Thermal Axion and Dark Radiation

Fazlollah Hajkarim

CETUP Workshop, Lead, South Dakota, June 24, 2024

Based on arXiv:2108.04259 (PRL) , arXiv:2108.05371 (JHEP),

arXiv:2205.07849 (JCAP) and

arXiv:2311.04974 (JCAP)

in collaboration with

F. D’Eramo, S. Yun

F. D’Eramo, E. Di Valentino, W. Giarè, A. Melchiorri, O. Mena, F. Renzi, S. Yun

F. D’Eramo, A. Lenoci

University of Oklahoma


HISTORY OF THE UNIVERSE
Particle era
Accelerators
RHIC & LHC heavy ions
LHC protons
High-energy cosmic rays
Inflation
Big Bang
Size of visible universe
Cosmic Microwave Background radiation is visible
Structure formation
Dark energy accelerated expansion
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s E = 10^16 GeV
t = 10^-10 s E = 10^2 GeV
t = 10^-4 s E = 10^-1 GeV
t = 10^2 s E = 10^-4 GeV
t = 3x10^5 y E = 3x10^-10 GeV
t = 10^9 y E = 10^-12 GeV
t = 13.8x10^9 y E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
Particle Data Group, LBNL © 2015
Supported by DOE

Dark Radiation

* There is a discrepancy $\Delta N_{\rm eff}$ from the theoretical value of number of effective neutrinos: $N_{\rm eff} = 3.044$ and observed value from cosmic microwave background (CMB) and big bang nucleosynthesis (BBN).

* Extra degrees of freedom from dark sector can be responsible for dark radiation (DR).

* Beyond the standard model physics have some proposals for DR like sterile neutrinos, axion, etc. Moreover, gravitational wave background may have a contribution to $\Delta N_{\rm eff}$.

* Future CMB experiments can put stronger bound on light relics and $\Delta N_{\rm eff}$.

* Relativistic degrees of freedom depending on their nature can decouple at different temperatures. They may be connected to the new physics!

* To estimate the accurate amount of dark radiation contributed to the CMB from a typical DR candidate we require to consider the precise thermal background in the early universe and all the possible production channels of DR.

arXiv:1303.5379 arXiv:2011.13874

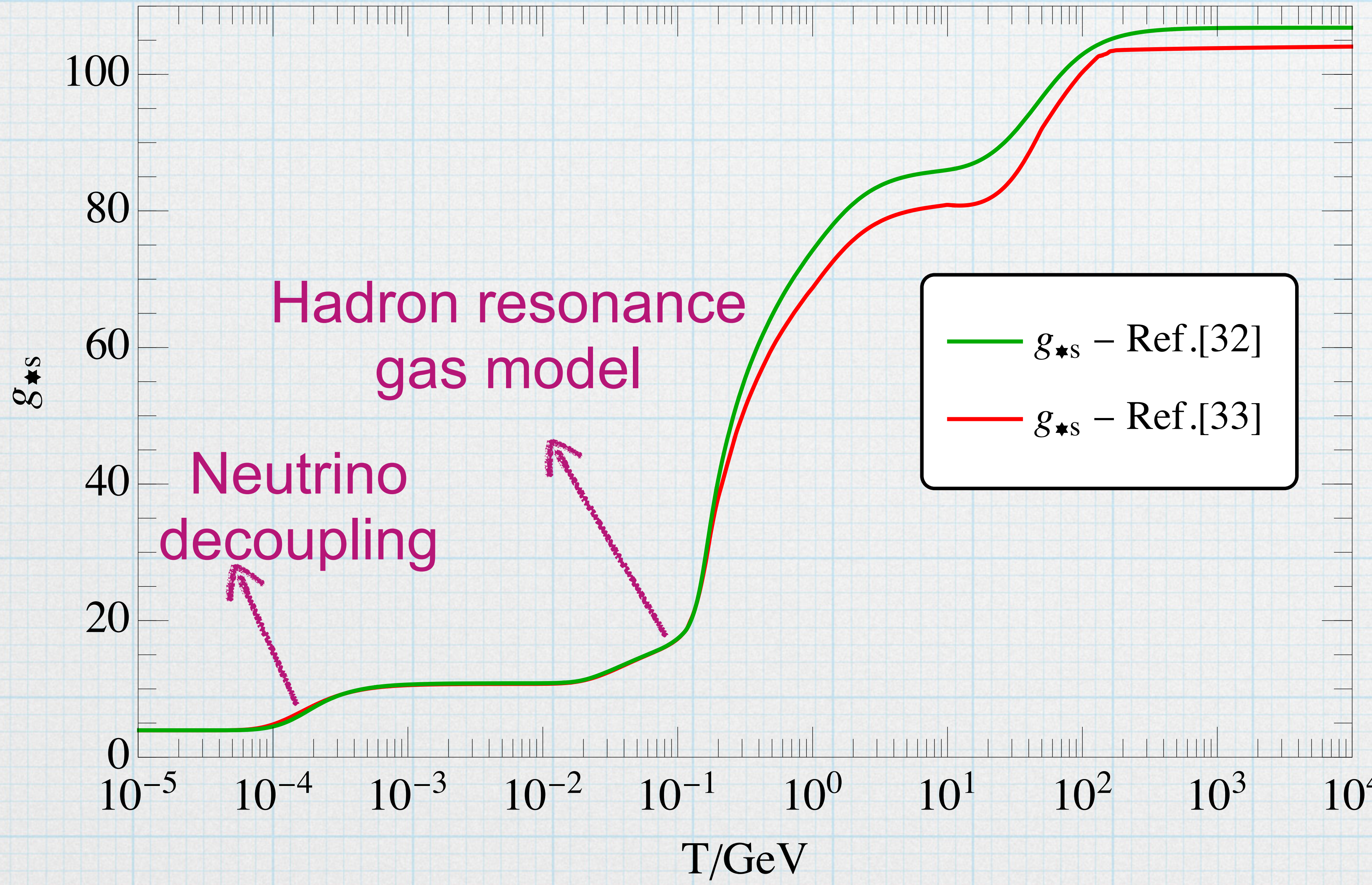
Thermal Background of SM Particles

We need precise equation of state for thermal bath of SM at different temperatures. Energy and entropy density of thermal bath:

$$s = \frac{2\pi^2}{45} g_{*s}(T)\, T^3 \qquad \rho = \frac{\pi^2}{30} g_*(T)\, T^4$$

Degrees of freedom

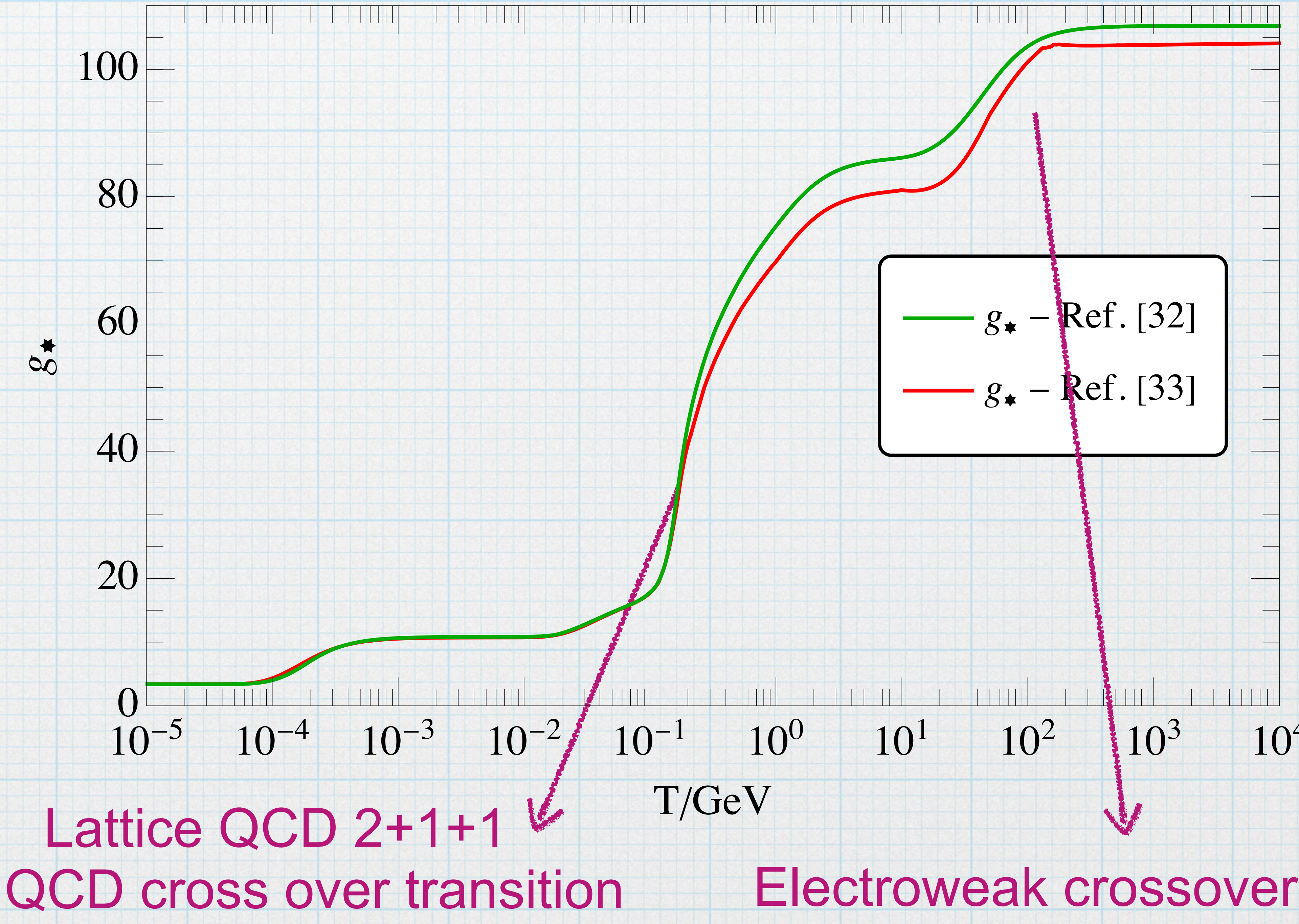
Entropy density degrees of freedom



arXiv:1503.03513
arXiv:1803.01038

The rest of SM particles considered free!

Energy density degrees of freedom

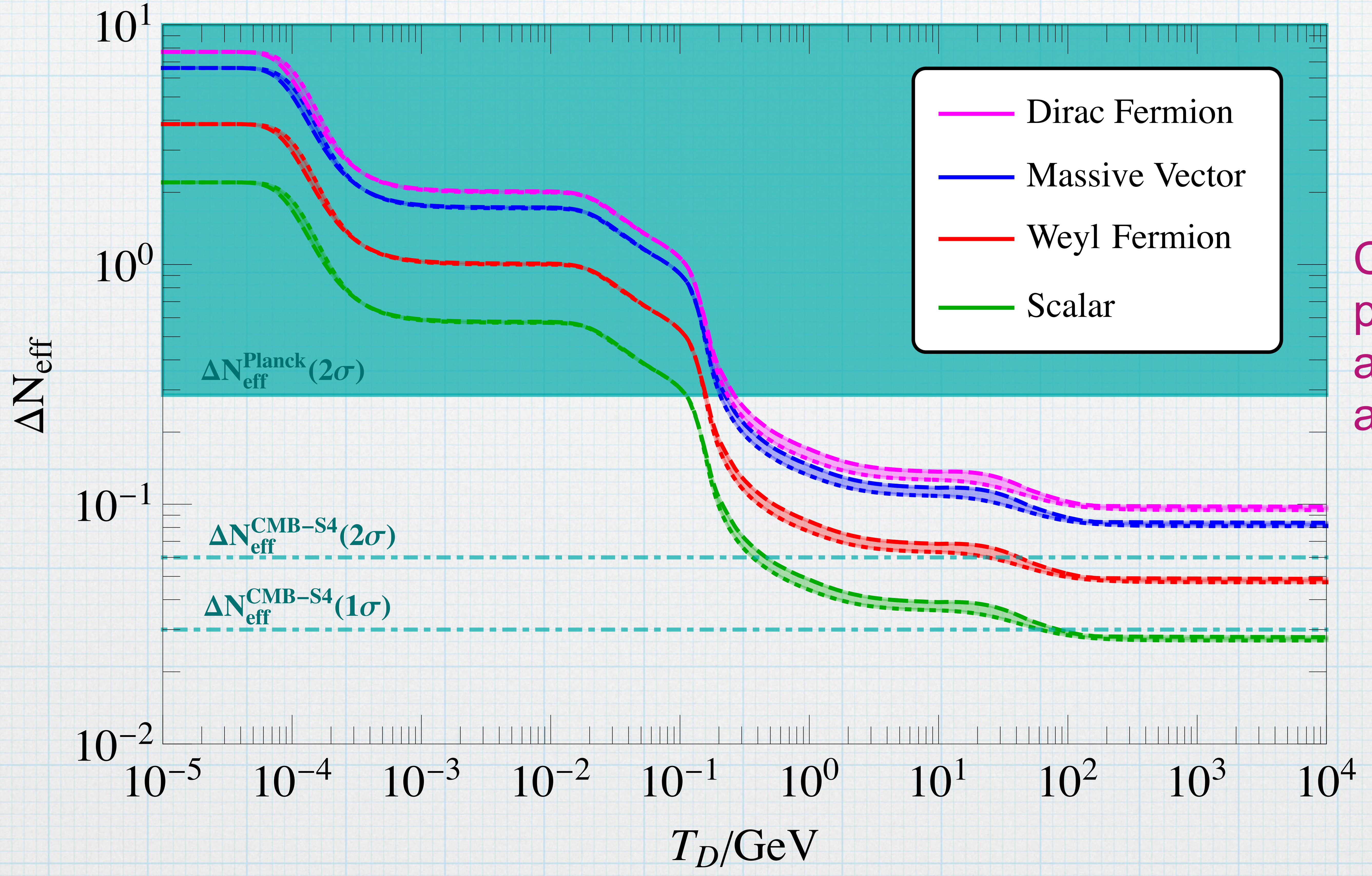


arXiv:1503.03513
arXiv:1803.01038

Different Types of Dark Radiation

Number of effective neutrinos $\Delta N_{\rm eff}$ versus decoupling temperature $T_{\rm D}$ of a typical DR:

$$\Delta N_{\rm eff} \equiv N_{\rm eff} - N_{\rm eff}^{\rm SM} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{\rho_{\rm DR}}{\rho_\gamma} \simeq g_{*\Phi} \times 13.69 \times g_{*s}^{\rm SM}(T_{\rm D})^{-4/3}$$



CMB-S4/future experiment can probe DR candidates decoupled around the QCD transition and above that!

