

Gauged Global Strings

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Outline

- Introduction
global strings and gauge strings
- Gauge $U(1)_Z \times$ global $U(1)_{PQ}$
and **string solutions**
- Cosmological implication
 - 1) **rich string structure / dynamics**
 - 2) **opening up QCD axion window**
 - 3) **gauge string radiating axions?**
- Conclusion

Global strings

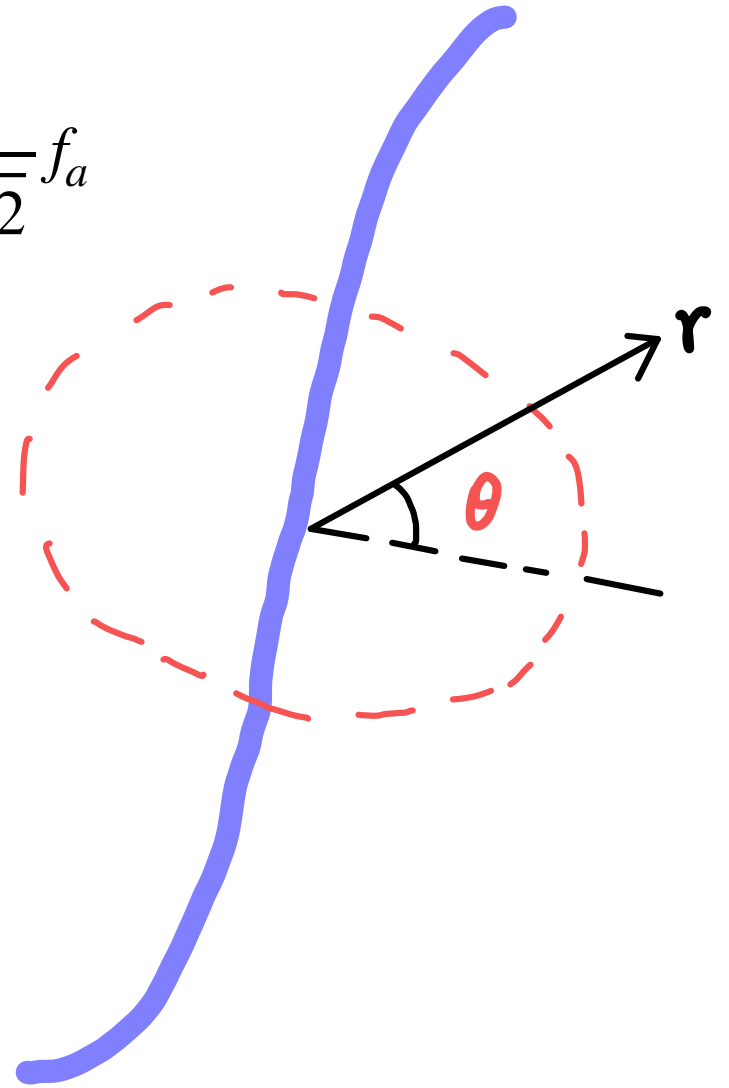
- global U(1) symmetry breaking $\langle \Phi \rangle = \frac{1}{\sqrt{2}} f_a$

- global string solution

$$\Phi(r, \theta) = \frac{1}{\sqrt{2}} f_a e^{i\theta}, \quad r \rightarrow \infty$$

- tension

$$\text{gradient term } \mu \simeq 2\pi \int_{m^{-1}}^L dr \frac{1}{r} |\partial_\theta \Phi(r, \theta)|^2 = \pi f_a^2 \ln(mL)$$

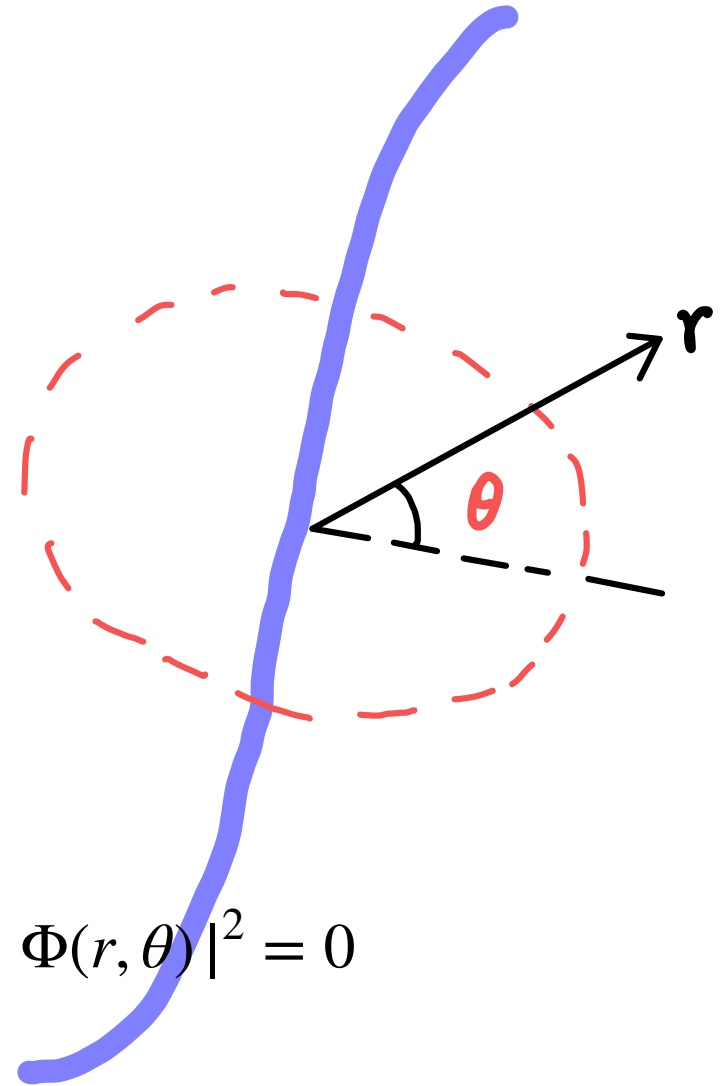


Gauge strings

- gauge string solution

$$\Phi(r, \theta) = \frac{1}{\sqrt{2}} f_a e^{i\theta}$$

$$Z_\mu = \frac{1}{e} \partial_\mu \theta \quad r \rightarrow \infty$$

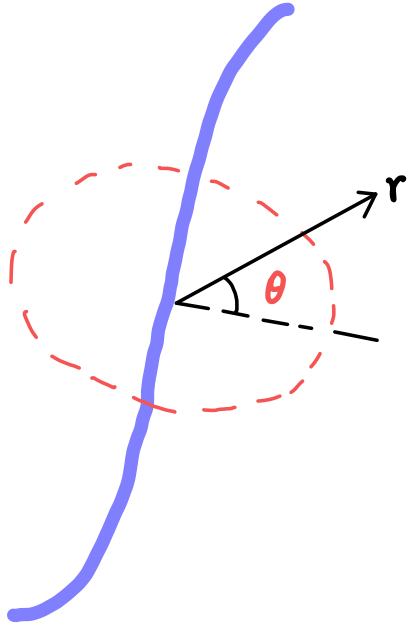


- tension

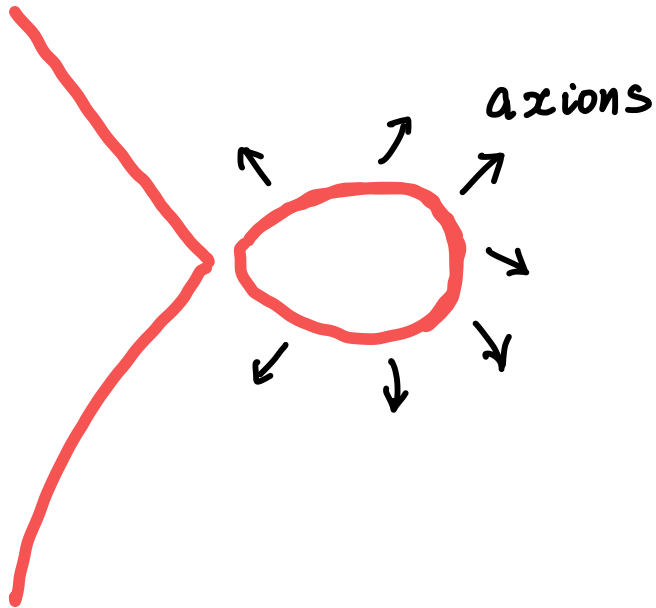
gradient term $\mu \simeq 2\pi \int_{m^{-1}}^L dr \left| \left(\frac{1}{r} \partial_\theta - ieZ_\mu \right) \Phi(r, \theta) \right|^2 = 0$

core $\mu \simeq \mathcal{O}(1) \pi f_a^2$

Motivation of cosmic strings



- theoretically interesting classical field solutions



- phenomenological rich cosmology (Kibble mechanism)
axion dark matter abundance
new observables (CMB, ...)

