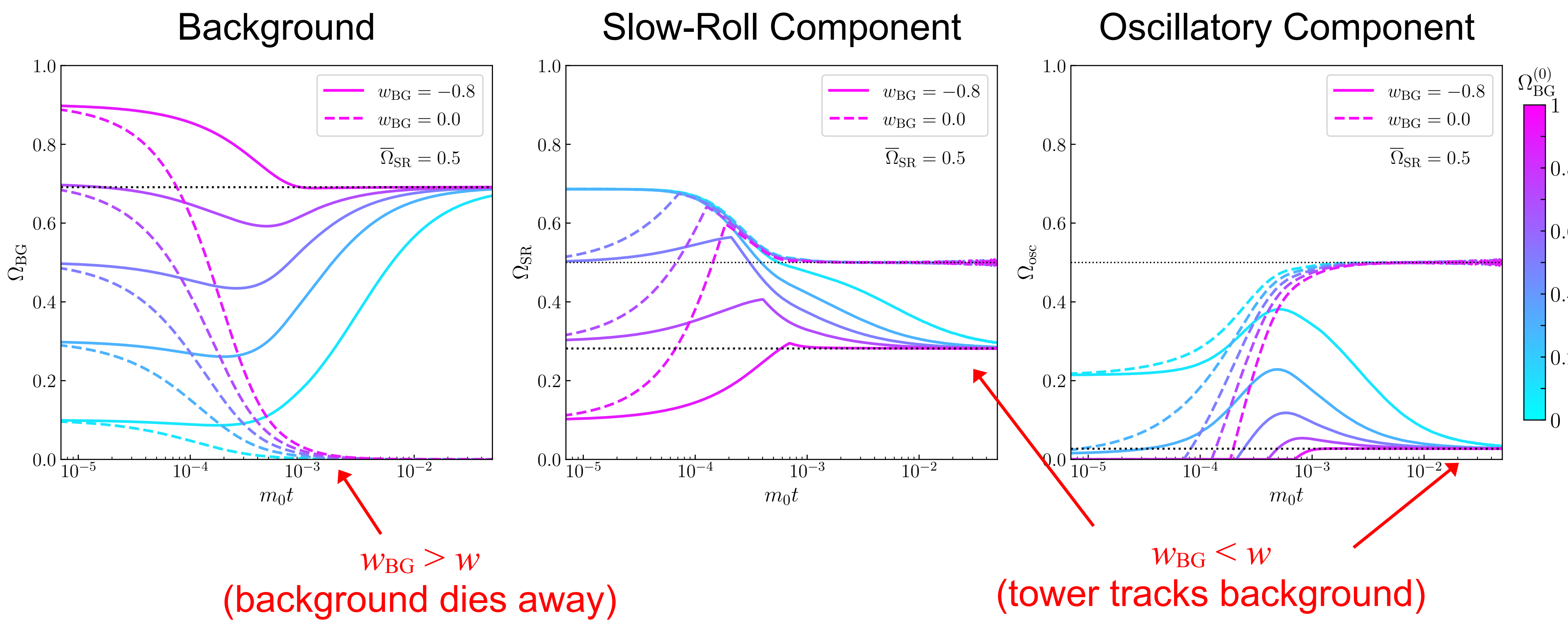
- It turns out that in the presence of such an energy component, the universe still evolves toward stasis (or something like it).
- However, the outcome depends on the relationship between $w_{\rm BG}$ and the equation-of-state parameter $\overline{w}$ the tower *would* have had during stasis if the background component weren't present.



Background Components and Tracking

- The **tracking phenomenon** which arises in situations in which $w_{\text{BG}} < \overline{w}$ has not been observed in other realizations of stasis.



- These results provide insight about how the universe might **enter into** – or exit from – an stasis epoch involving dynamical scalars.