CMBS4, arXiv:1610.02743 CMBS4, arXiv:1907.04473 Planck, arXiv:1807.06209 arXiv:2011.13874

QCD Axion and Strong CP problem

- QCD axion is a solution for the strong CP problem. It can explain the tiny value of neutron electric dipole moment!
- Connection to the UV completion physics through Peccei-Quinn symmetry restoration/breaking at high scales → giving mass to axions as pseudo Nambu-Goldstone bosons
- Possible contribution to the dark matter or dark radiation $(m_a \ll 1\ eV)$ depending on the production mechanisms. The relation between axion mass and axion decay constant is:

$$m_a \simeq 5.7 \times 10^{-6}\,\text{eV}\left(\frac{10^{12}\,\text{GeV}}{f_a}\right)$$

- There are some constraints from laboratory, astrophysical and cosmological experiments. The supernova constraint puts a lower bound on axion decay constant f_a. The upper bound on that comes from the condition on the overclosure of the universe. Axion constraints can be model dependent …

arXiv:2003.01100 R. Pecci, H. Quinn, 1977

Axion Production in the Early Universe

* Axion production from scatterings with hadrons using chiral perturbation theory
* Quarks and gluons scatterings with axions including QCD interactions
* Interactions from electroweak sector above and below the electroweak transition
* Possible interaction with leptons depending on the model

Different Thresholds!

arXiv:2003.01100 arXiv:1303.5379 arXiv:1008.4528 arxiv:2012.04736 arXiv:1604.08614

Axion Interactions with the SM Particles

A general Lagrangian for effective axion models:

Invariant under shift symmetry ← $a \to a + \text{const}$

$$\mathscr{L}^{(a)} = \mathscr{L}^{(a)}_{\text{gauge}} + \mathscr{L}^{(a)}_{\text{matter}}$$

Axion field — Fermion coupling

$$\mathscr{L}_{\text{int}} \supset \frac{1}{f_a}\left[a\, c_X \frac{\alpha_X}{8\pi} X^{a\mu\nu}\widetilde{X}^{a}_{\mu\nu} + \partial_\mu a\, c_\psi \overline{\psi}\gamma^\mu \psi\right]$$

Axion decay constant ← f_a

Gauge boson coupling

SM gauge bosons: $X = \{G, W, B\,(A)\}$

Quarks and leptons: $\psi = \{Q_L, u_R, d_R, L_L, e_R\}$

Below the QCD confinement scale (150 MeV) chiral perturbation theory for hadrons should be considered. For example axion-pions interaction $(1/f_a)\,\partial_\mu a\, \pi\, \partial^\mu \pi$.

arxiv:2012.04736 arXiv:2003.01100 arXiv:1801.06090 arXiv:1808.07430

Computation of Axion Yield

Boltzmann equation for the evolution of the number density of axions:

$$\frac{d}{dt}n_a + 3Hn_a = \gamma_a\left(1 - \frac{n_a}{n_a^{\rm eq}}\right)$$

$$n_i^{\rm eq} = \frac{m_i^2}{2\pi^2}TK_2\left(\frac{m_i}{T}\right) \propto T^3$$

Number density

Modified Bessel function 2nd kind

For relativistic case

$$s\,H\,x\frac{dY_a}{dx} = \left(1 - \frac{1}{3}\frac{\ln g_{*s}}{\ln x}\right)\gamma_a(x)\left(1 - \frac{Y_a}{Y_a^{\rm eq}}\right), \quad x = \frac{m}{T}, \quad Y_a = \frac{n_a}{s}, \quad \gamma_a \equiv n_a^{\rm eq}\Gamma_a$$

Entropy degrees of freedom

Sum of all interaction rates

Axion yield

Axion production rate from two body scattering of SM particles in a general case:

$$\Gamma_{a,S} = \frac{g_i g_j}{32\pi^4 n_a^{\rm eq}}T\int_{s_{min}}^{\infty} ds\,\frac{\lambda\left(s, m_i, m_j\right)}{\sqrt{s}}\,\sigma_{ij\to ka}(s)\,K_1\left(\frac{\sqrt{s}}{T}\right)$$

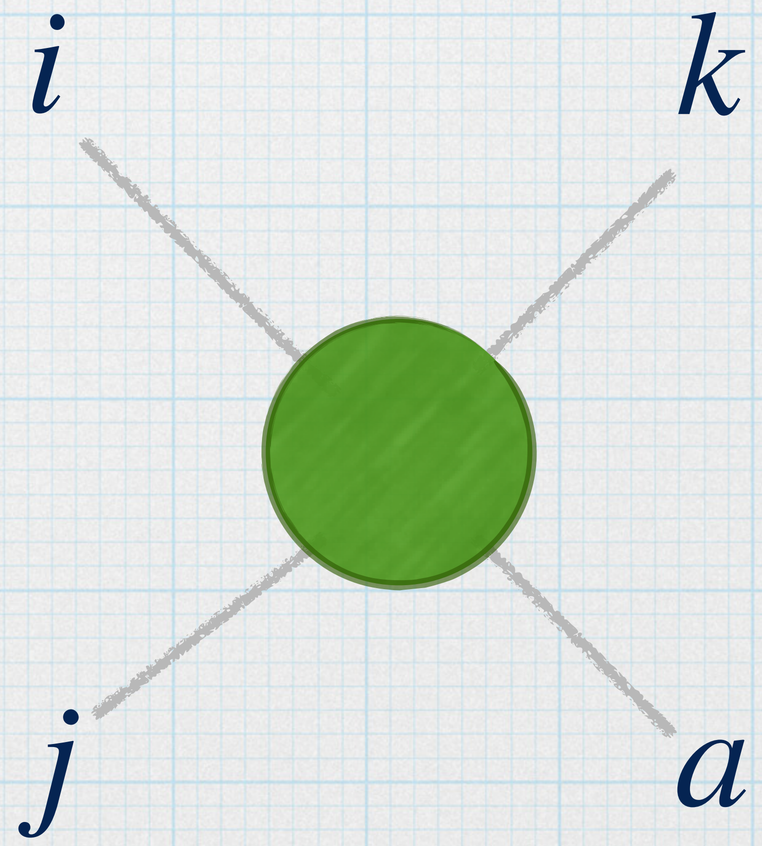
Degrees of freedom of incoming particles

Modified Bessel function 1st kind

$$s_{min} = {\rm Max}\left[\left(m_i + m_j\right)^2, m_k^2\right] \qquad \lambda\left(x, y, z\right) \equiv \left[x - \left(y+z\right)^2\right]\left[x - \left(y-z\right)^2\right]$$

Center of mass energy

Kaellen function



arxiv:2012.04736 arXiv:2003.01100 arXiv:1503.03513

Axion Production in the Early Universe

Axions can have different couplings with the SM particles depending on the axion model.

Lagrangian of KSVZ axion model:

$$\mathscr{L}_{aG} = \frac{\alpha_s}{8\pi}\frac{a}{f_a}G^A_{\mu\nu}\widetilde{G}^{A\mu\nu}$$

Coupling depends on light quark masses

$$\mathscr{L}_{a\pi} = \frac{\partial_\mu a}{f_a}\frac{c_{a\pi\pi\pi}}{f_\pi}\left[\pi^0\pi^+\partial^\mu\pi^- + \pi^0\pi^-\partial^\mu\pi^+ - 2\pi^+\pi^-\partial^\mu\pi^0\right]$$

Pion decay constant

Main interactions for the KSVZ axion case above and below the QCD transition:

$$T \gtrsim \Lambda_{\rm N} : \; q + \bar{q} \to g + a, \;\; q/\bar{q} + g \to q/\bar{q} + a, \;\; g + g \to g + a$$

$$T \lesssim \Lambda_{\rm ChPT} : \pi^+ + \pi^0 \to \pi^+ + a, \;\; \pi^- + \pi^0 \to \pi^- + a, \;\; \pi^+ + \pi^- \to \pi^0 + a$$

The production rate from rest of hadrons e.g. kaons, neutrons, protons, etc. contribute up to 10 % at $T_{\rm QCD} \simeq 150$ MeV. It can safely be ignored!

J. E. Kim, 1979 M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, 1980 arxiv:2012.04736 arXiv:0203221

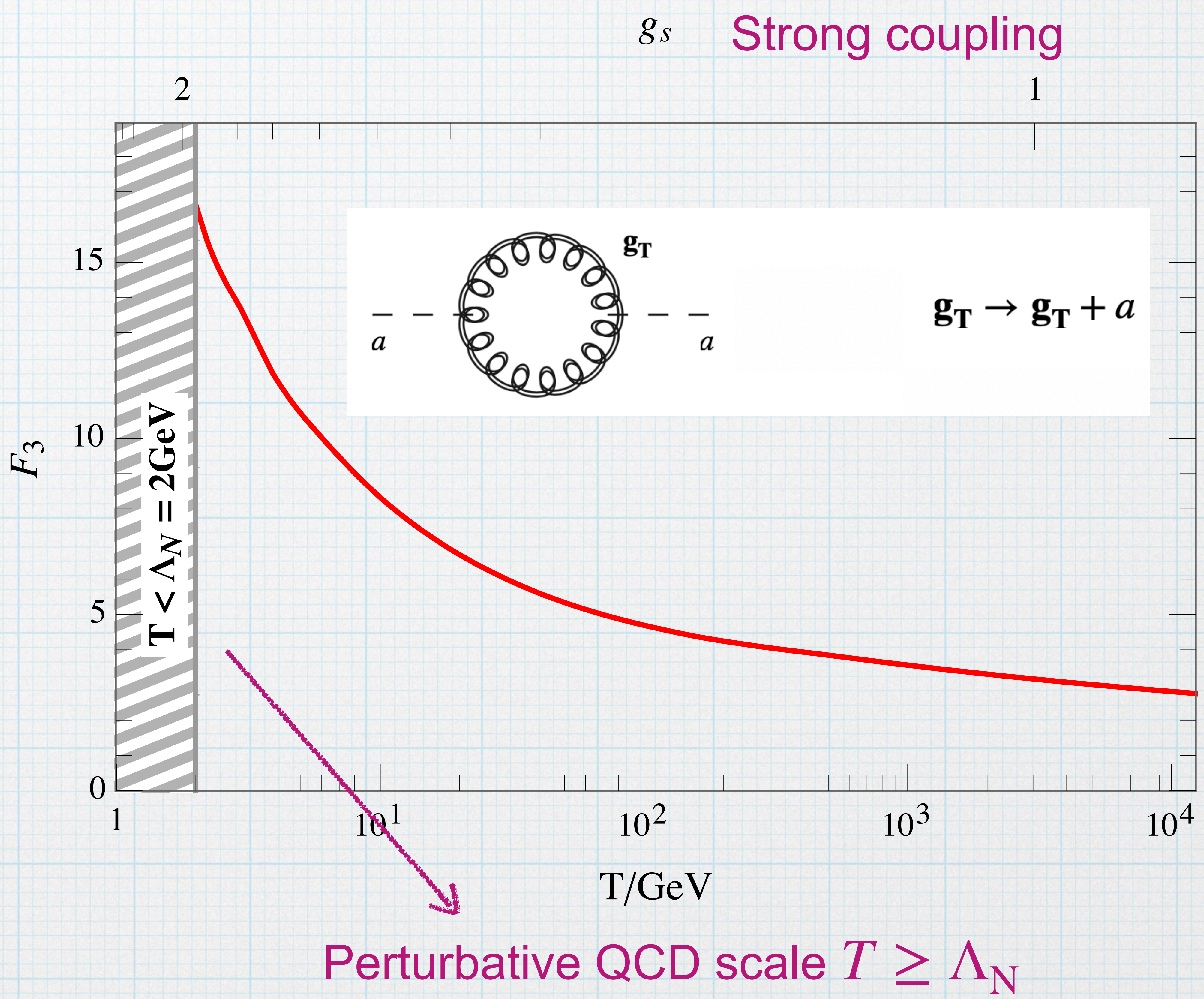
Thermal Effects on Gluon and Quark Scatterings

We consider the running of strong coupling g_s up to four loop orders for axion production from gluon and quark scatterings using `RunDec` package. Taking care of IR divergences due to massless gluons:

$$\gamma_{gg} = \frac{2\zeta(3)d_g}{\pi^3}\left(\frac{\alpha_s}{8\pi f_a}\right)^2 F_3(T)\, T^6$$

The only diagram contributing to the rate is the one-loop axion two-point function with virtual gluons exchanged, and with the tree-level gluon propagator replaced with the resummed thermal one.

Using continuum and pole approximations outside and inside the light cone for transverse and longitudinal parts of gluon propagator we can numerically calculate the function $F_3(T)$.



This calculation is also important for DFSZ model and contributes to the rate!

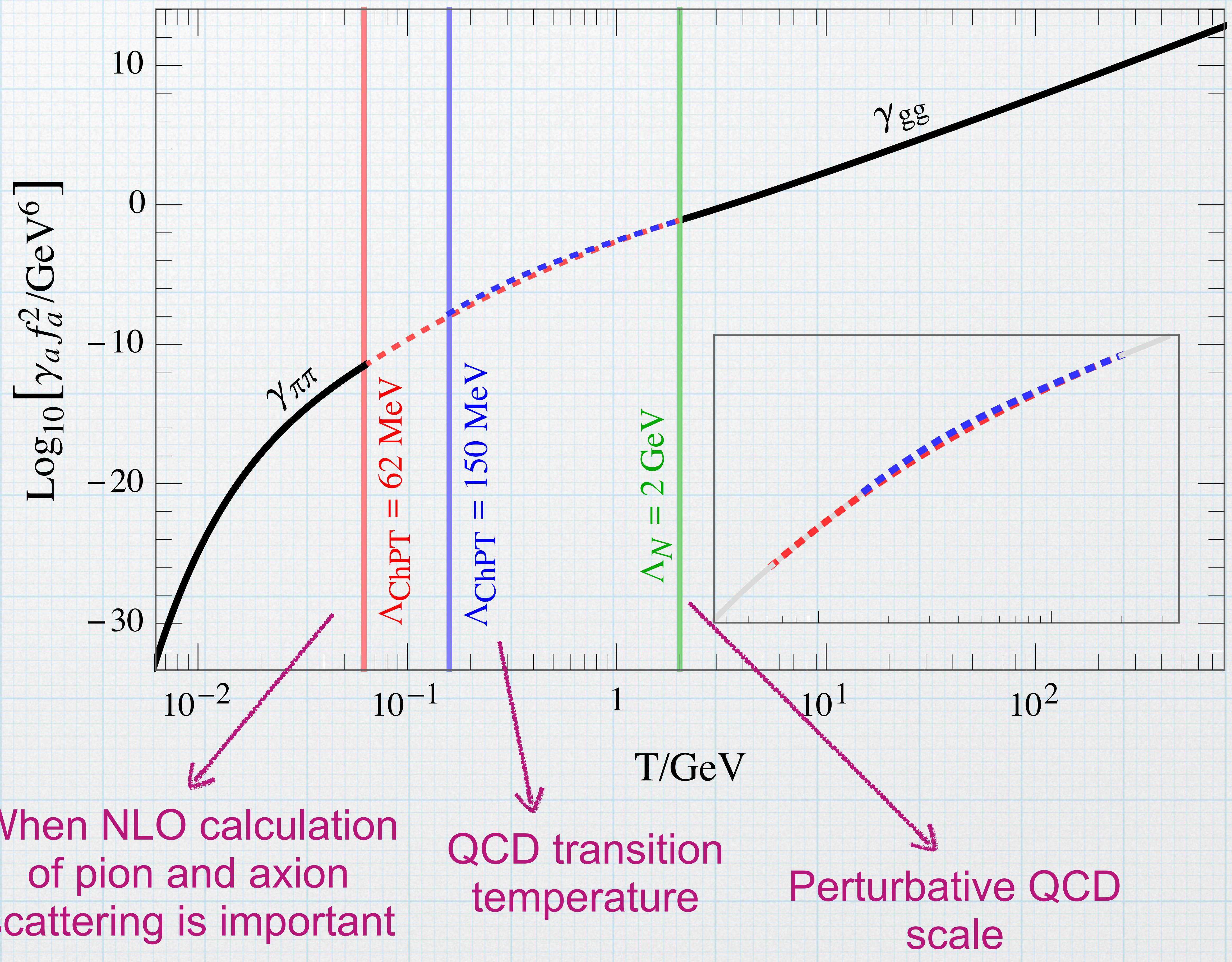
hep-ph/0004189 arXiv:1303.5379 arXiv:1008.4528 arXiv:2108.04259 arXiv:2108.05371

KSVZ Axion Production Rate

Calculating the total rate of KSVZ axion at low temperatures:

Due to crossover nature of QCD transition in the early universe at vanishing baryon asymmetry we can interpolate between the two regimes that gives a reasonable rate in the intermediate QCD confinement region.