$U(1)_Z \times U(1)_{PQ}$

- Lagrangian

$$\mathcal{L} = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + D_\mu\Phi_1^\dagger D^\mu\Phi_1 - \frac{\lambda_1}{4}\left(|\Phi_1|^2 - \frac{v_1^2}{2}\right)^2 + D_\mu\Phi_2^\dagger D^\mu\Phi_2 - \frac{\lambda_2}{4}\left(|\Phi_2|^2 - \frac{v_2^2}{2}\right)^2$$

$$D_\mu = \partial_\mu - ieZ_\mu$$

assume that $v_1 > v_2$

- $\Phi_1 \rightarrow \Phi_1 e^{i\alpha_Z + i\alpha_{PQ}}$

- $\Phi_2 \rightarrow \Phi_2 e^{i\alpha_Z - i\alpha_{PQ}}$

	Φ_1	Φ_2
$U(1)_Z$	1	1
$U(1)_{PQ}$	1	-1

Vacuum and fluctuations

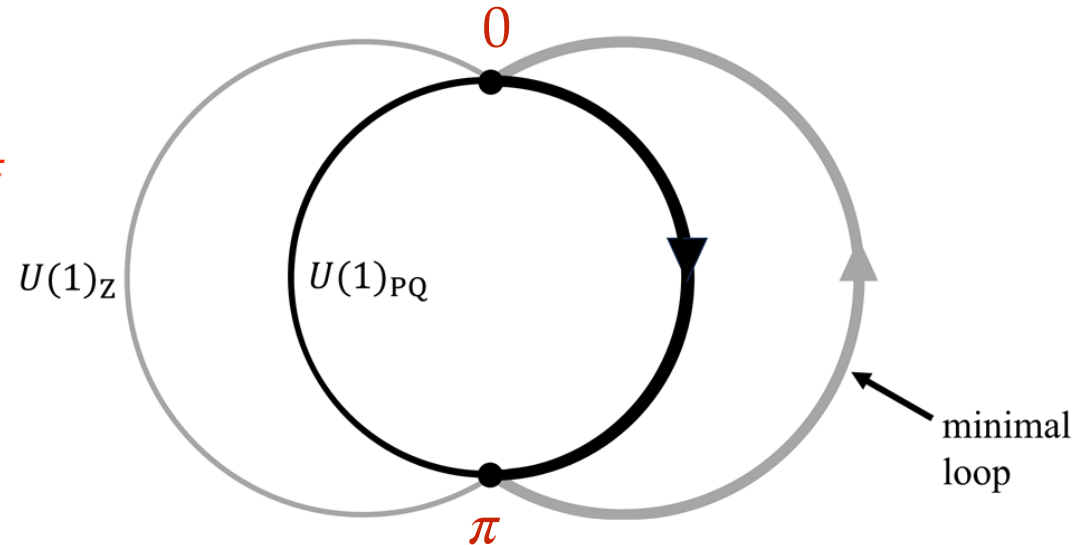
- cross section of vacuum manifold

$$\alpha_Z = \pi$$

$$\Phi_1 \rightarrow \Phi_1 e^{i\pi}, \Phi_2 \rightarrow \Phi_2 e^{i\pi} = \Phi_2 e^{-i\pi}$$

$$\sim \alpha_{PQ} = \pi$$

$$\Phi_1 \rightarrow \Phi_1 e^{i\pi}, \Phi_2 \rightarrow \Phi_2 e^{-i\pi}$$



- axion direction is orthogonal to the longitudinal mode of Z^μ

$$a(x) = v_a \alpha_{PQ}, \quad v_a = \frac{2v_1 v_2}{\sqrt{v_1^2 + v_2^2}} \sim 2v_2$$

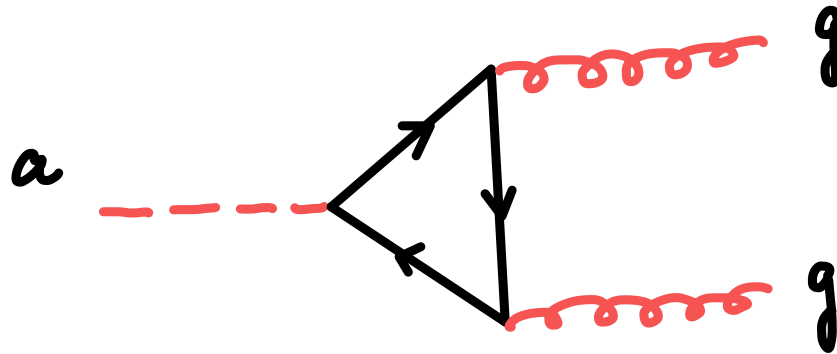
Integrating with QCD axion model

- KSVZ-like model

introduce Q_L and Q_R with color charge and $U(1)_{PQ}$ charge

$$\mathcal{L} = -\frac{y}{\Lambda} \left(\Phi_1 \Phi_2^* \bar{Q}_L Q_R + h.c. \right)$$

$$\mathcal{L} \supset \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a$$



- Barr and Seckel's model

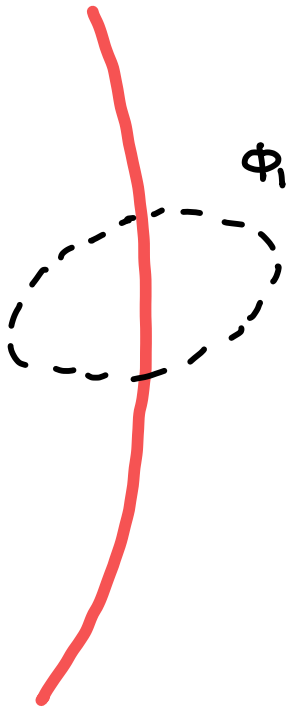
$Q_{1L} Q_{1R} Q_{2L} Q_{2R}$

color, $U(1)_Z$ and $U(1)_{PQ}$ charges

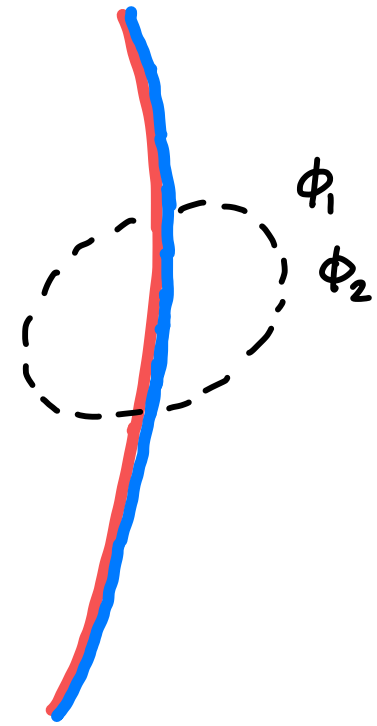
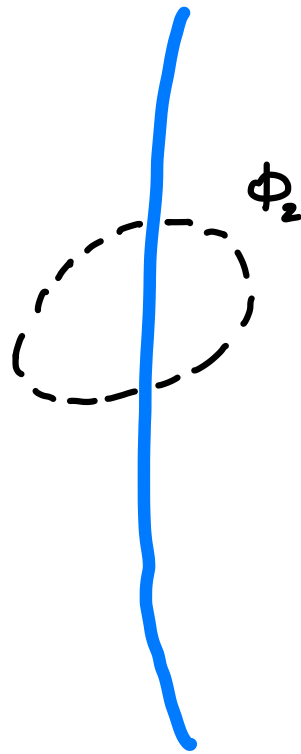
$$\mathcal{L} = \Phi_1 \bar{Q}_{1L} Q_{1R} + \Phi_2 \bar{Q}_{2L} Q_{2R} + h.c.$$