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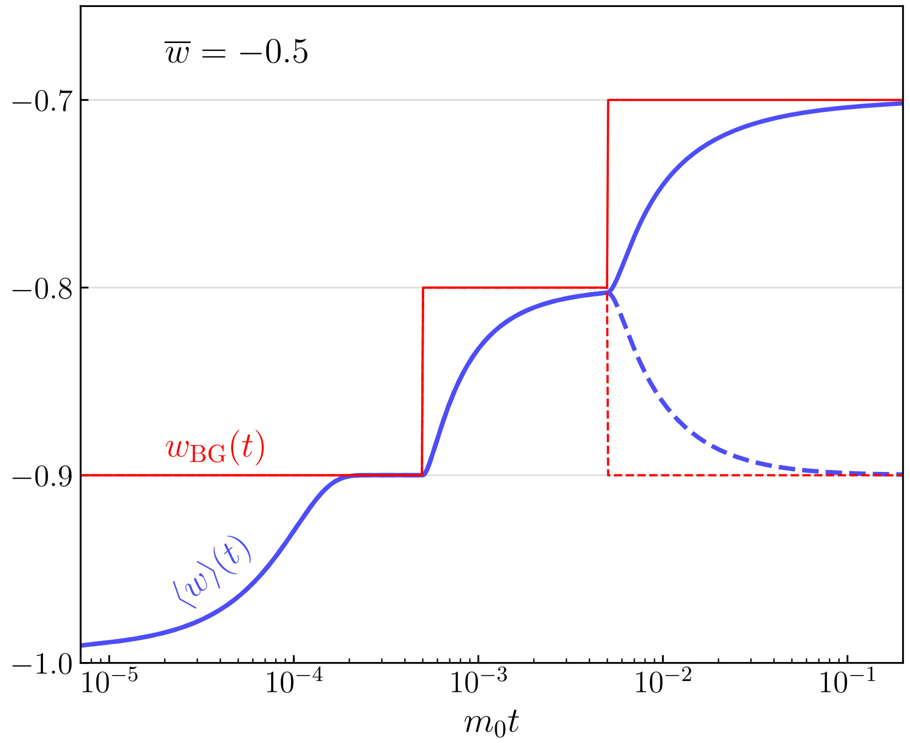
- It's also noteworthy that this tracking behavior is quite robust and persists even when $w_{\rm BG}$ **experiences an abrupt shift** (as might occur, for example, as the result of a phase transition).



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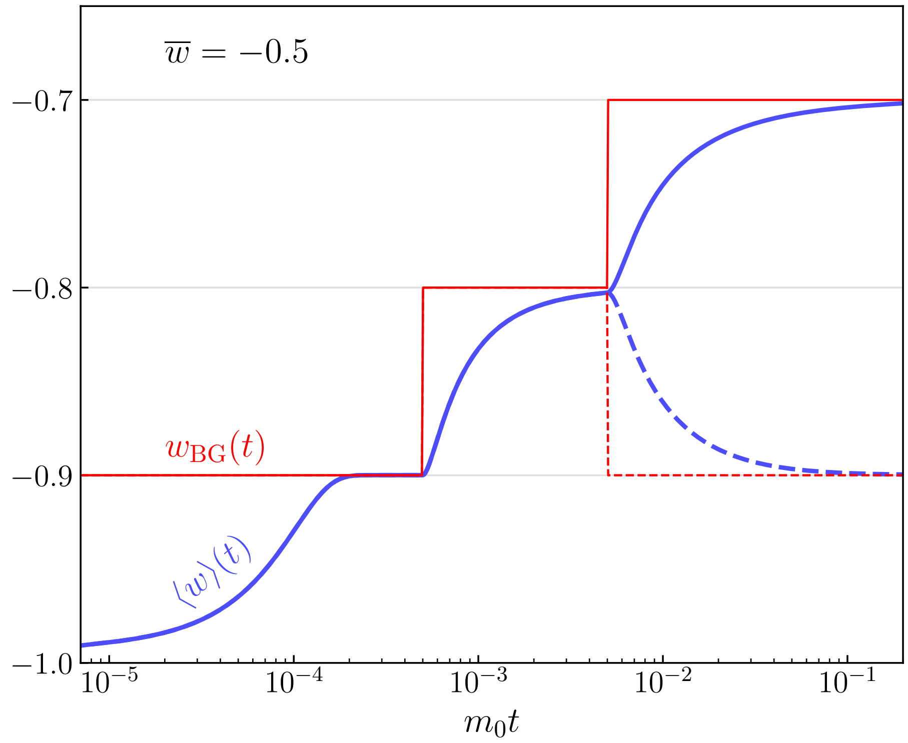