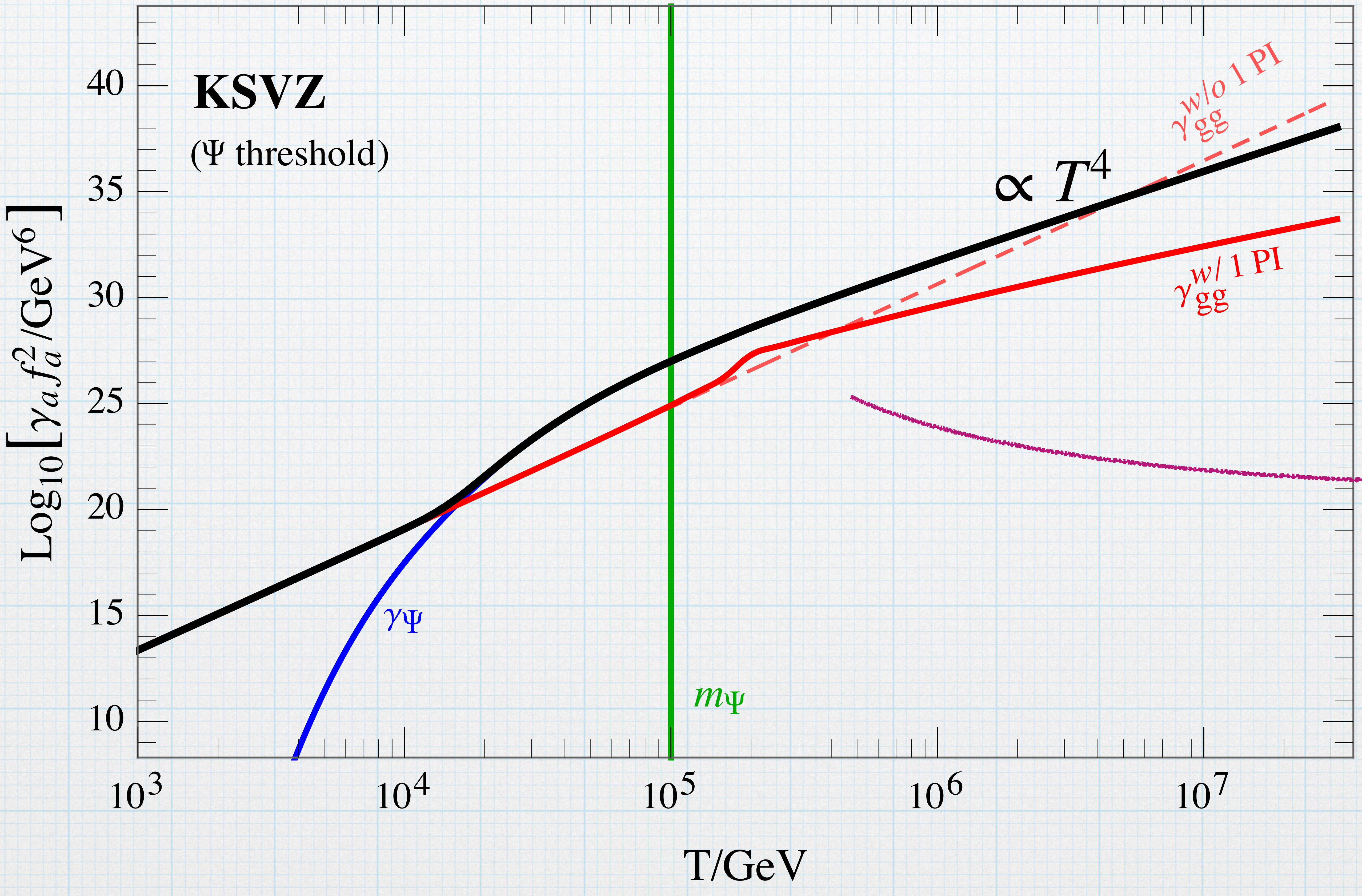
There is a small change in the rate due to different interpolation regime either choosing $\Lambda_{\rm ChPT} = 62$ or 150 MeV!



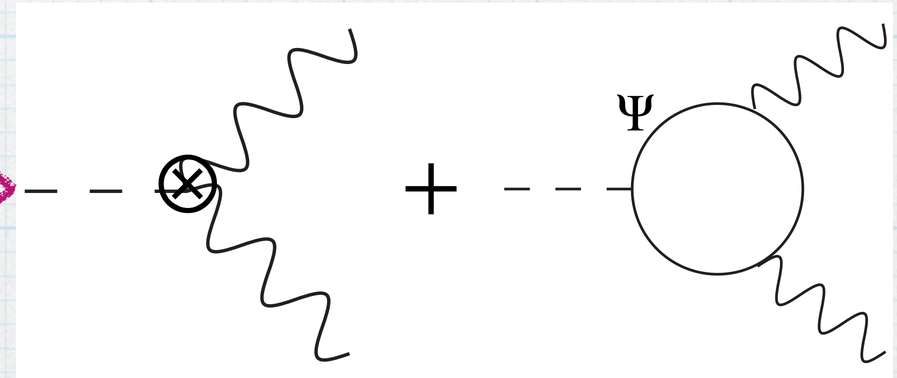
arXiv:1808.07430 arXiv:2108.04259 arXiv:2108.05371

KSVZ Axion Production Rate

Axion production rate for KSVZ model around the Peccei-Quinn fermion mass m_Ψ :



1PI - correction

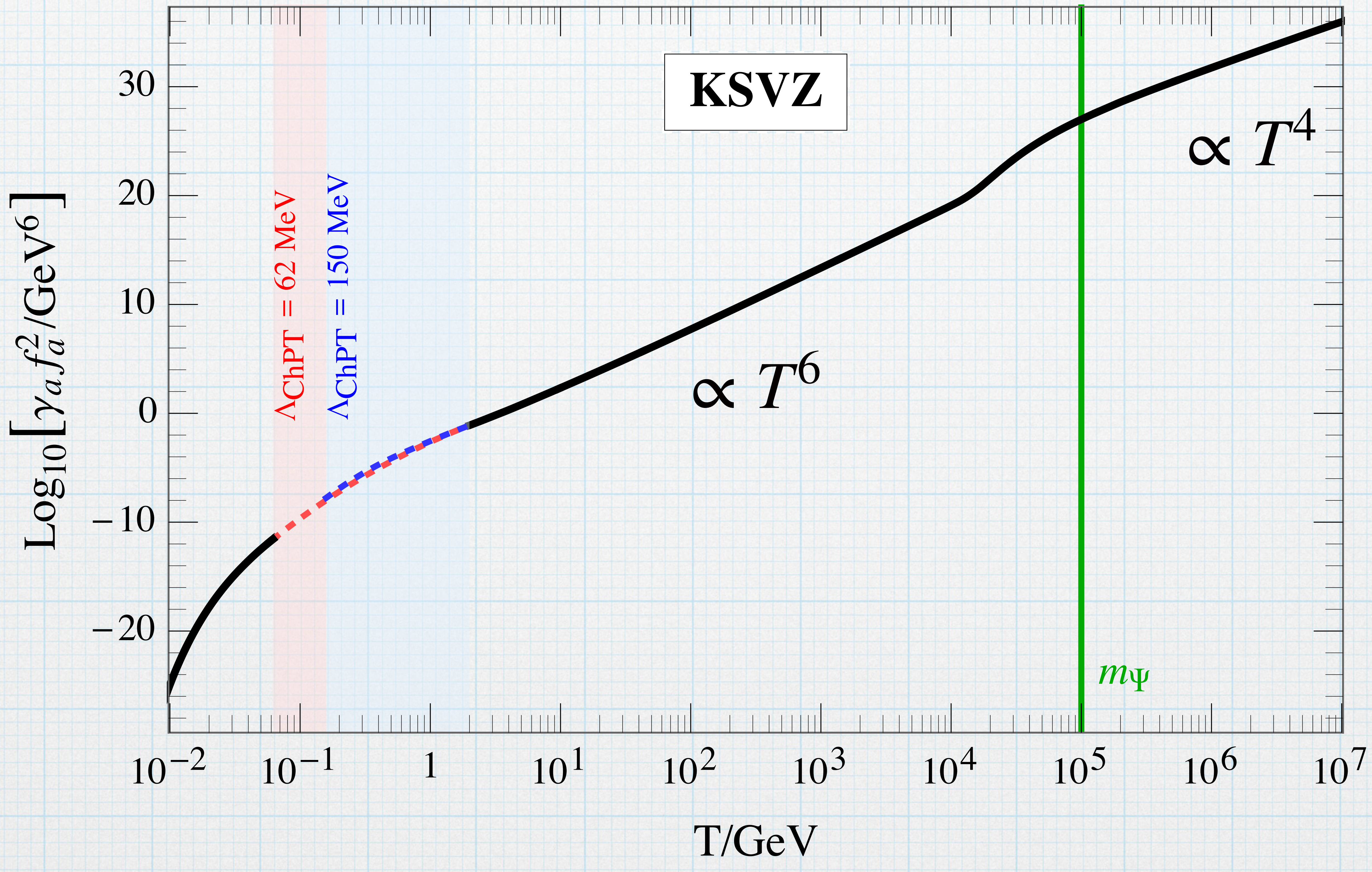


$$\mathscr{L}_{a\Psi} = i m_\Psi \frac{a}{f_a} \Psi_L^\dagger \Psi_R \rightarrow \quad T \gtrsim m_\Psi : \ \Psi + \bar{\Psi} \rightarrow g + a, \Psi/\bar{\Psi} + g \rightarrow \Psi/\bar{\Psi} + a$$

arXiv:2108.04259 arXiv:2108.05371

KSVZ Axion Production Rate

Total axion production rate for KSVZ model at different temperatures:



arXiv:2108.04259 arXiv:2108.05371

Effective Number of Relativistic Axion $\Delta N_{\rm eff}$

By calculating all axion interactions for any given model the number of effective extra degrees of freedom dominated by axions can be evaluated:

$$\Delta N_{\rm eff} \equiv N_{\rm eff} - N_{\rm eff}^{\rm SM} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_a}{\rho_\gamma} \simeq \frac{4}{7}\left(\frac{11}{4}\right)^{4/3} \left[\frac{2\pi^4}{45\zeta(3)}\, g_{*s}^{\rm SM}(T_{\rm CMB})\, Y_a^\infty\right]^{4/3}$$

$$\Delta N_{\rm eff} \simeq 75.6 \left[Y_a^\infty\right]^{4/3}$$

It can be constrained by CMB experiments which can falsify different axion models!

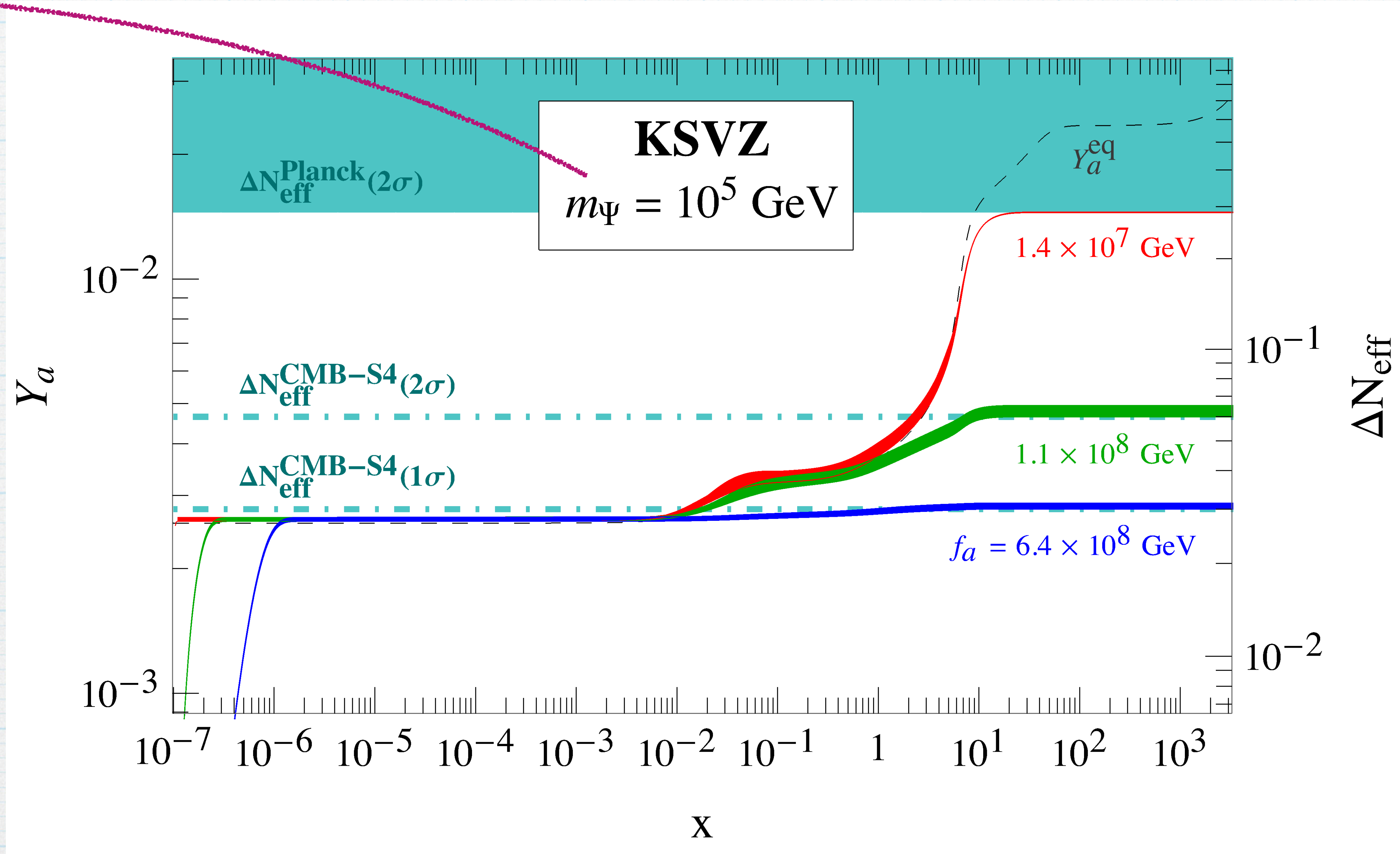
When axions thermalise $(\Gamma_a \gtrsim H)$ the value of $\Delta N_{\rm eff}$ gets its maximum. If axions do not reach the thermal equilibrium the final value of $\Delta N_{\rm eff}$ depends on the initial abundance of them!

arXiv:2003.01100 arxiv:2012.04736

Axion Yield in KSVZ Model

Axion yield evolution versus time or temperature in the KSVZ model:

Mass of Peccei-Quinn fermion



J. E. Kim, 1979 M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, 1980 arXiv:2108.04259 arXiv:2108.05371

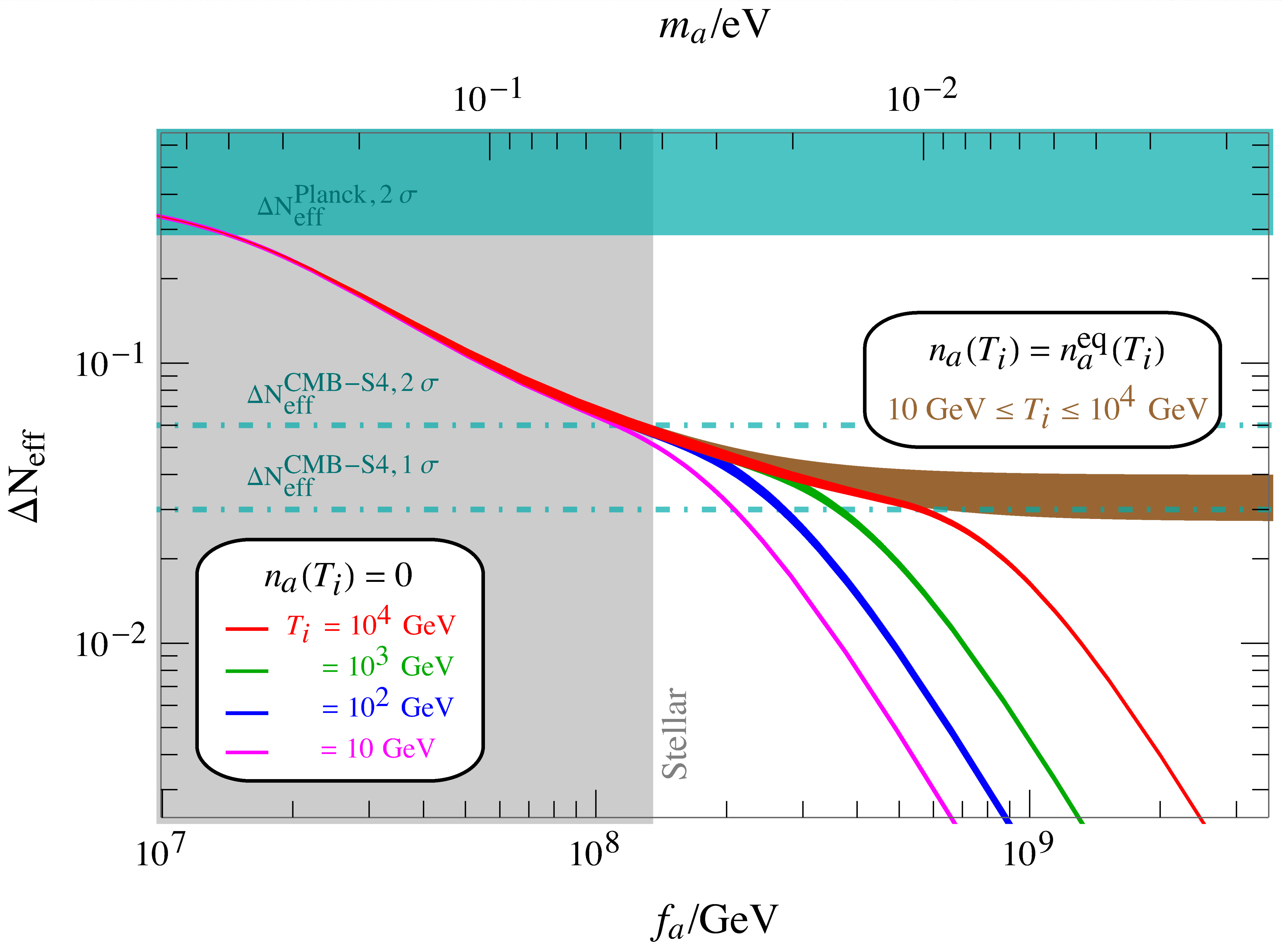
Detectability Perspective For Different Axion Decay Constants

Number of effective neutrinos $\Delta N_{\rm eff}$ versus axion decay constant f_a :

Planck is not sensitive enough for large f_a that is not constrained by supernova bound!

We consider different initial temperatures and initial axion densities.

CMB-S4 or a future experiment can probe a large portion of this parameter space in case $\Delta N_{\rm eff} \gtrsim 0.03$ for different f_a 's!



Stellar bound from supernova SN1987A $f_a \gtrsim 1.4 \times 10^8$ GeV !

arXiv:2108.04259 arXiv:2108.05371

DFSZ Model

Different parts of Lagrangian of DFSZ axion after Peccei-Quinn symmetry breaking and field redefinitions:

$$\mathscr{L}_{aG} = \frac{\alpha_s}{8\pi}\frac{a}{f_a} G^A_{\mu\nu}\widetilde{G}^{A\mu\nu}$$

Coupling depends on light quark masses and Higgs vev's

$$\mathscr{L}_{a\pi} = \frac{\partial_\mu a}{f_a}\frac{c_{a\pi\pi\pi}}{f_\pi}\left[\pi^0\pi^+\partial^\mu\pi^- + \pi^0\pi^-\partial^\mu\pi^+ - 2\pi^+\pi^-\partial^\mu\pi^0\right]$$

Pion decay constant

Strong interactions for DFSZ axion case above and below the QCD transition:

$$T \gtrsim \Lambda_{\mathrm{N}} :\; q + \bar{q} \to g + a,\, q/\bar{q} + g \to q/\bar{q} + a,\, g + g \to g + a$$

$$T \lesssim \Lambda_{\mathrm{ChPT}} : \pi^+ + \pi^0 \to \pi^+ + a,\, \pi^- + \pi^0 \to \pi^- + a,\, \pi^+ + \pi^- \to \pi^0 + a$$

In the DFSZ model axion also couples to leptons, quarks and weak gauge bosons and we have two Higgs doublets. Coupling of axion with leptons leads to leptons and photons interactions:

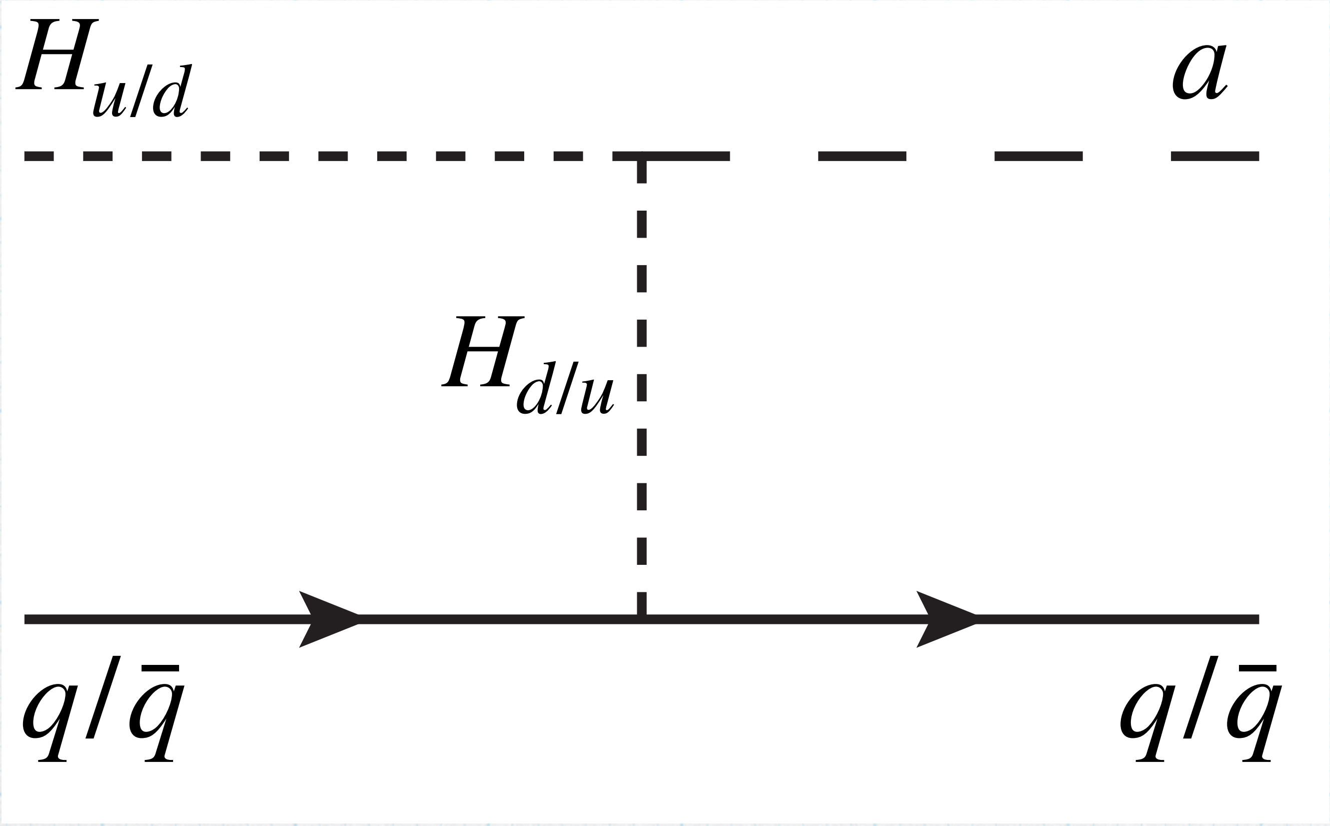
$$\mathscr{L}_{al} = \frac{\partial_\mu a}{f_a}\sum_f c_f \bar{f}\gamma^\mu f$$

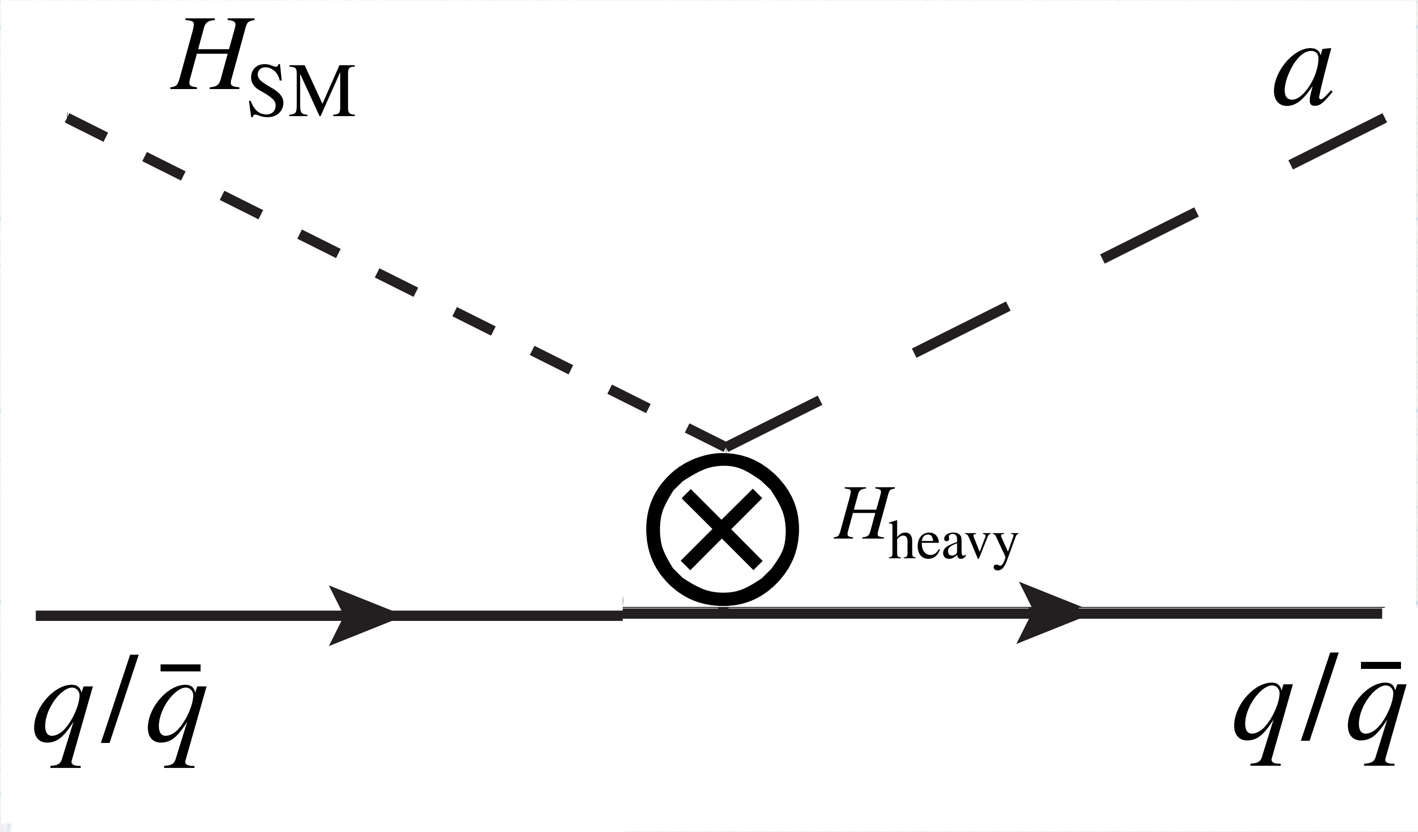
A. R. Zhitnitsky, 1980   M. Dine, W. Fischler, and M. Srednicki, 1981   arxiv:2012.04736   arXiv:0203221

DFSZ Model

Electroweak sector interaction for axion production assuming the decoupling limit for the two Higgs doublet model:

$$\mathcal{L}_{\mathrm{DFSZ}}^{\mathrm{linear}} \supset i \frac{a}{f_a} \frac{B}{3} \left(H_u^T i\sigma^2 H_d \right) + \mathrm{h.c.}$$





Below the pseudo-scalar mass scale one can integrate out the heavy Higgses:

$$\mathcal{L}_{\mathrm{DFSZ}}^{\mathrm{linear}}\Big|_{T<m_A} = i\frac{a}{f_a}\left(-\frac{\cos\alpha\cos\beta}{3}\bar{Q}_L \widetilde{H}_{\mathrm{SM}} Y^{(u)} u_R - \frac{\sin\alpha\sin\beta}{3}\bar{Q}_L H_{\mathrm{SM}} Y^{(d)} d_R - \frac{\sin\alpha\sin\beta}{3}\bar{L}_L H_{\mathrm{SM}} Y^{(e)} d_R \right) + \mathrm{h.c.}$$

arXiv:2108.04259 arXiv:2108.05371

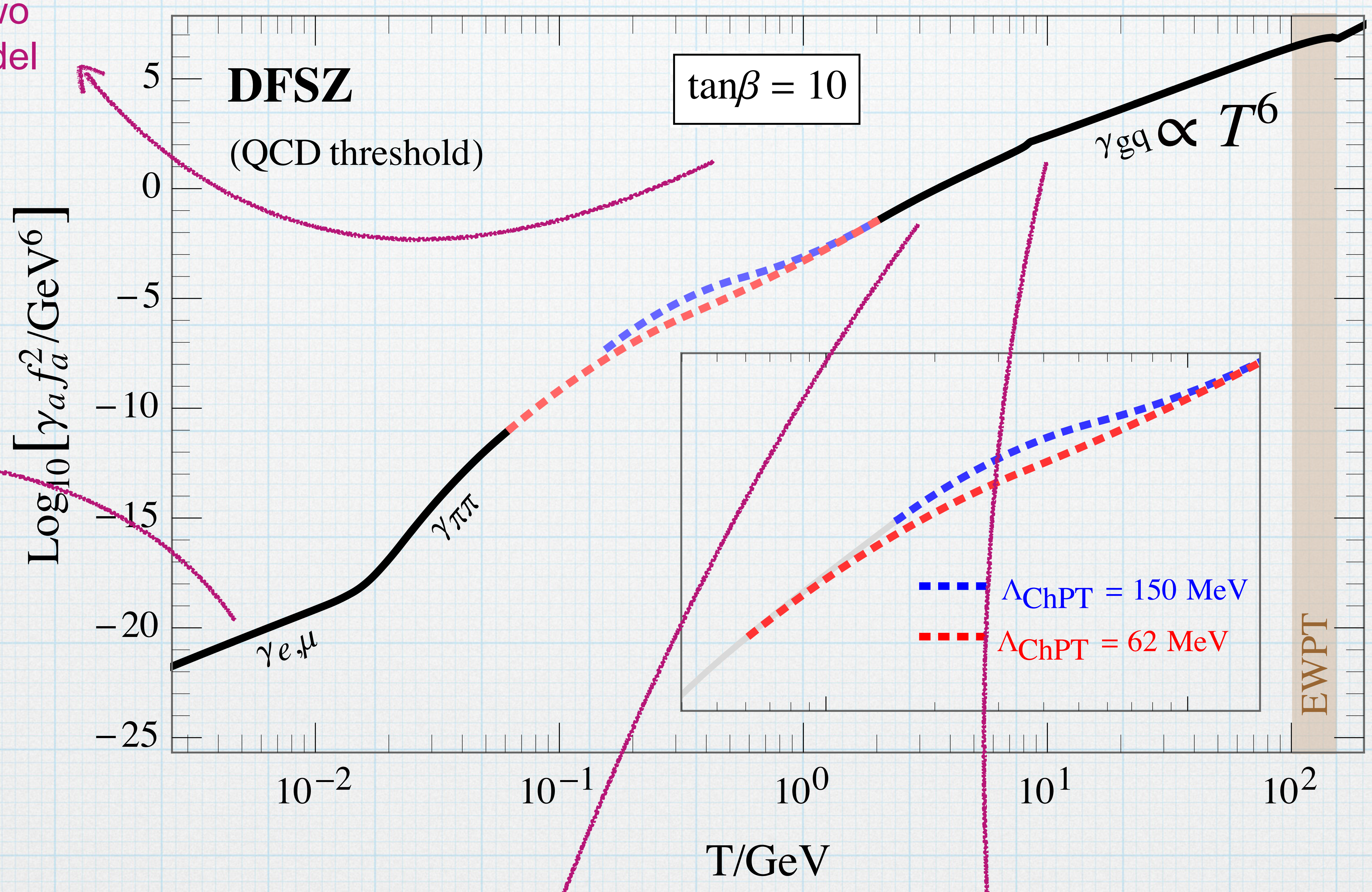
Axion Production Rate in DFSZ Model

Axion production rate for DFSZ model at low temperatures below the EWPT:

Ratio of vevs in two Higgs doublet model

Lepton interactions

$$l + \bar{l} \rightarrow \gamma + a$$
$$l/\bar{l} + \gamma \rightarrow l/\bar{l} + a$$



Quark interactions with the EW sector

1-PI correction for the SM quarks

[arXiv:2108.04259](https://arxiv.org/abs/2108.04259) [arXiv:2108.05371](https://arxiv.org/abs/2108.05371)

Axion Production Rate in DFSZ Model

Axion production rate for DFSZ model around the psuedo-scalar mass and above the EW scale:

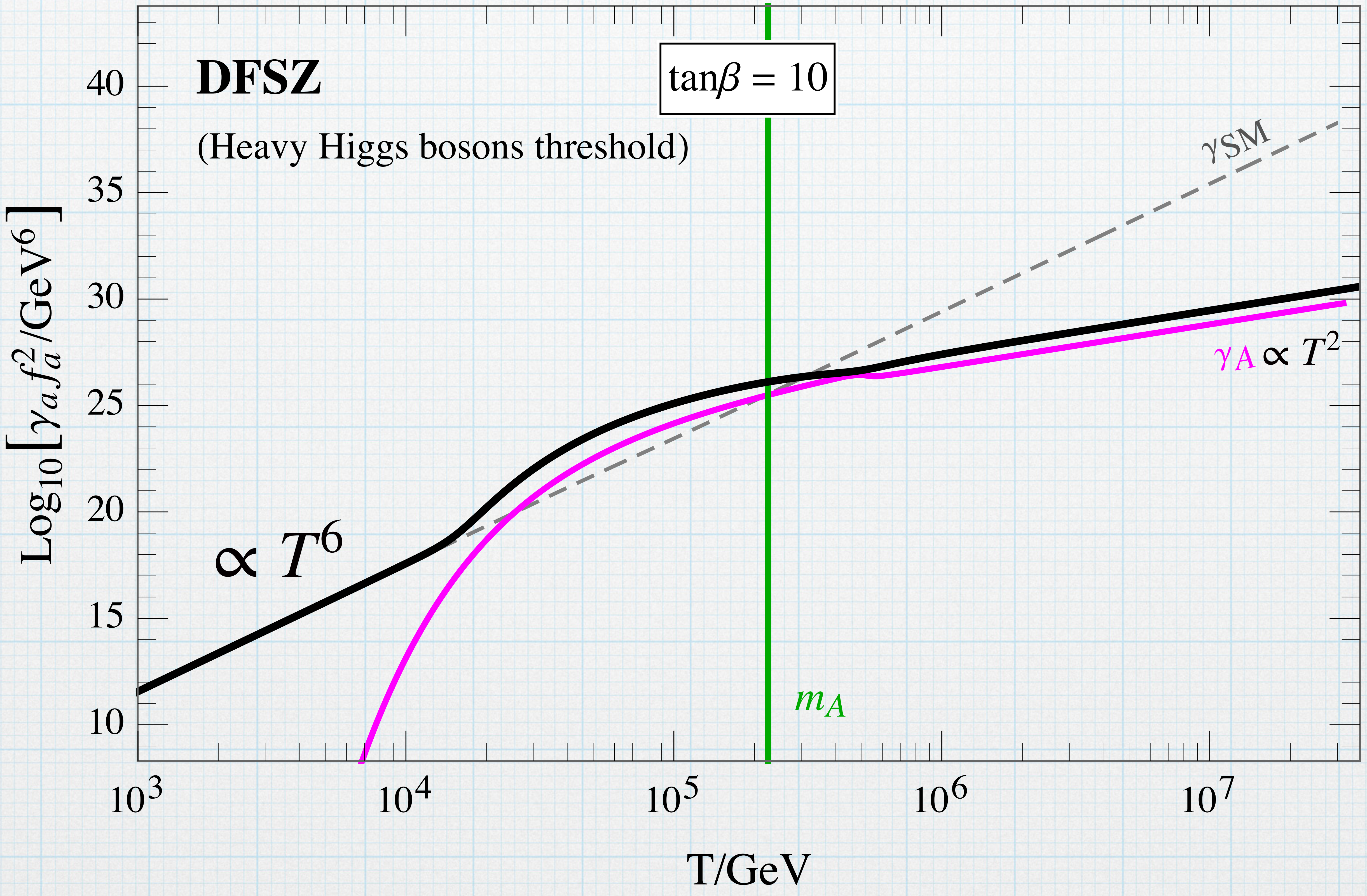
Fermion interaction with the EW sector

$$f+\bar{f} \to h^0/G^0 + a$$

$$f+\bar{f}' \to G^\pm + a$$

$$f/\bar{f} + h^0/G^0 \to f\bar{f} + a$$

$$f/\bar{f} + G^\pm \to f'/\bar{f}' + a$$



SM fermion interactions with the EW sector at high temperatures

$$u/d + \bar{u}/\bar{d} \to H^{(\dagger)}_{d/u} + a$$

$$u/d(\bar{u}/\bar{d}) + H^{(\dagger)}_{d/u} \to u/d(\bar{u}/\bar{d}) + a$$

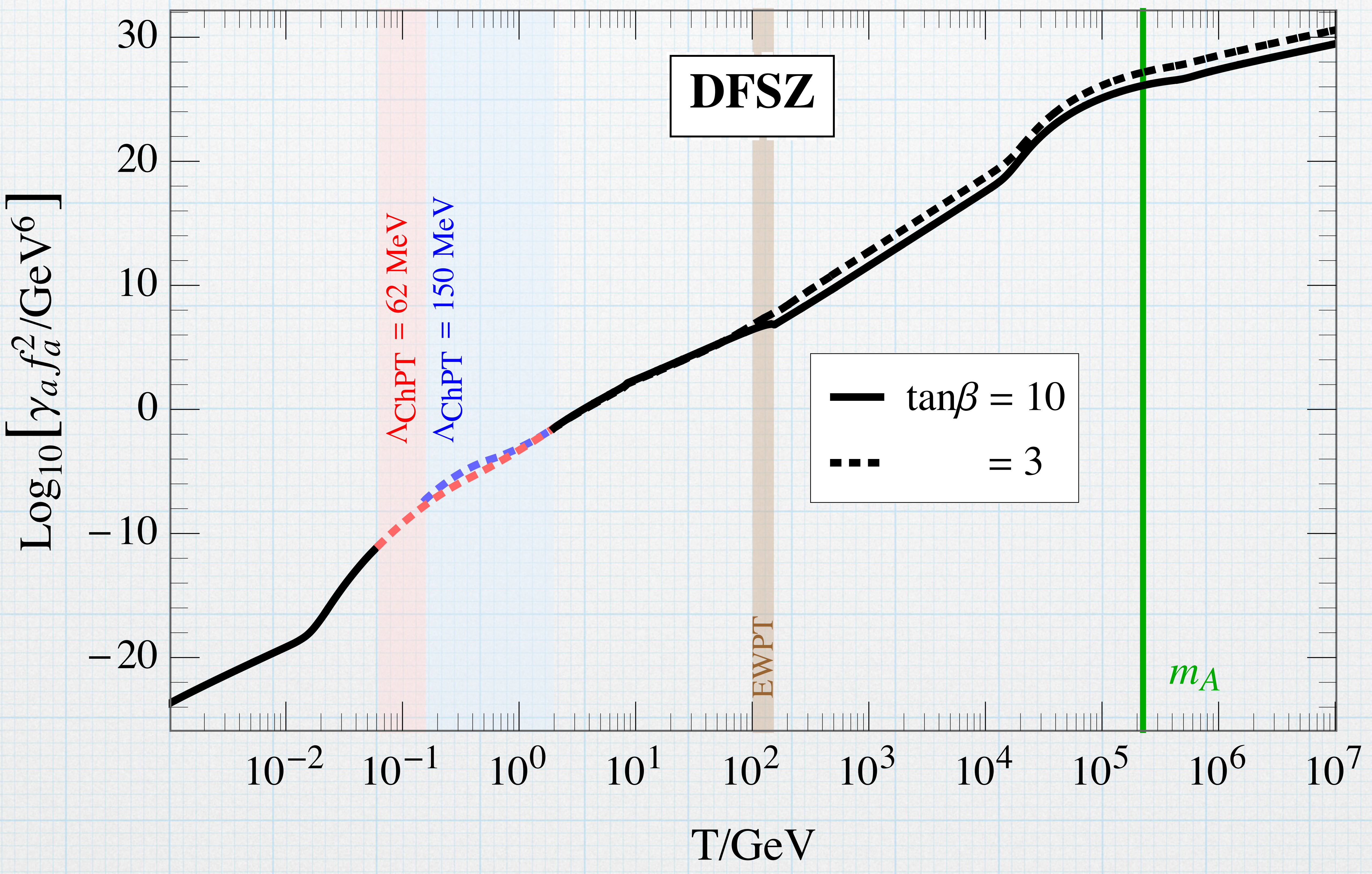
$$H^{(\dagger)}_{u/d} + V_\mu \to H^{(\dagger)}_{d/u} + a$$

$$H^{(\dagger)}_{u/d} + H^{(\dagger)}_{d/u} \to V_\mu + a$$

arXiv:2108.04259 arXiv:2108.05371

Axion Production Rate in DFSZ Model

Total axion production rate for DFSZ model at different temperatures:

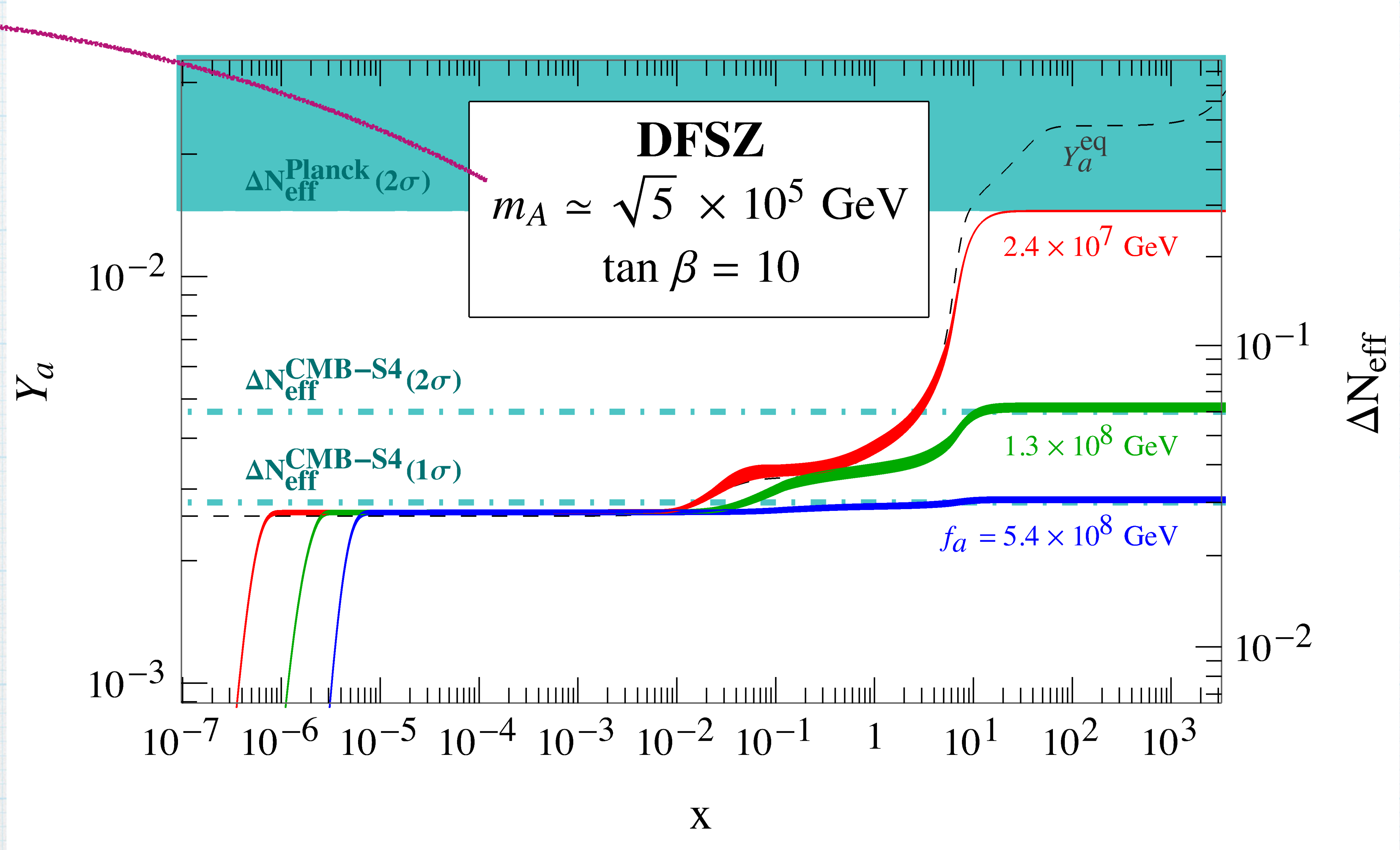


arXiv:2108.04259 arXiv:2108.05371

Axion Yield in DFSZ Model

Axion yield evolution versus time or temperature:

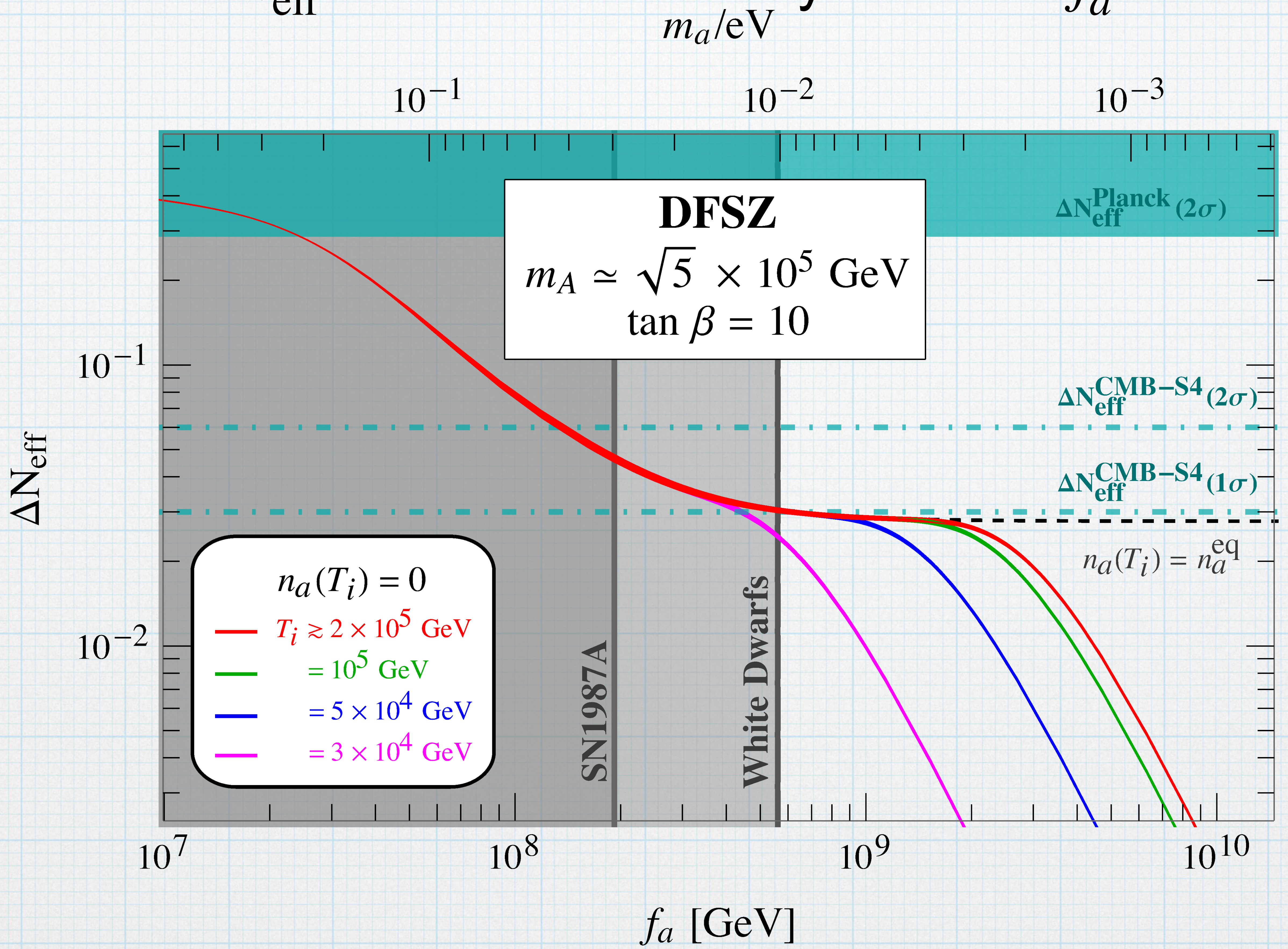
Pseudo-scalar mass



arXiv:2108.04259 arXiv:2108.05371

Detectability Perspective For Different Axion Decay Constants

Number of effective neutrinos $\Delta N_{\rm eff}$ versus axion decay constant f_a for DFSZ model:

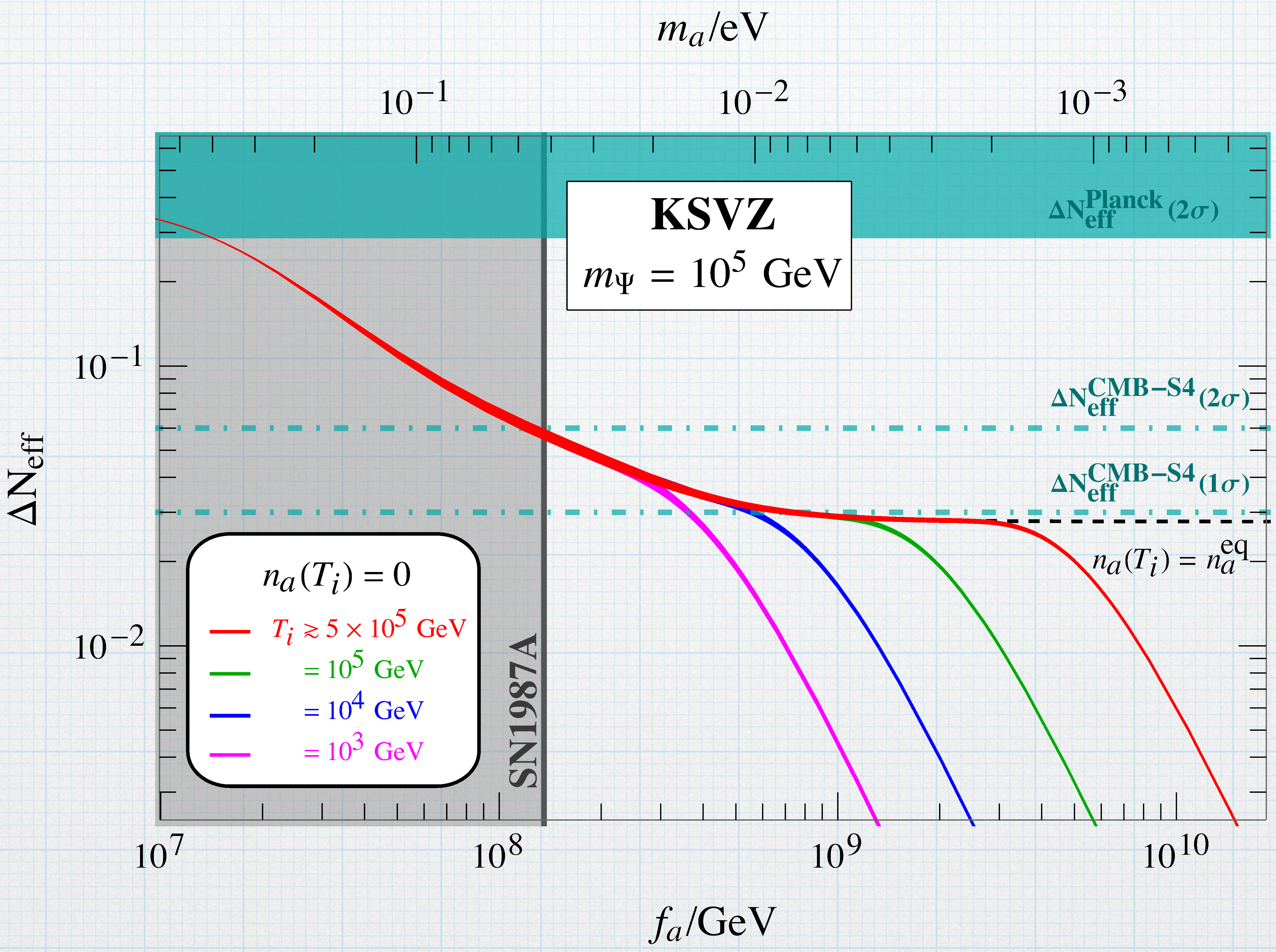


The astrophysical bounds on f_a in the DFSZ model are different from the KSVZ case due to different couplings with SM particles.

arXiv:2108.04259 arXiv:2108.05371

Detectability Perspective of KSVZ Axion

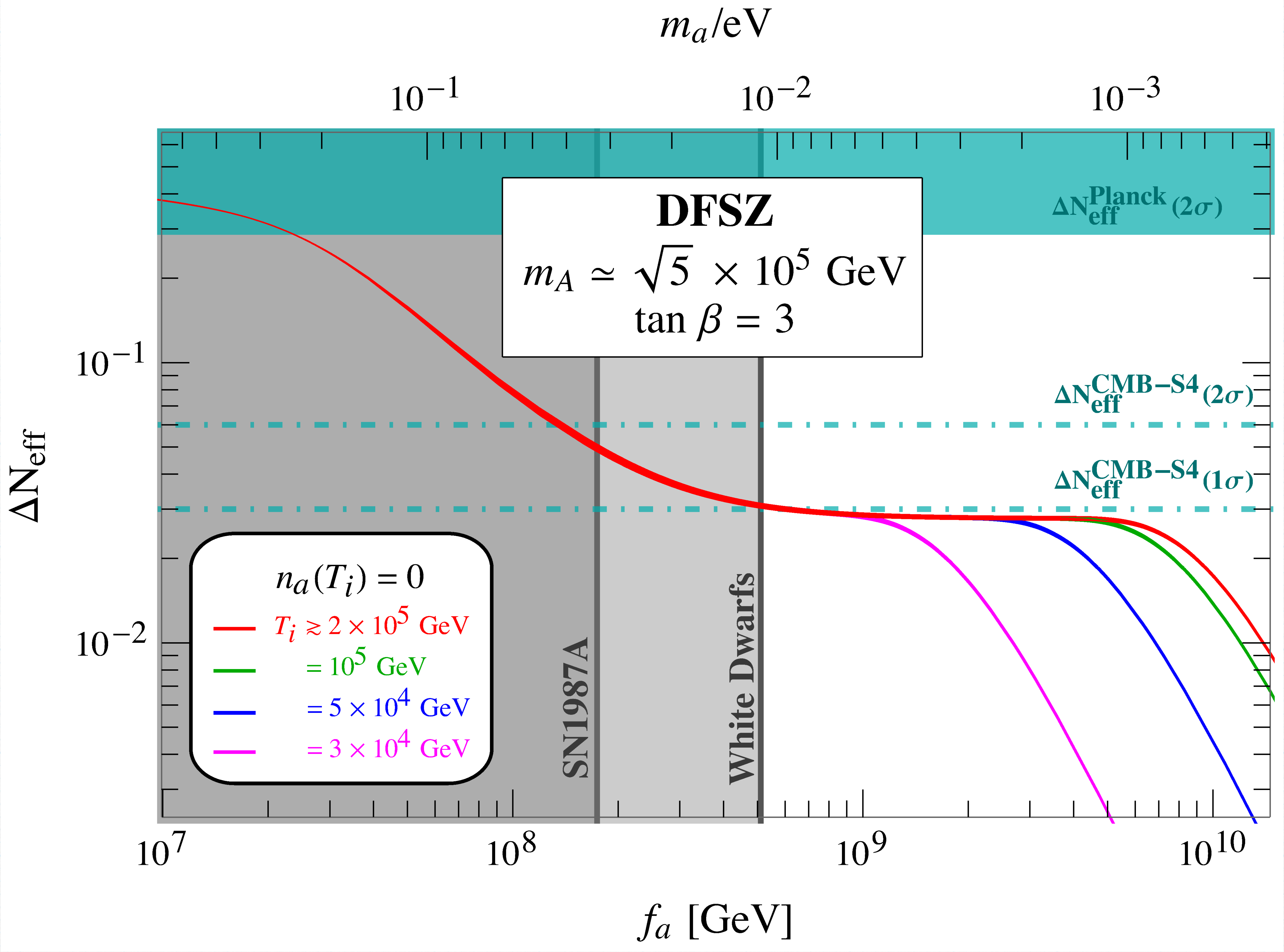
Number of effective neutrinos ΔN_{eff} versus axion decay constant f_a for KSVZ axion:



arXiv:2108.04259 arXiv:2108.05371

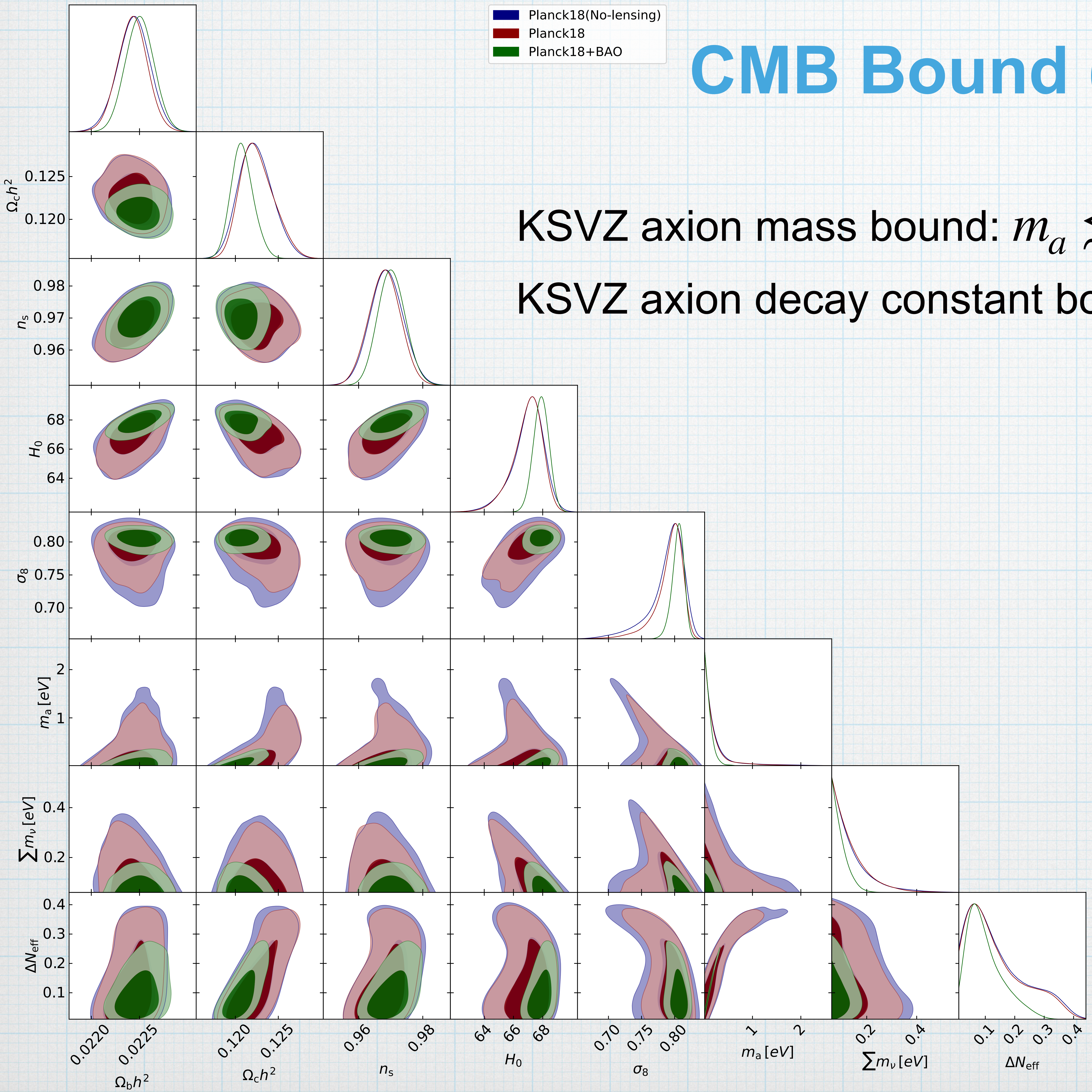
Detectability Perspective of DFSZ Axion

The astrophysical bounds on f_a in the DFSZ model are different from the KSVZ case due to different couplings with SM particles.



arXiv:2108.04259 arXiv:2108.05371

CMB Bound (KSVZ)