String Solutions

(1, 0)



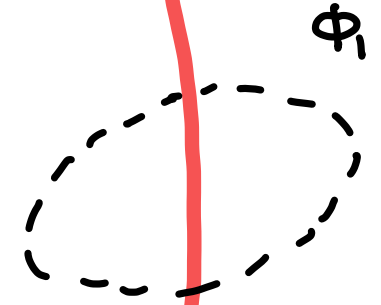
(0, 1)



(1,0) strings

- (1,0) string

$$\Phi_1 = \frac{1}{\sqrt{2}} v_1 e^{i\theta}, \quad \Phi_2 = \frac{1}{\sqrt{2}} v_2, \quad Z_\mu = c \partial_\mu \theta, \quad r \rightarrow \infty$$



- gradient energy

$$\begin{aligned} \mu_{k,(1,0)} &= \int_0^{2\pi} d\theta \int_\delta^L dr r \left(\left| \left(\frac{1}{r} \partial_\theta - ieZ_\theta \right) \Phi_1 \right|^2 + \left| (-ieZ_\theta) \Phi_2 \right|^2 \right) \\ &= \pi \ln\left(\frac{L}{\delta}\right) [v_1^2 (1 - ec)^2 + v_2^2 (ec)^2] \end{aligned}$$

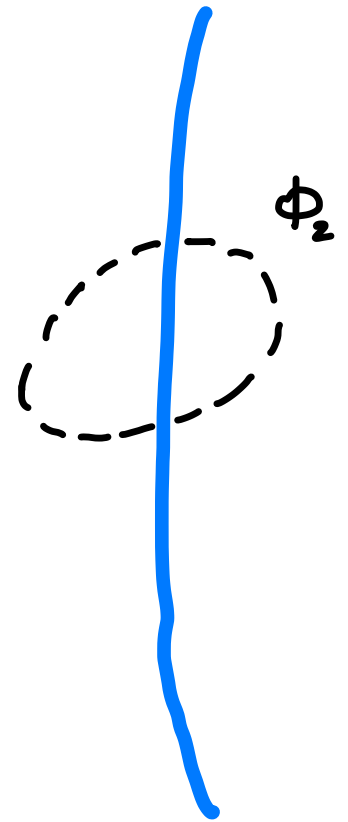
- outside core (minimize it by varying c)

$$\mu_{k,(1,0)} = \pi \frac{v_1^2 v_2^2}{v_1^2 + v_2^2} \ln\left(\frac{L}{\delta}\right) = \pi f_a^2 \ln\left(\frac{L}{\delta}\right)$$

(0,1) strings

- (0,1) string

$$\Phi_1 = \frac{1}{\sqrt{2}}v_1, \quad \Phi_2 = \frac{1}{\sqrt{2}}v_2 e^{i\theta}, \quad Z_\mu = c \partial_\mu \theta, \quad r \rightarrow \infty$$



- gradient energy of (0,1) = gradient energy of (1,0)

$$\mu_{k,(0,1)} = \mu_{k,(1,0)}$$

- outside core region

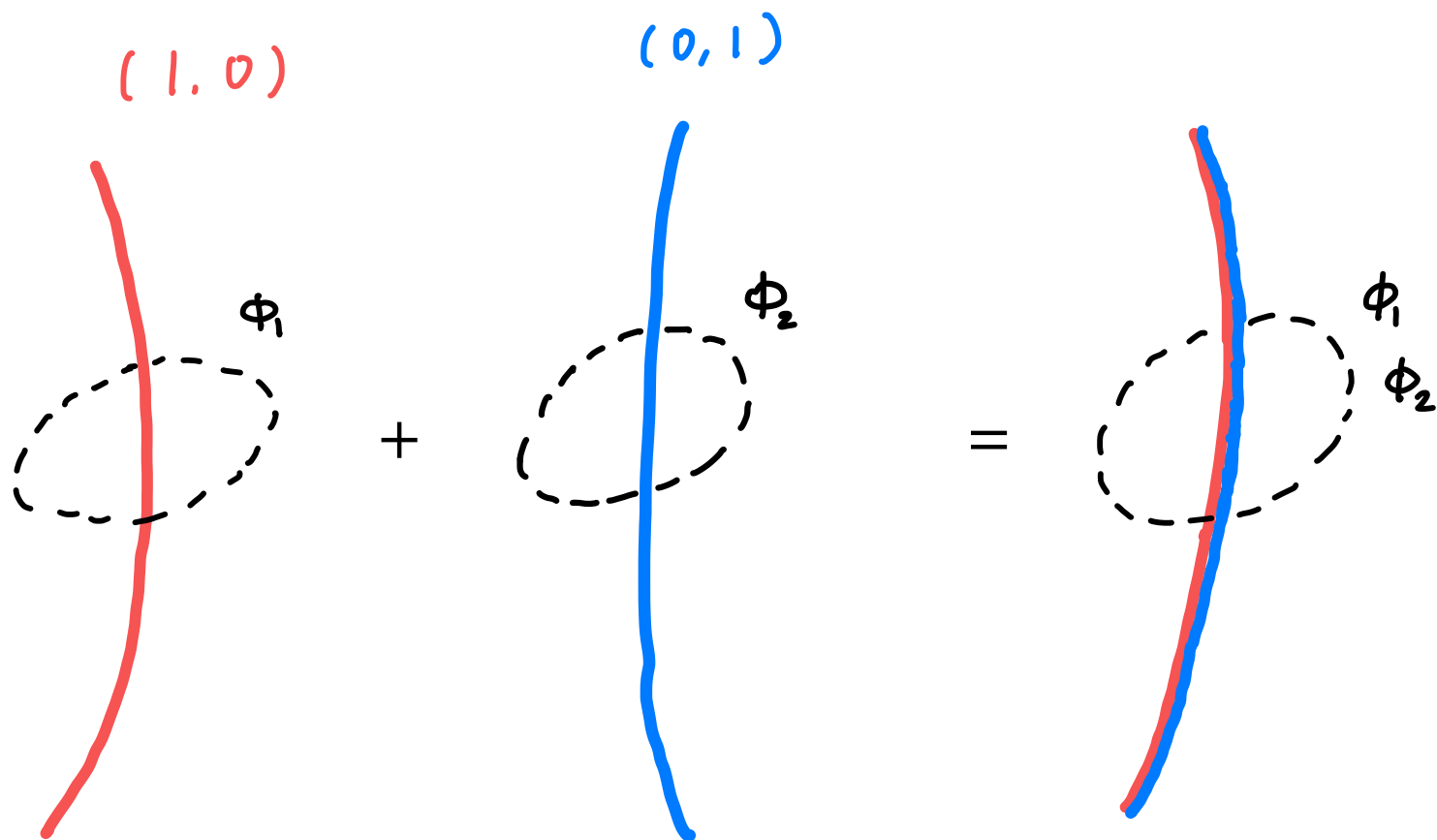
(1,0) string is equivalent to (0,-1) string through a gauge transformation

$$\left(\Phi_1 = \frac{1}{\sqrt{2}}v_1 e^{i\theta}, \Phi_2 = \frac{1}{\sqrt{2}}v_2 \right) \xrightarrow{\alpha_Z \rightarrow \alpha_Z - \theta} \left(\Phi_1 = \frac{1}{\sqrt{2}}v_1, \Phi_2 = \frac{1}{\sqrt{2}}v_2 e^{-i\theta} \right)$$

- outside core region

(1,0) can be viewed as an anti-string of (0,1)

$(1,0) + (0,1) \rightarrow ?$



(1,1) strings

- gradient energy of (1,1) string

$$\Phi_1 = \frac{1}{\sqrt{2}} v_1 e^{i\theta}, \quad \Phi_2 = \frac{1}{\sqrt{2}} v_2 e^{i\theta}, \quad Z_\mu = c \partial_\mu \theta, \quad r \rightarrow \infty$$