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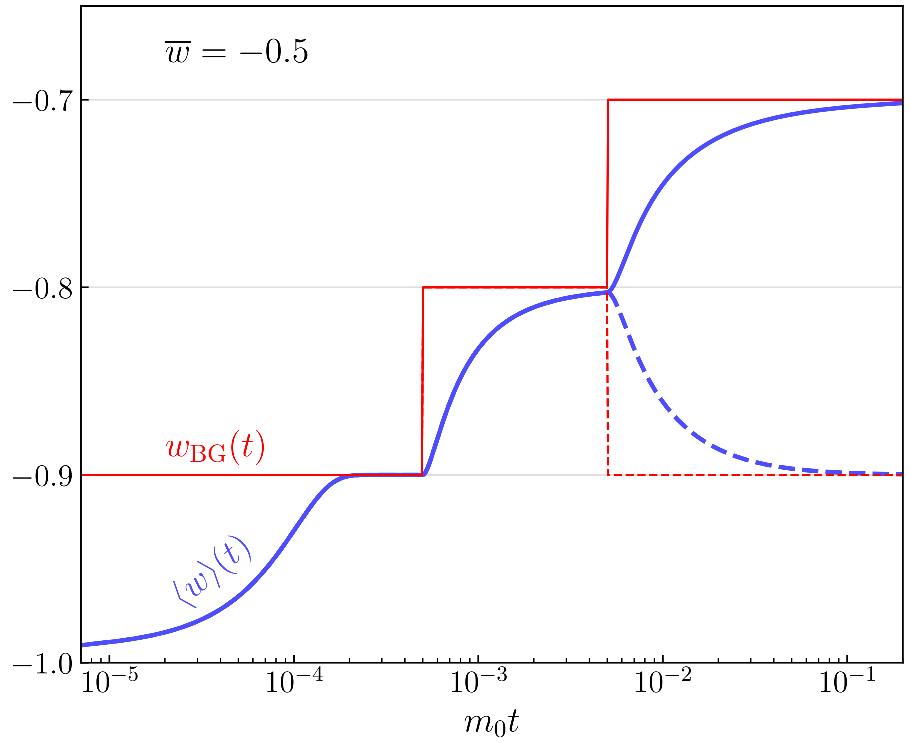

- Moreover, $\langle w \rangle$ tracks $w_{\rm BG}$ even in the regime in which $w_{\rm BG}$ **evolves continuously** with time.

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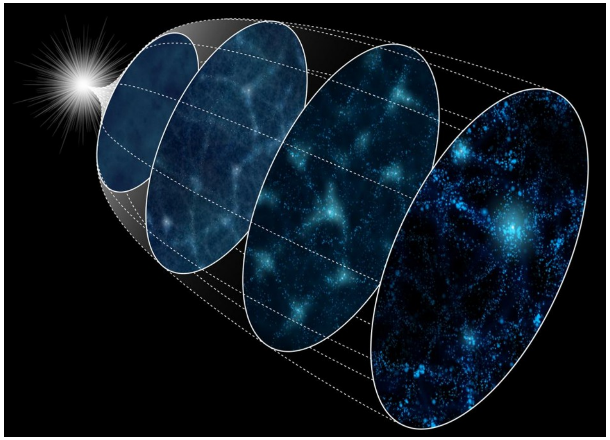