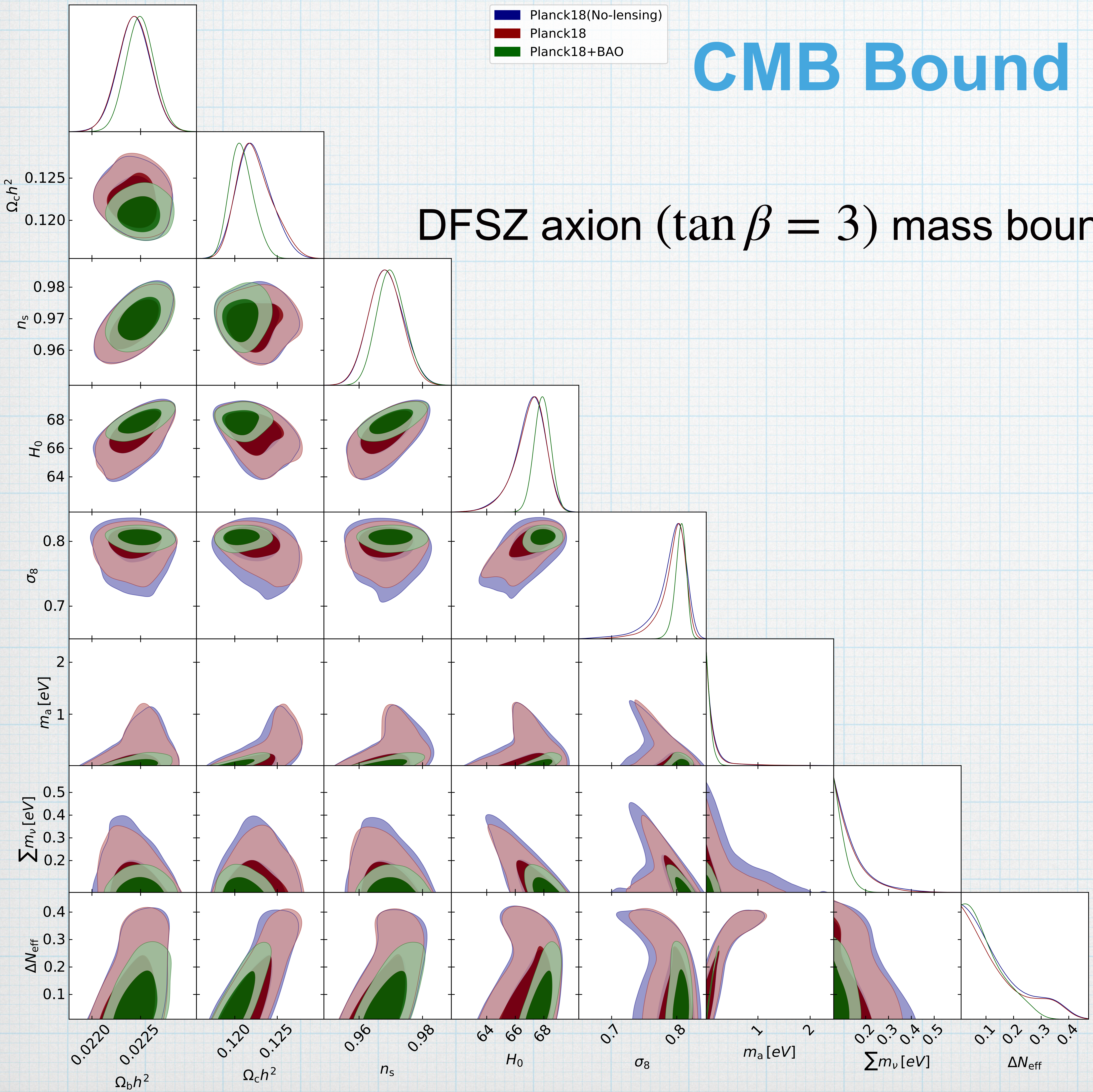


KSVZ axion mass bound: $m_a \lesssim 0.282\ (0.420)$ eV

KSVZ axion decay constant bound: $f_a \gtrsim 2.02\ (1.35) \times 10^7$ GeV

D'Eramo, Di Valentino, Giarè, Hajkarim, Melchiorri, Mena, Renzi, Yun, arXiv:2205.07849

CMB Bound (DFSZ)



DFSZ axion ($\tan\beta = 3$) mass bound $m_a \lesssim 0.209\ (0.293)$ eV

D'Eramo, Di Valentino, Giarè, Hajkarim, Melchiorri, Mena, Renzi, Yun, arXiv:2205.07849

Dark Radiation in the Phase Space

Boltzmann equation for distribution function of dark radiation X:

Collision operator

$$\omega_1 \frac{df_X(k_1,t)}{dt} = C[f_X(k_1,t)]$$
$$\frac{d\rho_{\mathcal{B}}}{dt} + 3H(1+w_{\mathcal{B}})\rho_{\mathcal{B}} = -g_X \int \frac{d^3k_1}{(2\pi)^3} C[f_X(k_1,t)]$$
$$H = \frac{\sqrt{\rho_{\mathcal{B}}+\rho_X}}{\sqrt{3}M_{\rm Pl}}$$

$$C[f_X(k_1,t)] = \frac{\ell}{2}\int \prod_{i=1}^{n} d\Pi_i \prod_{j=n+1}^{n+m} d\Pi_j \prod_{r=2}^{\ell} d\mathcal{K}_r\, (2\pi)^4\delta^4\left(P_{\rm in}-P_{\rm fin}\right)|\mathcal{M}|^2$$
$$\times \prod_{i=1}^{n} f_{\mathcal{B}_i} \prod_{j=n+1}^{n+m}(1\pm f_{\mathcal{B}_j}) \prod_{r=1}^{\ell}(1\pm f_X(k_r)) \left[1-\prod_{r=1}^{\ell} e^{\omega_r/T}\frac{f_X(k_r)}{1\pm f_X(k_r)}\right]$$

Background radiation

Distribution function

$$\frac{df_X(k,t)}{dt} = \mathcal{C}(k,t)\left(1-\frac{f_X(k,t)}{f_X^{\rm eq}(k,t)}\right)$$
$$\frac{d\rho_{\mathcal{B}}}{dt} + 3H(1+w_{\mathcal{B}})\rho_{\mathcal{B}} = -g_X\int\frac{d^3k}{(2\pi)^3}\, k\,\mathcal{C}(k,t)\left(1-\frac{f_X(k,t)}{f_X^{\rm eq}(k,t)}\right)$$
$$H = \frac{\sqrt{\rho_{\mathcal{B}}+\rho_X}}{\sqrt{3}M_{\rm Pl}}$$

$$\frac{df_X(q,A)}{d\log A} = \frac{\mathcal{C}(q,A)}{H(A)}\left(1-\frac{f_X(q,A)}{f_X^{\rm eq}(q,A)}\right)$$
$$\frac{dR_{\mathcal{B}}}{d\log A} + (3w_{\mathcal{B}}-1)R_{\mathcal{B}} = -\frac{g_X}{2\pi^2}\int dq\, q^3\left(1-\frac{f_X(q,A)}{f_X^{\rm eq}(q,A)}\right)\frac{\mathcal{C}(q,A)}{H(A)}$$
$$H(A) = \frac{\sqrt{R_{\mathcal{B}}(A)+R_X(A)}}{\sqrt{3}M_{\rm Pl}}\frac{T_I^2}{A^2}$$

$$R_{\mathcal{B}} \equiv \frac{\rho_{\mathcal{B}}a^4}{T_I^4 a_I^4} = \frac{\rho_{\mathcal{B}}}{T_I^4}A^4\,, \qquad R_X \equiv \frac{\rho_X a^4}{T_I^4 a_I^4} = \frac{\rho_X}{T_I^4}A^4$$

$$R_X(A) = \frac{g_X}{2\pi^2}\int dq\, q^3 f_X(q,A)$$

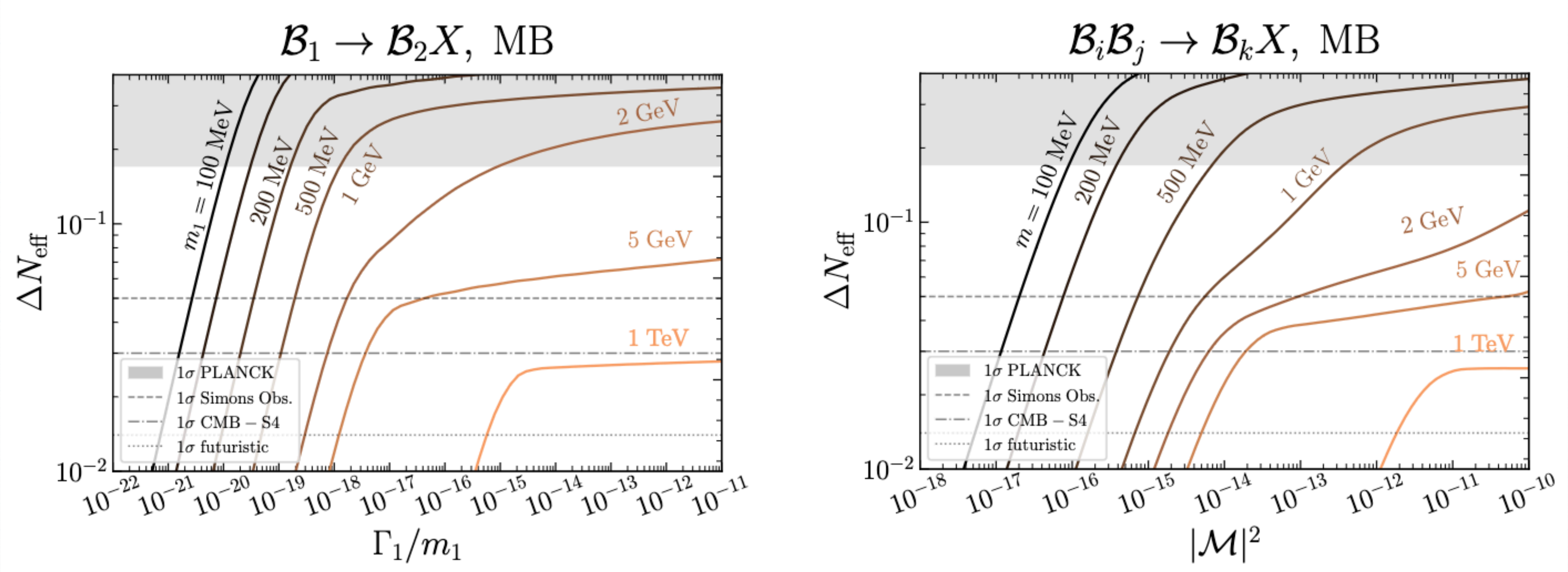
$$q \equiv \frac{ka}{(a_I T_I)} = \frac{kA}{T_I}$$

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3}\frac{R_X(A_F)}{2R_{\mathcal{B}}(A_F)/g_{*\rho}(T_F)}$$

Comoving momentum

arXiv:2311.04974

Dark Radiation in the Phase Space



arXiv:2311.04974

Dark Radiation from Energy Density

Boltzmann equation for energy density:

$$\frac{d\rho_X}{dt} + 4H\rho_X = \ell\left[1 - \left(e^{\mu_X/T}\right)^\ell\right] \int \prod_{i=1}^{n} d\Pi_i \prod_{j=n+1}^{n+m} d\Pi_j \prod_{r=1}^{\ell} d\mathcal{K}_r \,(2\pi)^4 \delta^4\left(P_{\rm in} - P_{\rm fin}\right)$$
$$k_1 \,|\mathcal{M}|^2 \prod_{i=1}^{n} f_{\mathcal{B}_i} \prod_{j=n+1}^{n+m} \left(1 \pm f_{\mathcal{B}_j}\right) \prod_{r=1}^{\ell} \left(1 \pm f_X(k_r)\right) \,.$$

$$\epsilon_X \equiv \ell \int \prod_{i=1}^{n} d\Pi_i \prod_{j=n+1}^{n+m} d\Pi_j \prod_{r=1}^{\ell} d\mathcal{K}_r (2\pi)^4 \delta^4\left(P_{\rm in} - P_{\rm fin}\right) k_1 |\mathcal{M}|^2 \prod_{i=1}^{n} f_{\mathcal{B}_i} \prod_{j=n+1}^{n+m} (1 \pm f_{\mathcal{B}_j})$$

$$\frac{d\rho_X}{dt} + 4H\rho_X = \epsilon_X \left[1 - \left(\frac{\rho_X}{\rho_X^{\rm eq}}\right)^\ell\right]$$

$$R_{\mathcal{B}}(A) = \frac{\pi^2}{30} g_{*\rho}^{\mathcal{B}}(T) A^4 \left(\frac{T}{T_I}\right)^4$$

$$\frac{d\rho_X}{dt} + 4H\rho_X = \epsilon_X \left[1 - \left(\frac{\rho_X}{\rho_X^{\rm eq}}\right)^\ell\right]$$
$$\frac{d\rho_{\mathcal{B}}}{dt} + 3(1 + w_{\mathcal{B}})H\rho_{\mathcal{B}} = -\epsilon_X \left[1 - \left(\frac{\rho_X}{\rho_X^{\rm eq}}\right)^\ell\right]$$
$$H = \frac{\sqrt{\rho_X + \rho_{\mathcal{B}}}}{\sqrt{3}M_{\rm Pl}}$$

$$\frac{dR_X}{d\log A} = \frac{A^4}{H}\frac{\epsilon_X}{T_I^4}\left[1 - \left(\frac{R_X}{R_X^{\rm eq}}\right)^\ell\right]$$
$$\frac{dR_{\mathcal{B}}}{d\log A} + (3w_{\mathcal{B}} - 1)R_{\mathcal{B}} = -\frac{A^4}{H}\frac{\epsilon_X}{T_I^4}\left[1 - \left(\frac{R_X}{R_X^{\rm eq}}\right)^\ell\right]$$
$$H = \frac{\sqrt{R_{\mathcal{B}} + R_X}}{\sqrt{3}M_{\rm Pl}}\frac{T_I^2}{A^2}$$

$$\Delta N_{\rm eff} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_X(t_{\rm CMB})}{\rho_\gamma(t_{\rm CMB})}$$

arXiv:2311.04974

Dark Radiation from Number Density

Boltzmann equation for number density:

$$\frac{dn_X}{dt} + 3Hn_X = \ell \times \int \prod_{i=1}^{n} d\Pi_i \prod_{j=n+1}^{n+m} d\Pi_j \prod_{r=1}^{\ell} d\mathcal{K}_r \, (2\pi)^4 \delta^4 \left(P_{\rm in} - P_{\rm fin}\right) |\mathcal{M}|^2$$
$$\times \prod_{i=1}^{n} f_{\mathcal{B}_i} \prod_{j=n+1}^{n+m} (1 \pm f_{\mathcal{B}_j}) \prod_{r=1}^{\ell} (1 \pm f_X(k_r)) \left[1 - \prod_{r=1}^{\ell} e^{\omega_r/T} \frac{f_X(k_r)}{1 \pm f_X(k_r)}\right]$$

Number density

$$n_X = g_X \int \frac{d^3k}{(2\pi)^3} f_X(k,t)$$

$$\frac{dn_X}{dt} + 3Hn_X = \gamma_X \left[1 - \left(\frac{n_X}{n_X^{\rm eq}}\right)^{\ell}\right]$$

Energy density

$$\rho_X = g_X \xi_{\rho X} \frac{\pi^2}{30} \left(\frac{\pi^2}{\zeta(3)} \frac{n_X}{g_X \xi_{n X}}\right)^{4/3}$$

$$\Delta N_{\rm eff} = g_X \xi_{\rho X} \frac{4}{7} \left(\frac{11}{4}\right)^{4/3} \left(\frac{\frac{2\pi^4}{45\zeta(3) g_X \xi_{n X}} Y_X(T_{\rm CMB})}{1 - \frac{2\pi^4}{45\zeta(3)} \frac{\xi_{\rho X}}{\xi_{n X}} Y_X(T_{\rm CMB})} g_{*s}^{\mathcal{B}}(T_{\rm CMB}) \right)^{4/3}$$

$$\Delta N_{\rm eff} = g_X \xi_{\rho X} \frac{4}{7} \left(\frac{11}{4}\right)^{4/3} \left(\frac{2\pi^4}{45\zeta(3)} \frac{g_{*s}^{\mathcal{B}}(T_{\rm CMB})}{g_X \xi_{n X}}\right)^{4/3} Y_X(T_{\rm CMB})^{4/3}$$