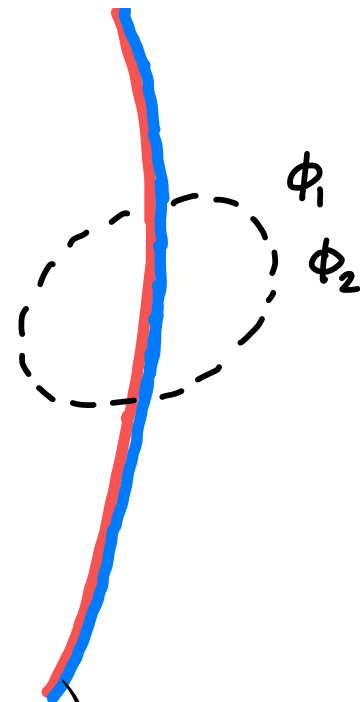
$$\mu_{k,(1,1)} = \int_0^{2\pi} d\theta \int_\delta^L dr r \left(\left| \left(\frac{1}{r} \partial_\theta - ieZ_\theta \right) \Phi_1 \right|^2 + \left| \left(\frac{1}{r} \partial_\theta - ieZ_\theta \right) \Phi_2 \right|^2 \right)$$

- the profile of Z_θ can simultaneously cancel the gradient energy of Φ_1 and Φ_2

$$\mu_{k,(1,1)} = 0$$

- (1,1) gauge string

(1,0) and (0,1) global strings



The full tension

- magnetic self-energy, scalar potential energy, and gradient energy

- (1,0) string

$$\mu_{(1,0)} \simeq \pi v_1^2 + \pi v_1^2 \ln \left(\frac{m_1}{m_Z} \right) + \pi v_2^2 \ln \left(\frac{m_Z L}{2} \right)$$

- (0,1) string

$$\mu_{(0,1)} \simeq \frac{\pi}{2} v_2^2 + \pi v_2^2 \ln \left(\frac{m_2}{m_Z} \right) + \pi v_2^2 \ln \left(\frac{m_Z L}{2} \right)$$

- (1,1) string

$$\mu_{(1,1)} = \pi v_1^2 + \pi v_1^2 \ln \left(\frac{m_1}{m_Z} \right) + 0$$

- heavy core of (1,0) string $\mu_{(1,0)} > \mu_{(0,1)}$

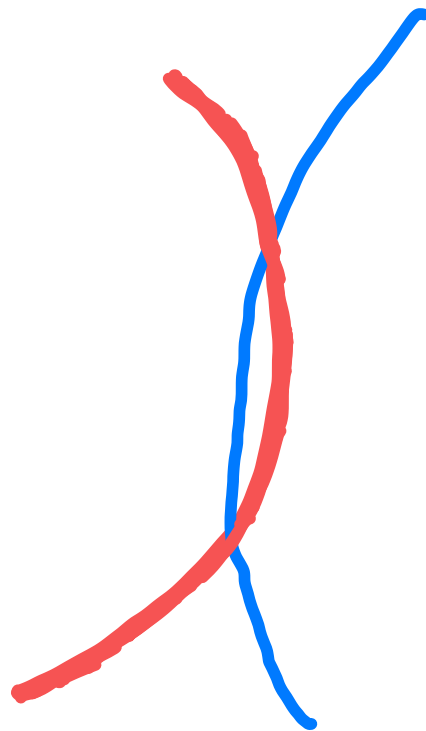
- binding energy of (1,1) string

$$\mu_{(1,0)} + \mu_{(0,1)} - \mu_{(1,1)} = \pi v_2^2 \left[2 \ln \left(\frac{m_Z L}{2} \right) - 1 \right]$$

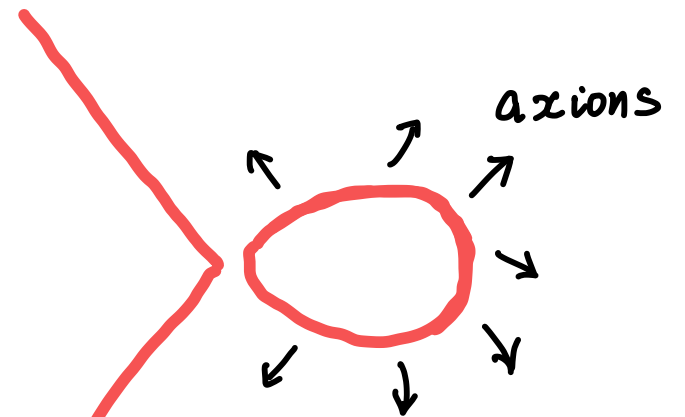
Cosmological Implication



formation



evolution



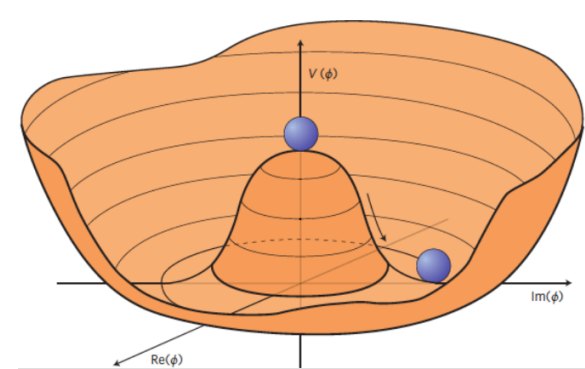
radiation

First phase transition and string formation

- consider $v_1 \gg v_2$

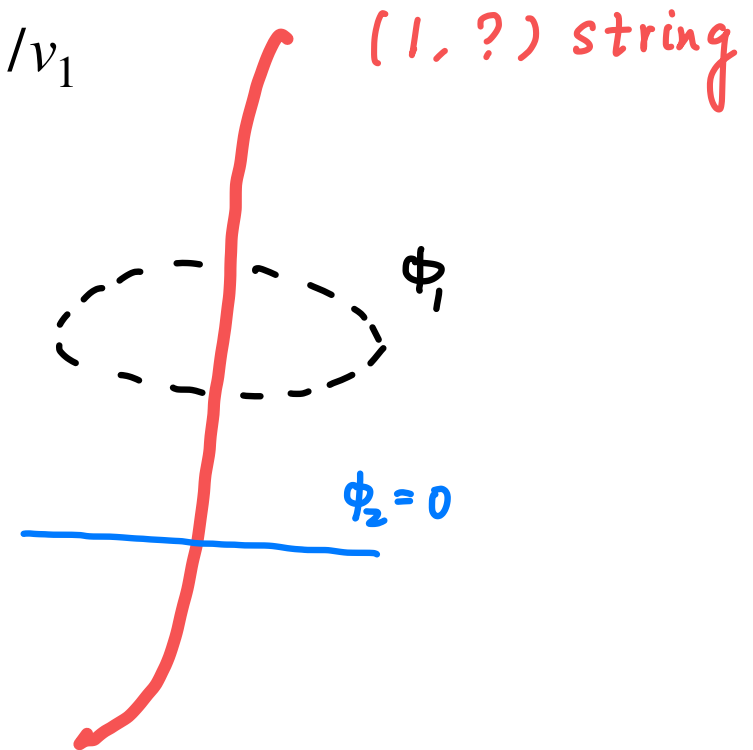
first phase transition

$$\langle \Phi_1(x) \rangle = \frac{v_1}{\sqrt{2}} \quad \text{and} \quad \langle \Phi_2(x) \rangle = 0$$