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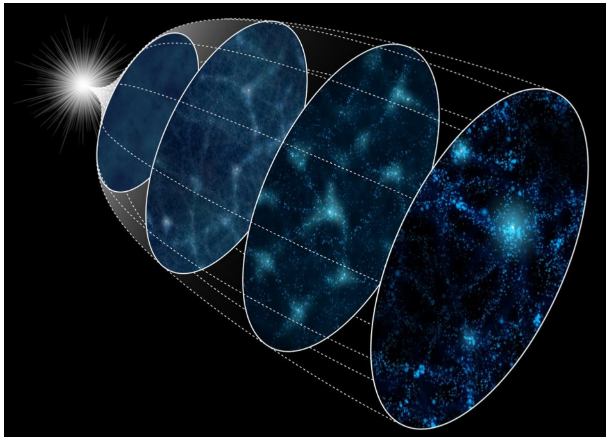

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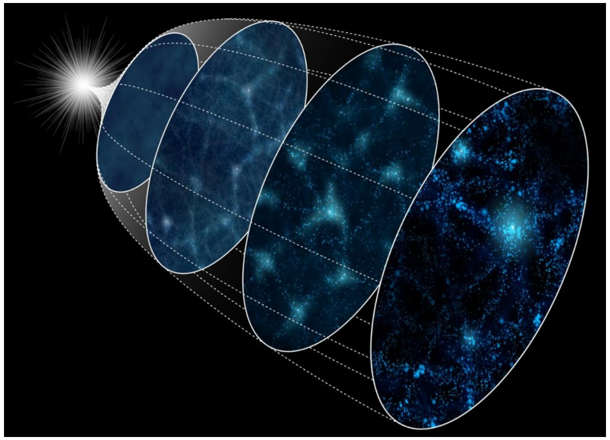

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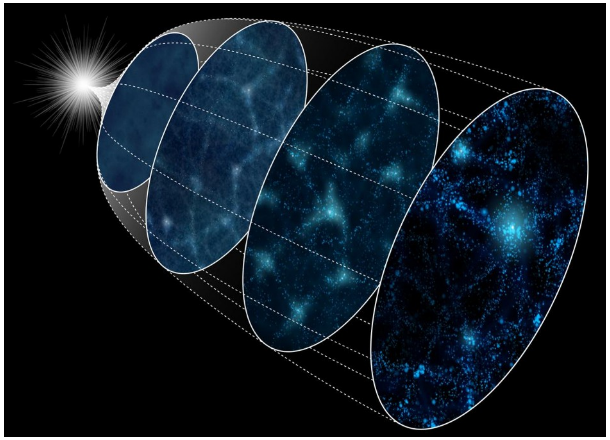

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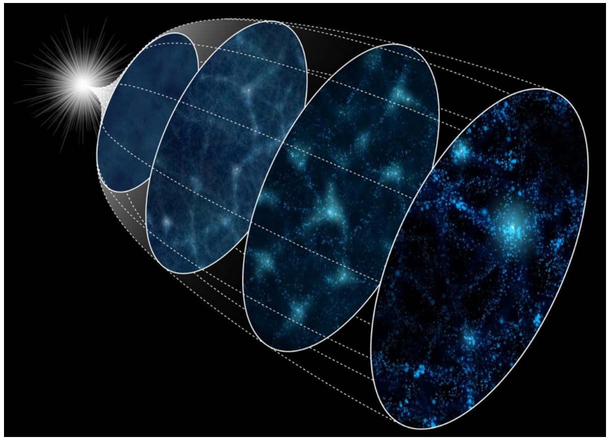

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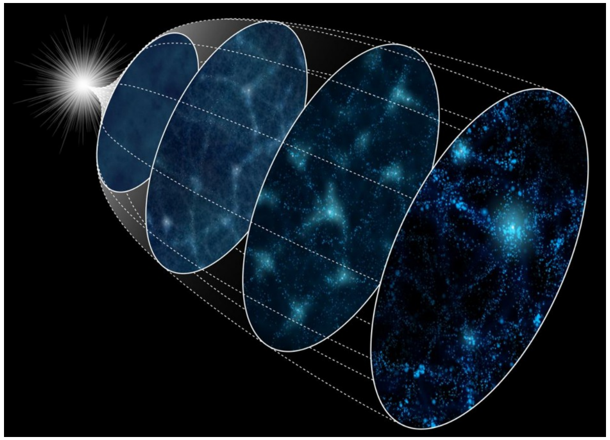