- string formation, the correlation length $\sim 1/v_1$

- U(1) gauge strings form
(1, **n**) string



Second phase transition

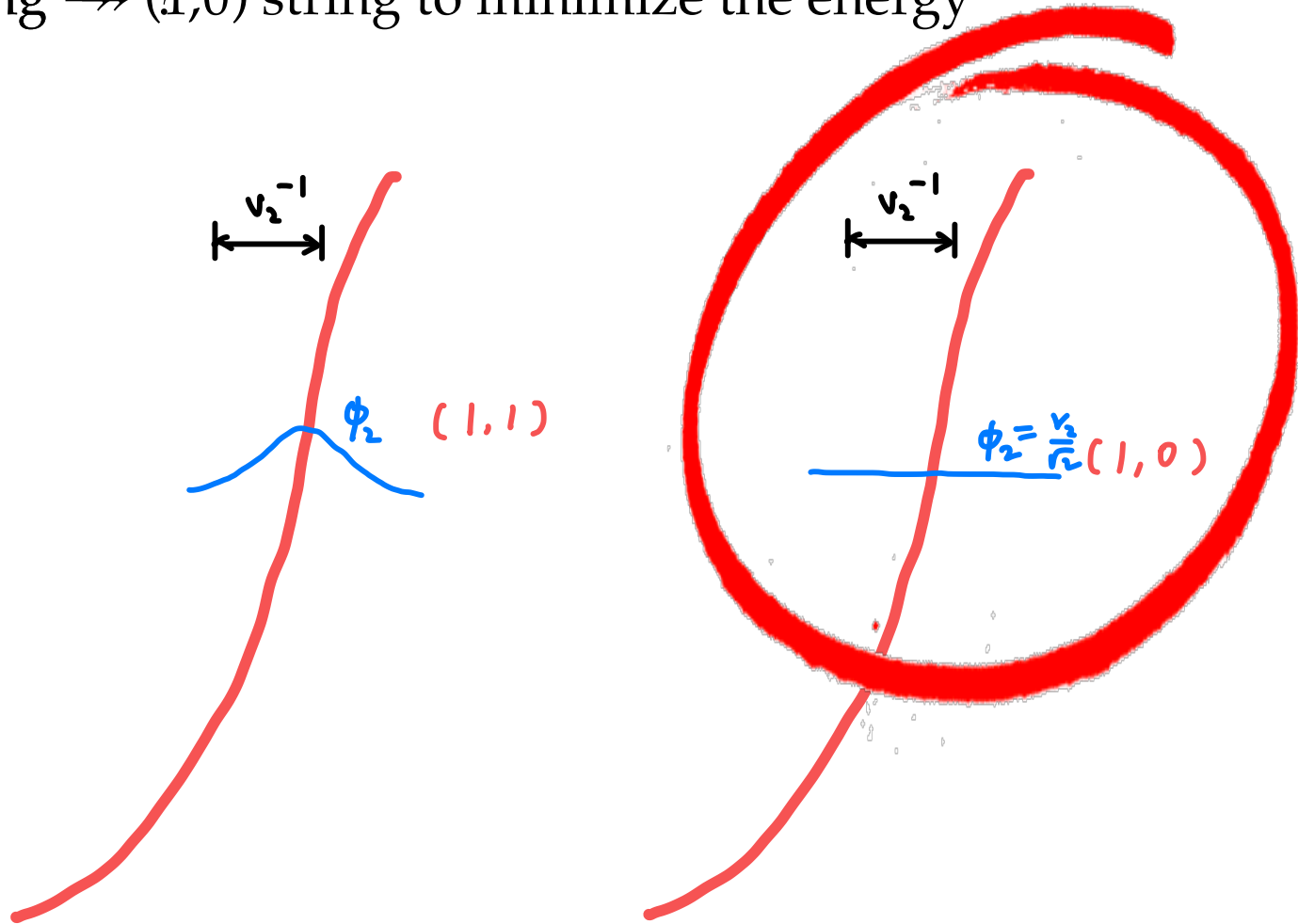
- second phase transition

$$\langle \Phi_1(x) \rangle = \frac{v_1}{\sqrt{2}} \quad \text{and} \quad \langle \Phi_2(x) \rangle = \frac{v_2}{\sqrt{2}}$$

- string formation, the correlation length $\sim 1/v_2$
- $(0,1)$ strings form via Kibble mechanism

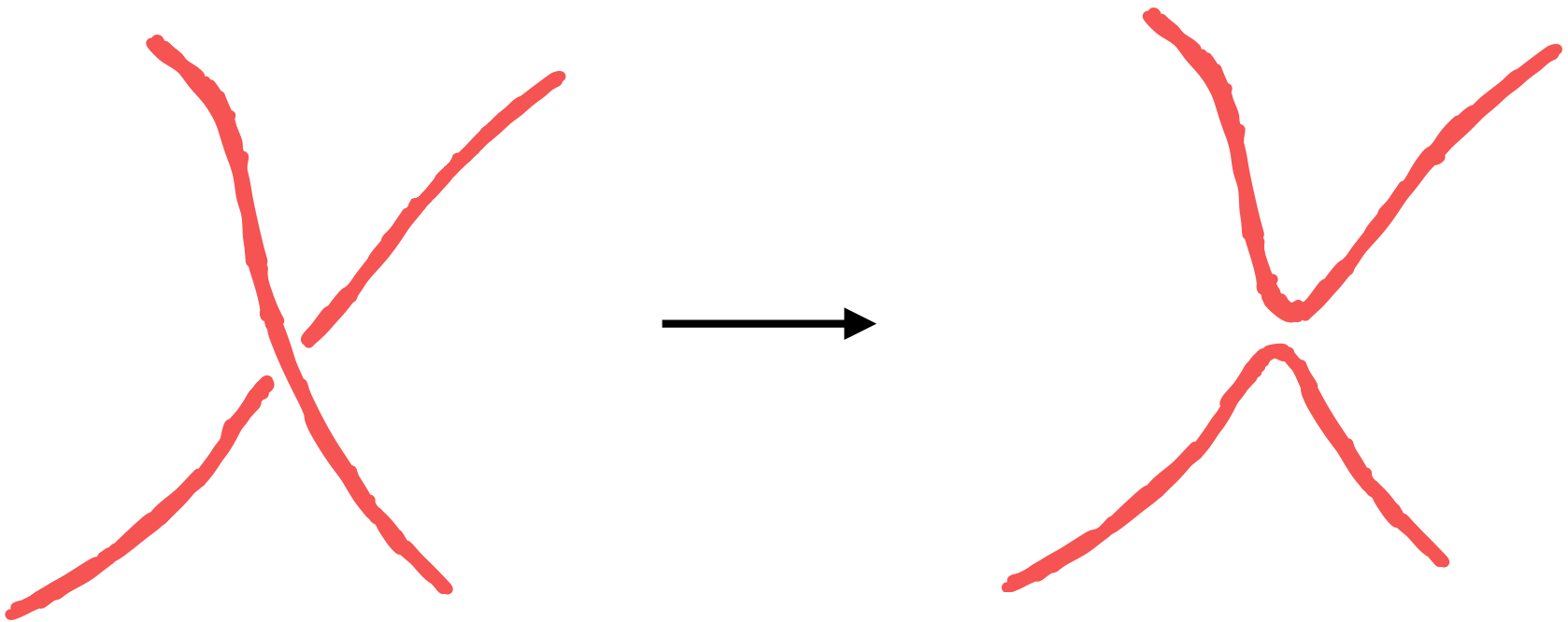
(1,n) string in the second phase transition

- (1, n) string \rightarrow (1,0) string to minimize the energy



String network evolution

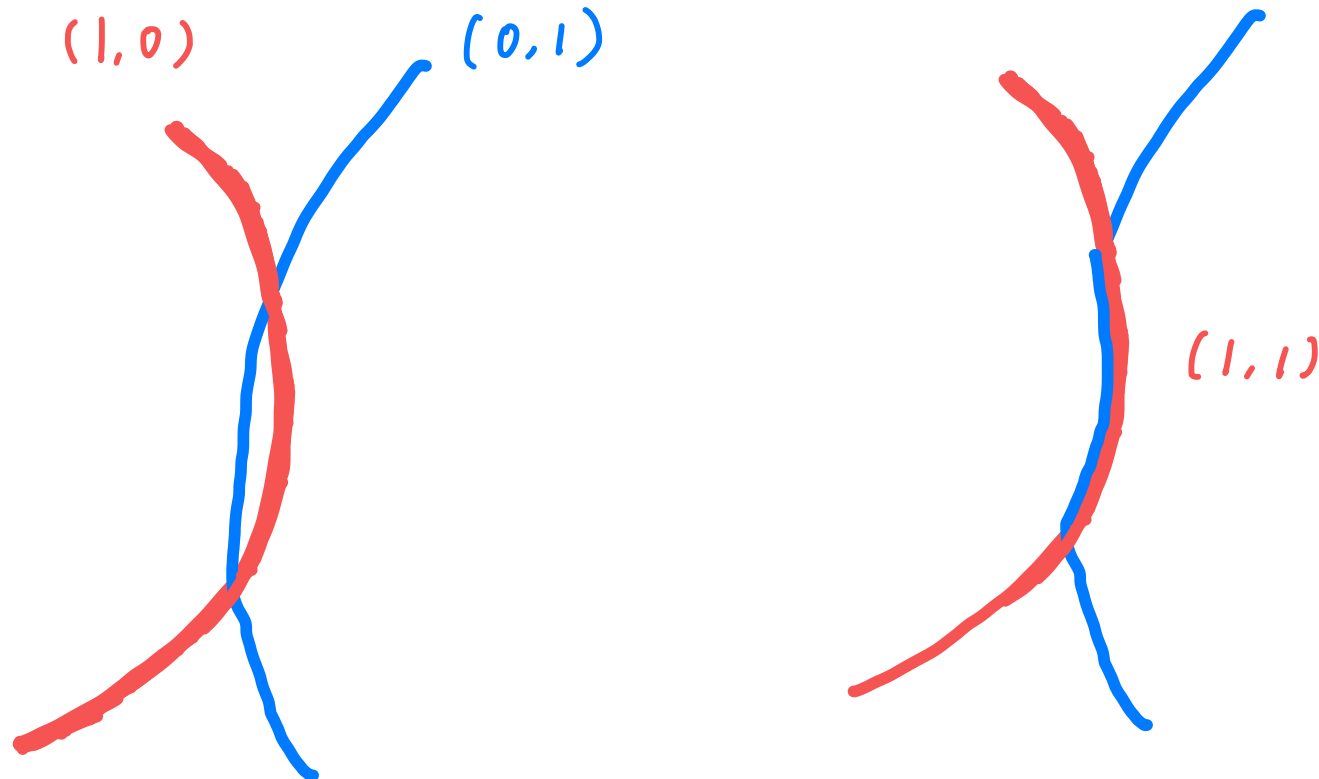
- $(1,0)$ string encounters $(1,0)$ string