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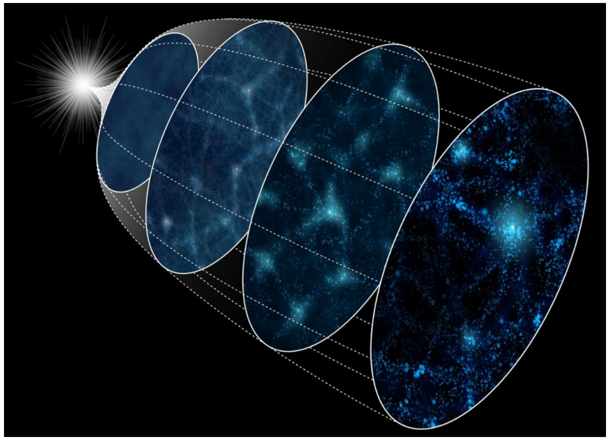

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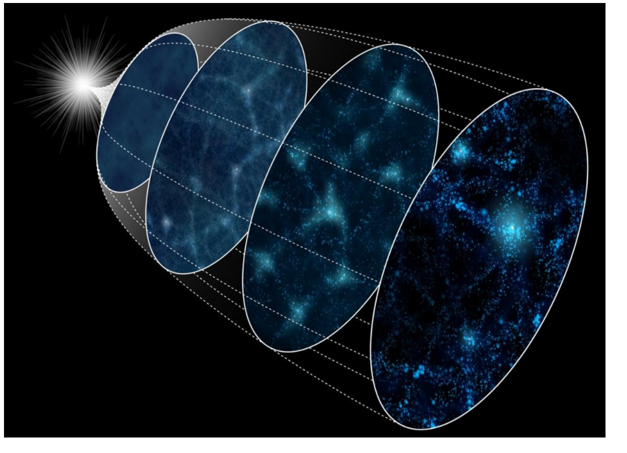

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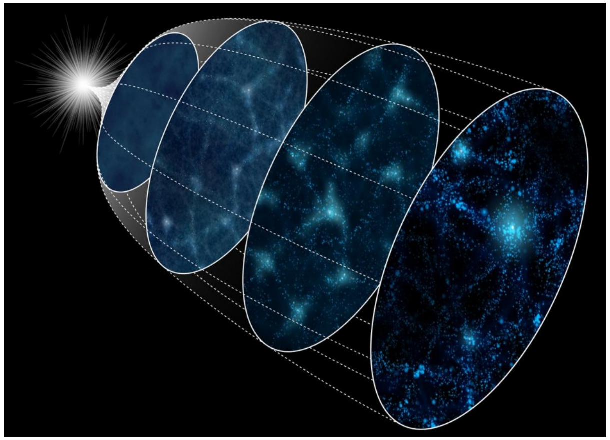

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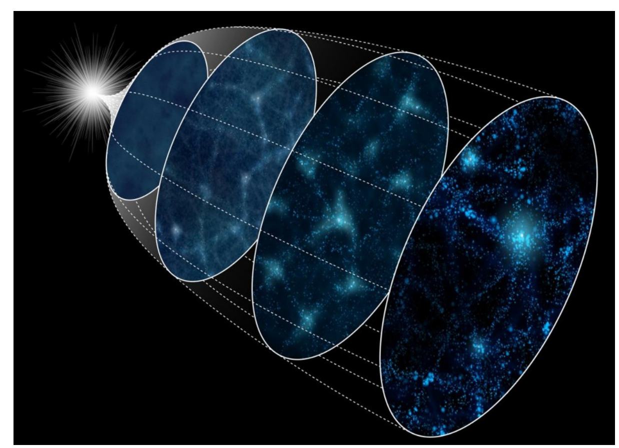