String network evolution

- $(1,0)$ string encounters $(0,1)$ string \rightarrow $(1,1)$ bound state

Y-junctions



- Other works on simulations of Y-junctions found 1) some fraction of Y-junctions remain
2) scaling solution

Urrestilla, Vilenkin JHEP(2008)

Rajantie, Skellariodou, Stoica, JCAP (2007)

Copeland, Saffin JHEP (2005)

QCD axion

- dark matter abundance

misalignment + **string radiation** + domain wall collapse

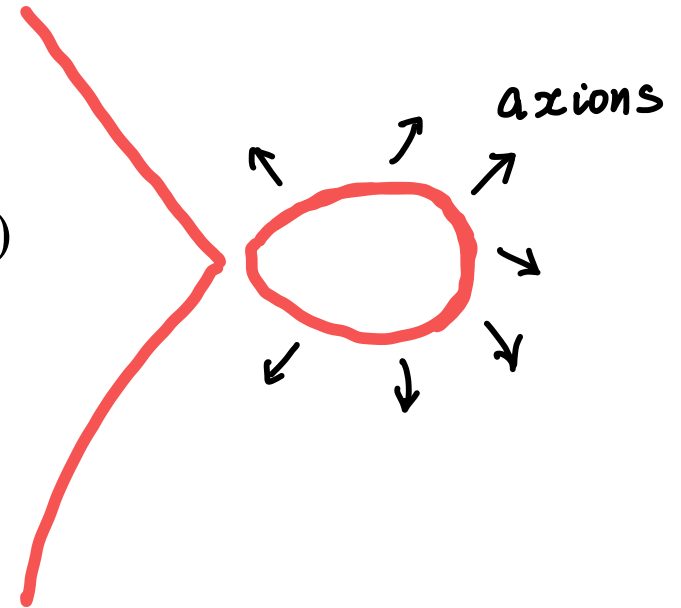
$$\rho_{a,0} = \rho_a^{\text{vac}}(t_0) + m_a n_a^{\text{str}}(t_0) + m_a n_a^{\text{DW}}(t_0)$$

- uncertainty from string radiation

Scenario A: IR spectrum $\frac{dE}{d\omega} \propto \delta(\omega - 2\pi t^{-1})$

Scenario B: flat spectrum $\frac{dE}{d\omega} \propto \frac{1}{\omega}$

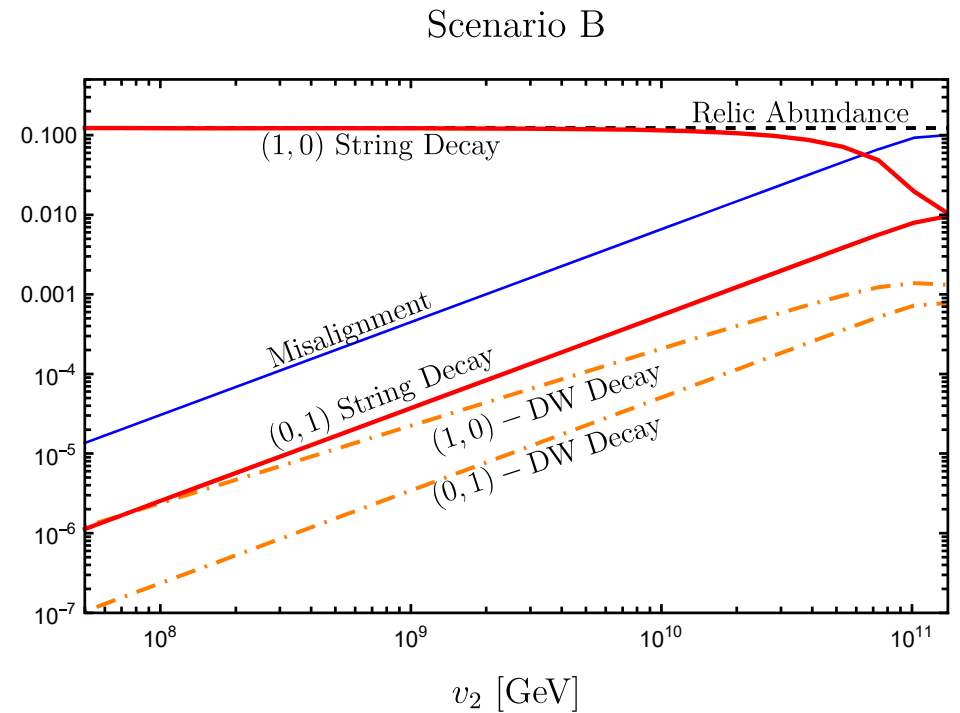
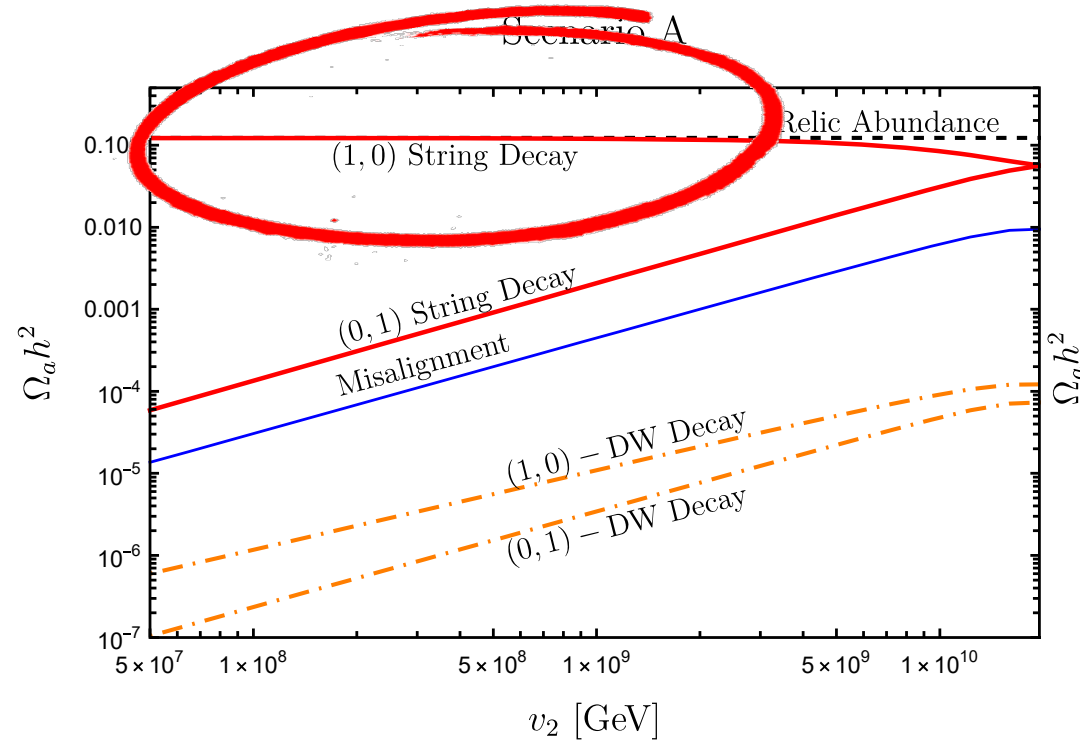
- $n_a^{\text{str}} \propto \frac{1}{\langle \omega \rangle}$