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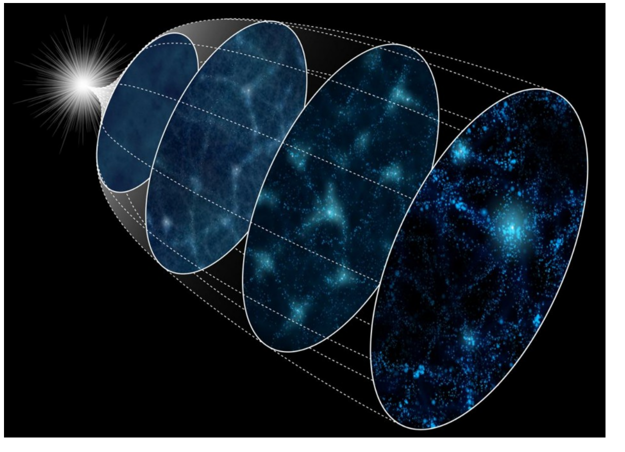
This is an intriguing possibility – and one that warrants further exploration!



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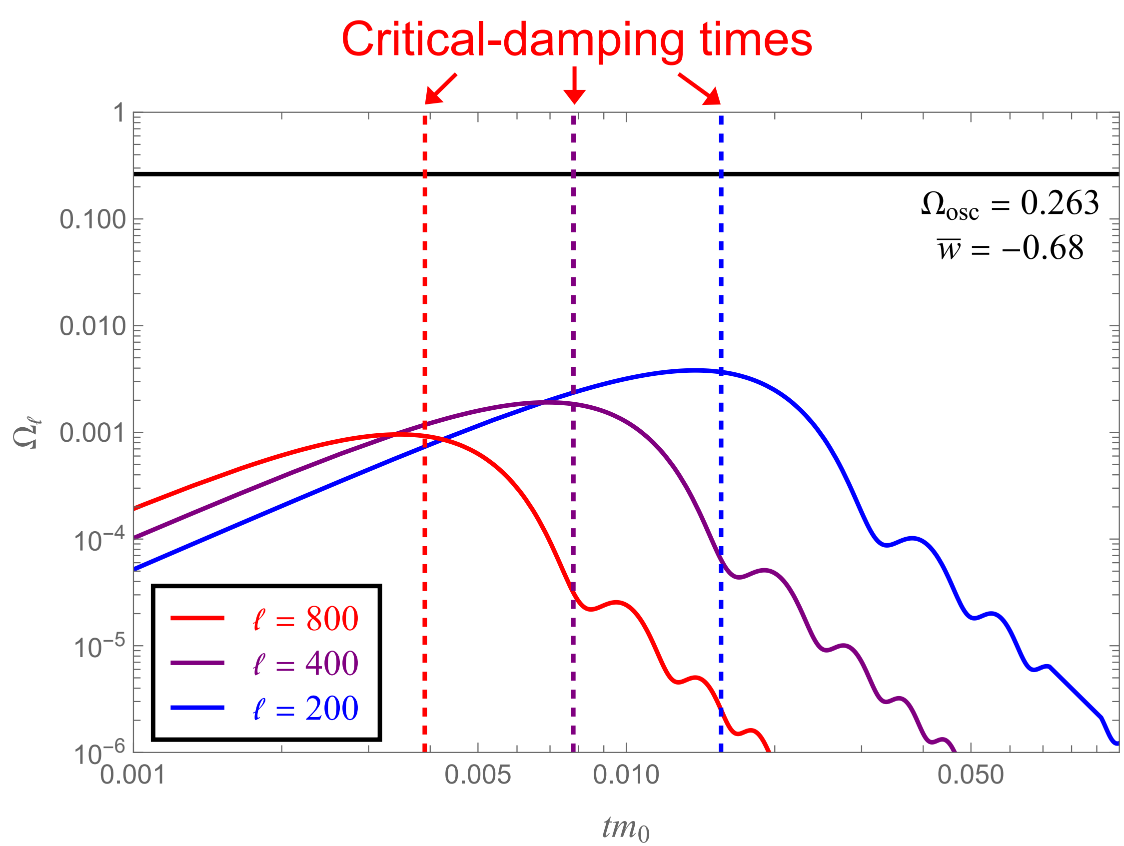