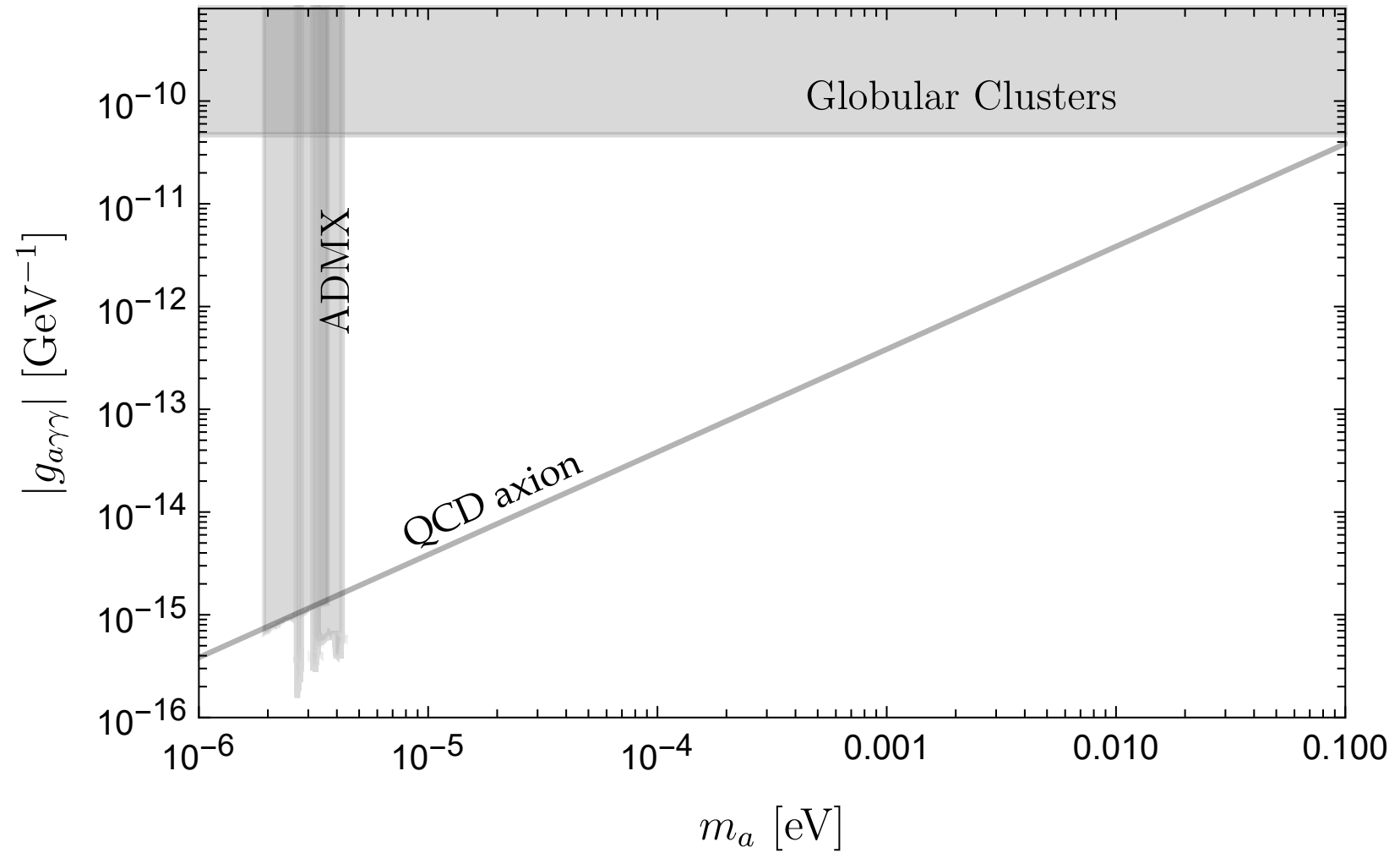
Gauged global string

- (0,1) string tension is the same as a standard QCD axion string
- **heavy core** of (1,0) string

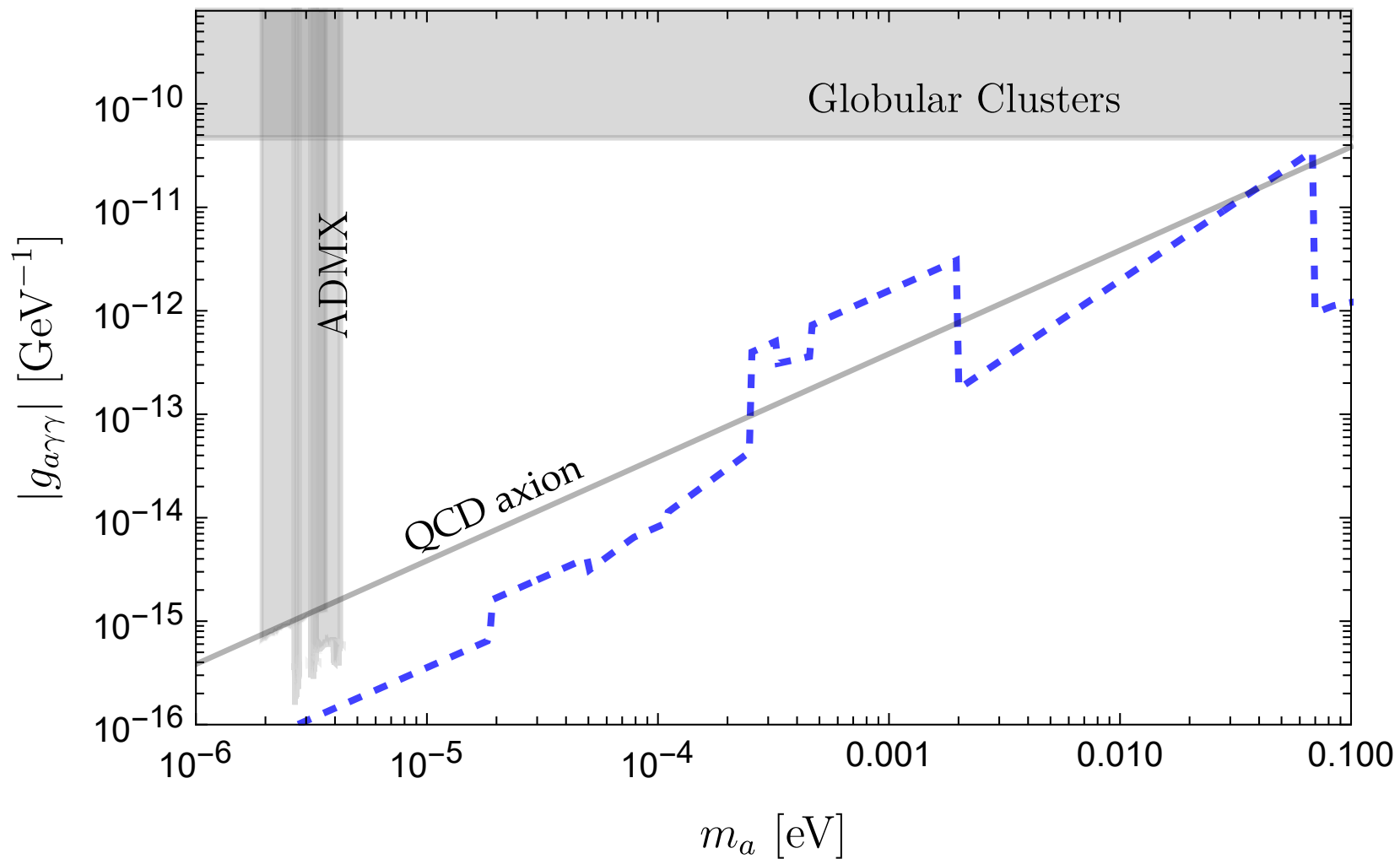
$$\mu_{(1,0)}(t) \simeq \pi v_1^2 \ln \left(\frac{m_1}{m_Z} \right) + \pi f_a^2 \ln \left(\frac{m_Z t}{2} \right)$$