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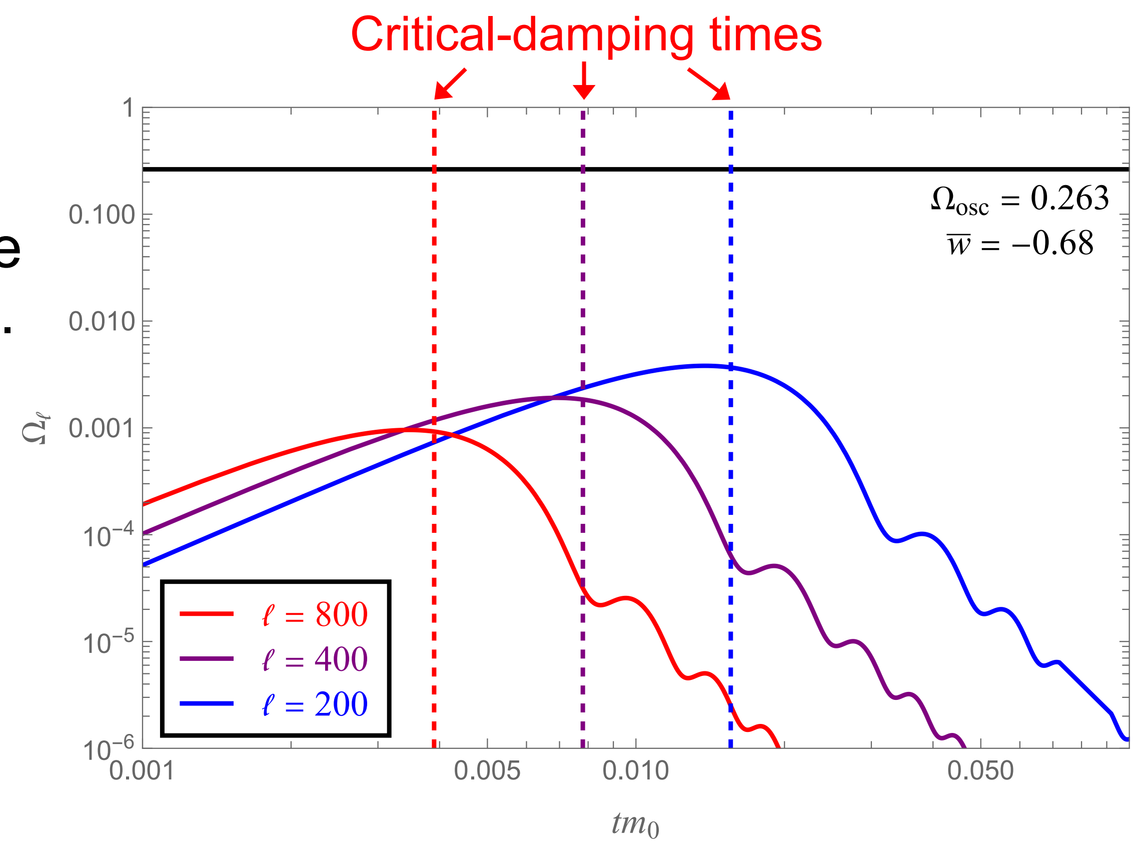
- The abundance Ω_ℓ of each ϕ_ℓ initially rises while it's slowly rolling, but then **falls precipitiously** once its critical-damping time is reached.
- In this way, stasis inflation features an "**undertow**" which suppresses the abundances of unwanted relics and isocurvature contributions from the heavier ϕ_ℓ.



Summary

- ***Stable, mixed-component cosmological eras*** – *i.e.* ***stasis eras*** – are indeed a viable cosmological possibility – and one that can arise naturally in many extensions of the Standard Model.
- A **tower of scalar fields** which undergo a transition from overdamped to underdamped evolution can give rise to stasis.
- Stasis itself is an **attractor** in these systems, but several fundamental characteristics of the stasis epoch toward which the universe evolves depend on the initial conditions.
- In the presence of an additional background component with equation-of-state parameter $w_{\rm BG}$, the tower exhibits a **tracking behavior** in which its own equation-of-state parameter evolves toward $w_{\rm BG}$.