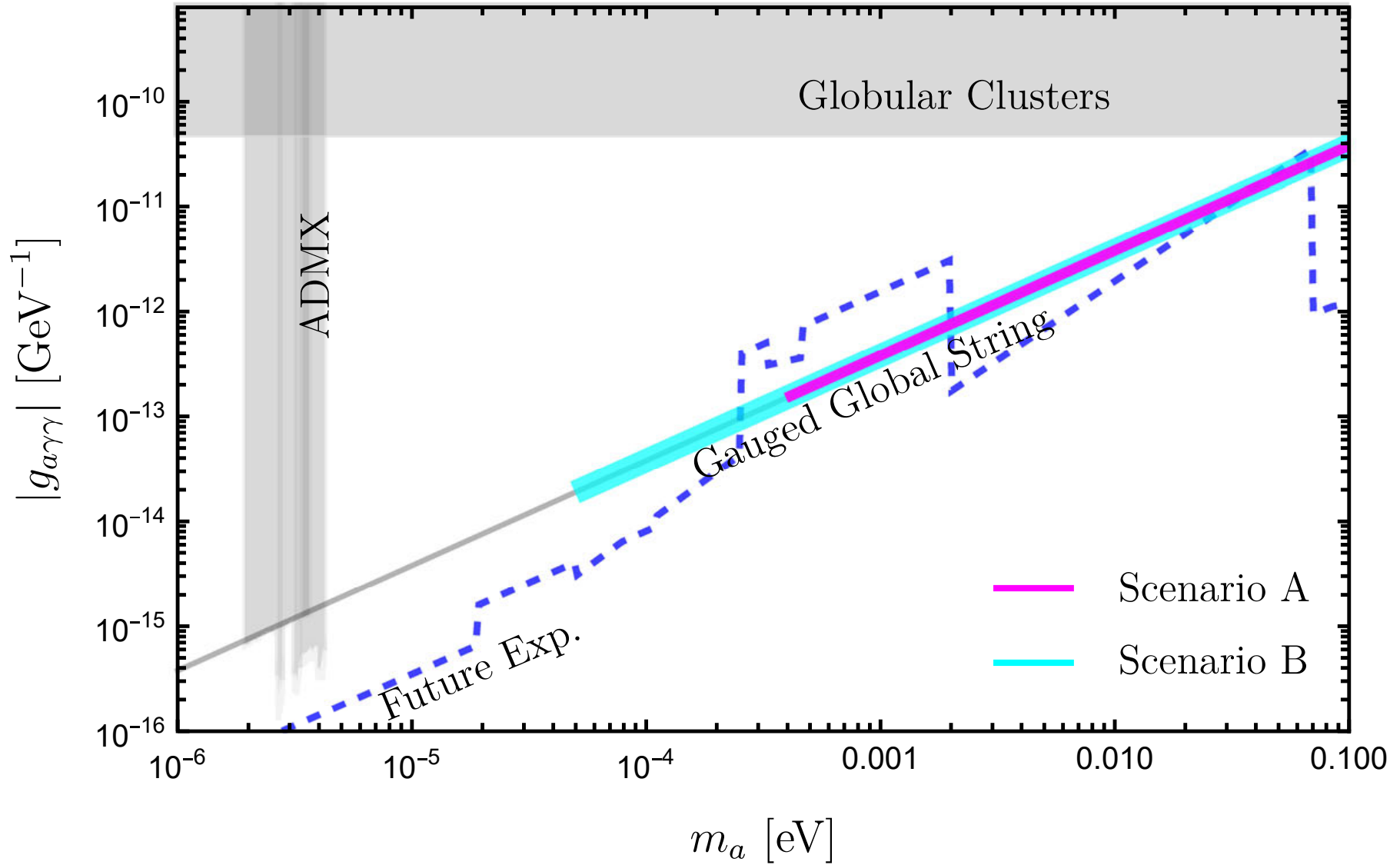
QCD axion window



Future explorations

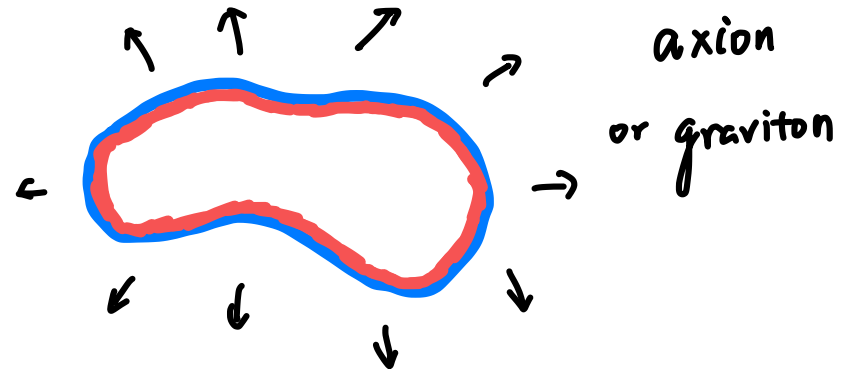


Gauged global string



Gauge string (1,1) radiation

- gauge strings radiate gravitons
- How about (1,1) gauge strings?



- (1,1) string is gauge string, but it also has axion as light d.o.f

$$\mathcal{L} = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}e^2(\phi_1^2 + \phi_2^2)Z_\mu^2 - g(\phi_1, \phi_2) e Z^\mu \partial_\mu a + \frac{1}{2} f(\phi_1, \phi_2) (\partial_\mu a)^2$$

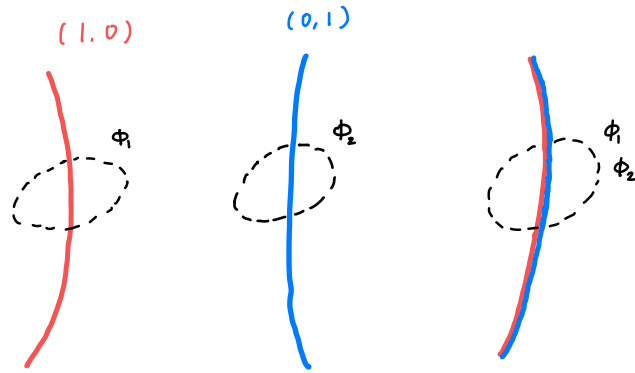
$$g(\phi_1, \phi_2) = f_a \frac{\phi_1^2}{v_1^2} - f_a \frac{\phi_2^2}{v_2^2}$$

- Kalb-Ramond field $B^{\mu\nu}$, $\partial_\mu a = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial^\nu B^{\alpha\beta}$

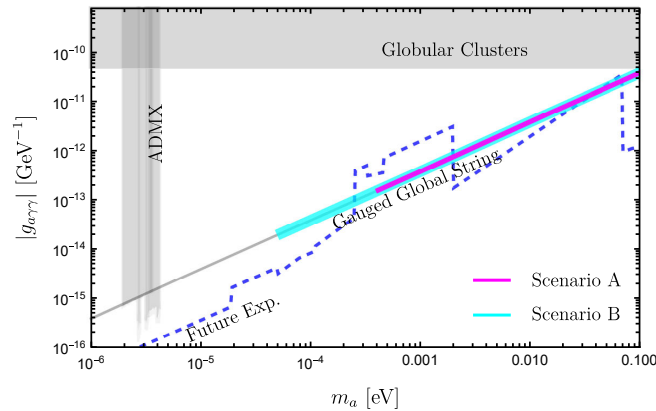
- radiation power

$$\frac{dP_a}{d\Omega} \sim e^2 f_a^2$$

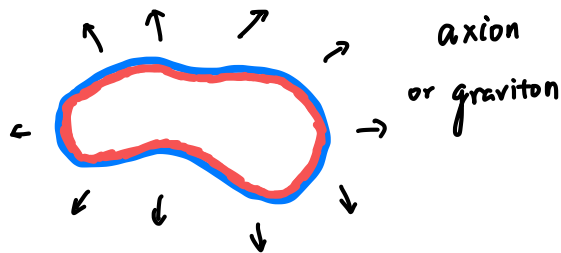
Conclusion



- $U(1)_Z \times U(1)_{PQ}$
 $(1,0)$, $(0,1)$ and $(1,1)$ strings



- Cosmology
Y-Junctions
 opening QCD axion mass windows



- $(1,1)$ gauge string
 radiating axions and gravitons