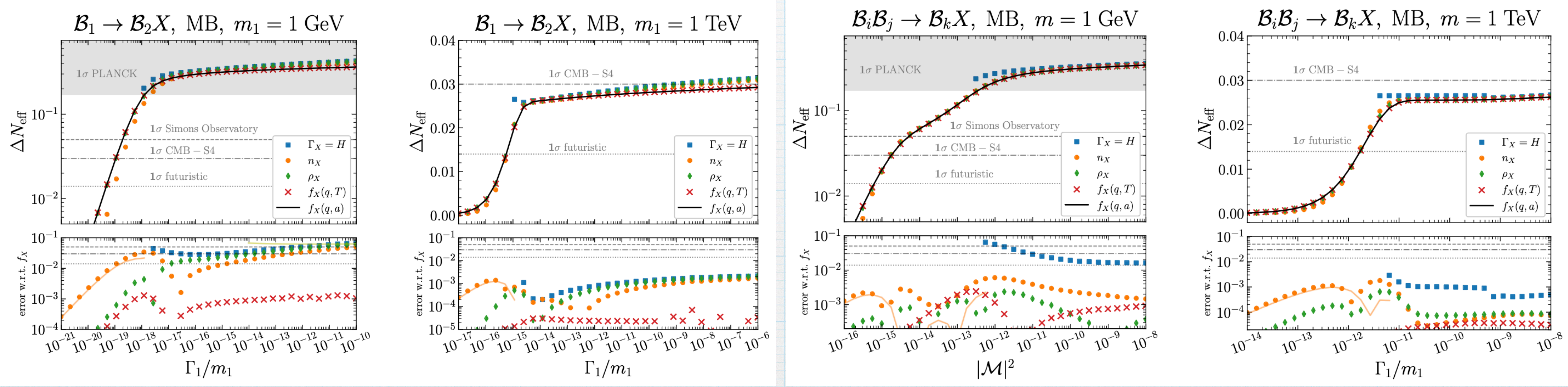
arXiv:2311.04974

Dark Radiation from Interaction Rate and Uncertainties

Interaction rate:

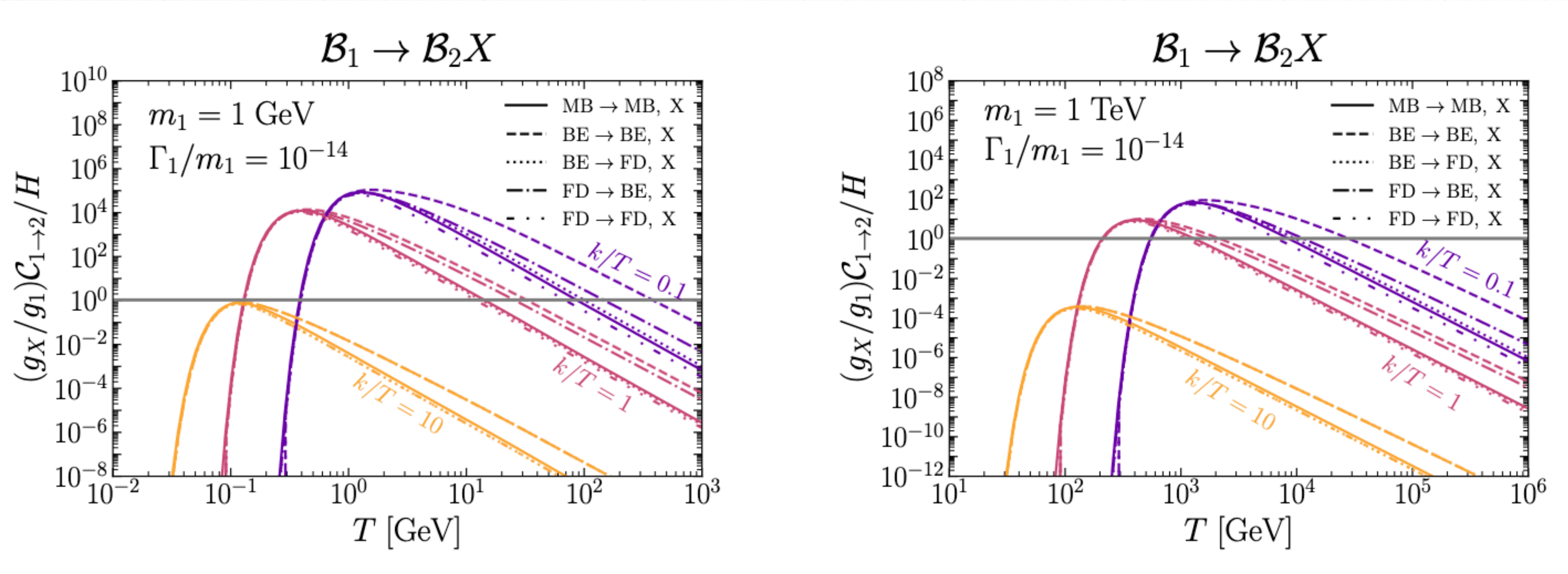
$$\Gamma_X \equiv \frac{\ell}{n_X^{\rm eq}} \int \prod_{i=1}^{n} d\Pi_i \prod_{j=n+1}^{n+m} d\Pi_j \prod_{r=1}^{\ell} d\mathcal{K}_r \, (2\pi)^4 \delta^4 \left(P_{\rm in} - P_{\rm fin}\right) |\mathcal{M}|^2$$
$$\times \prod_{i=1}^{n} f_{\mathcal{B}_i} \prod_{j=n+1}^{n+m} \left(1 \pm f_{\mathcal{B}_j}\right) \prod_{r=1}^{\ell} \left(1 \pm f_X^{\rm eq}(k_r)\right) \; .$$

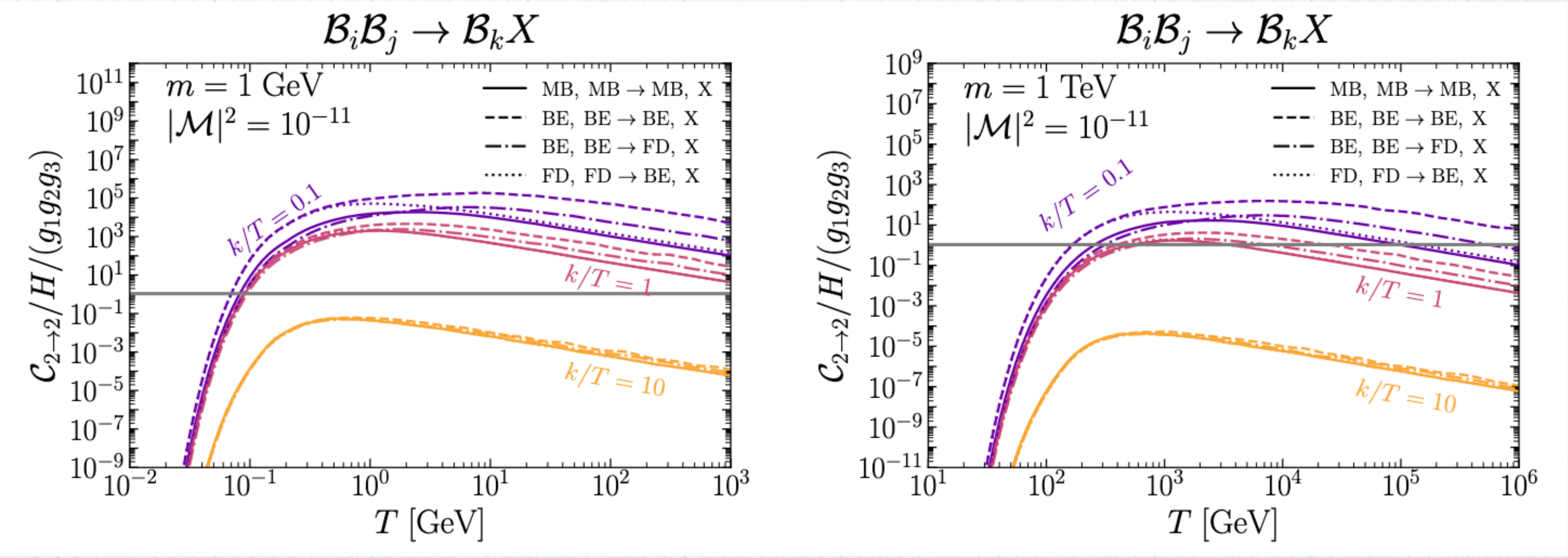
$$\Gamma_X(T_{\rm D}) = H(T_{\rm D})$$

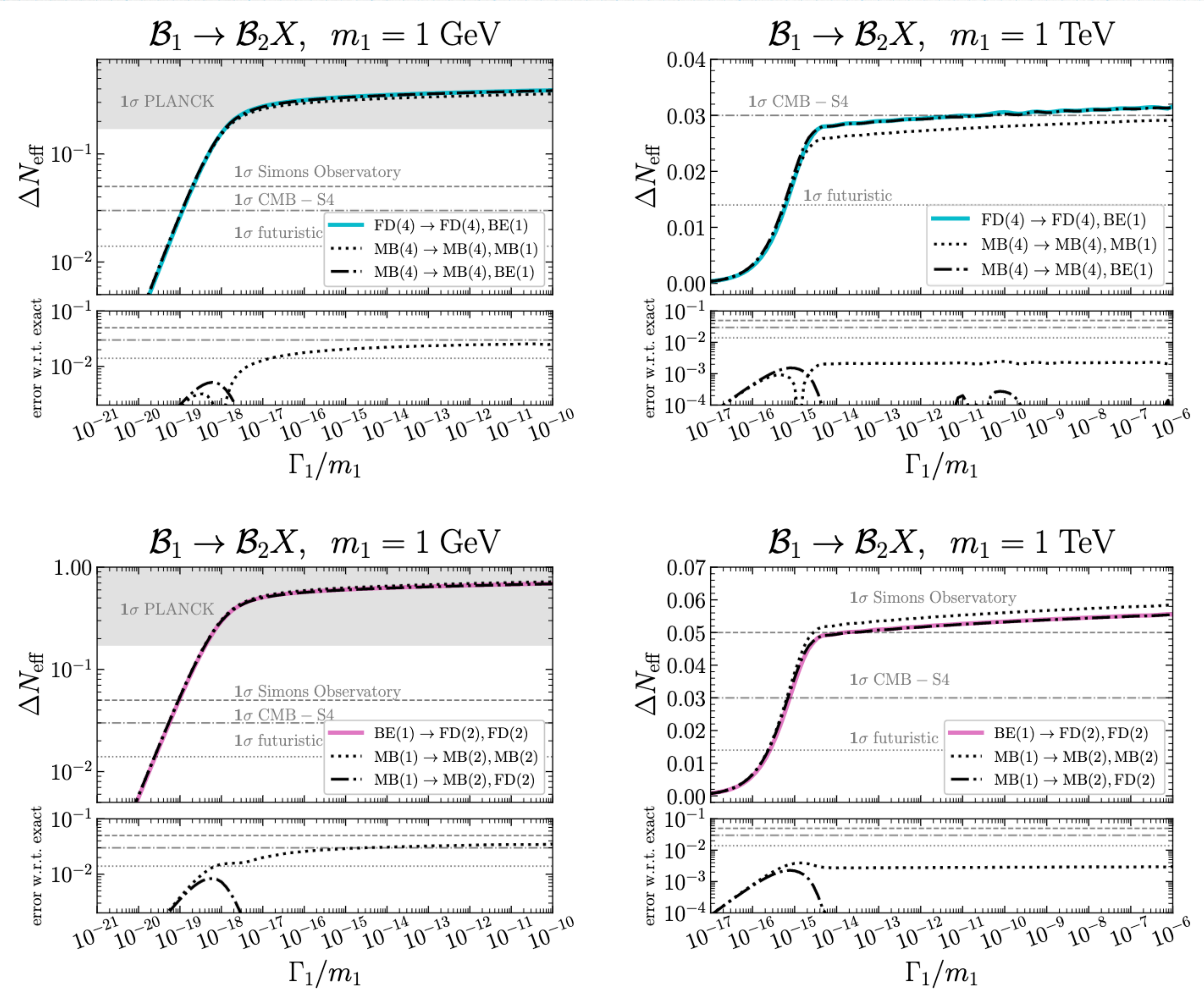


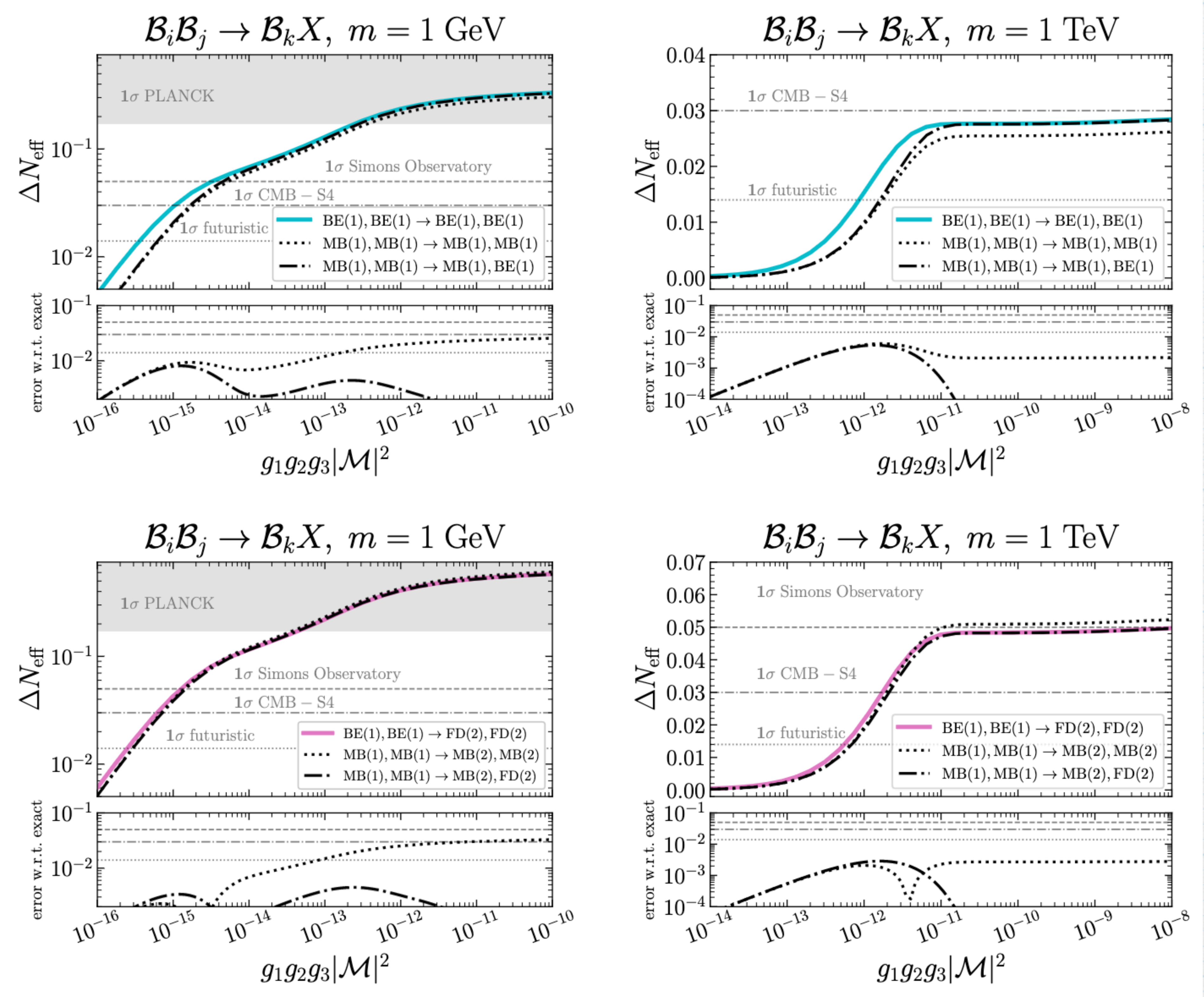
arXiv:2311.04974

Uncertainties in Classical and Quantum Statistics









arXiv:2311.04974

Summary and Conclusions

* Computing the axion production rate below and above the QCD crossover transition and matching the two limits with the interpolation that is physically meaningful. Also we did calculation across the electroweak transition and above that…

* Considering precise interaction rates of axion including thermal effects at different scales that improves the theoretical prediction for dark radiation from KSVZ and DFSZ axion models.

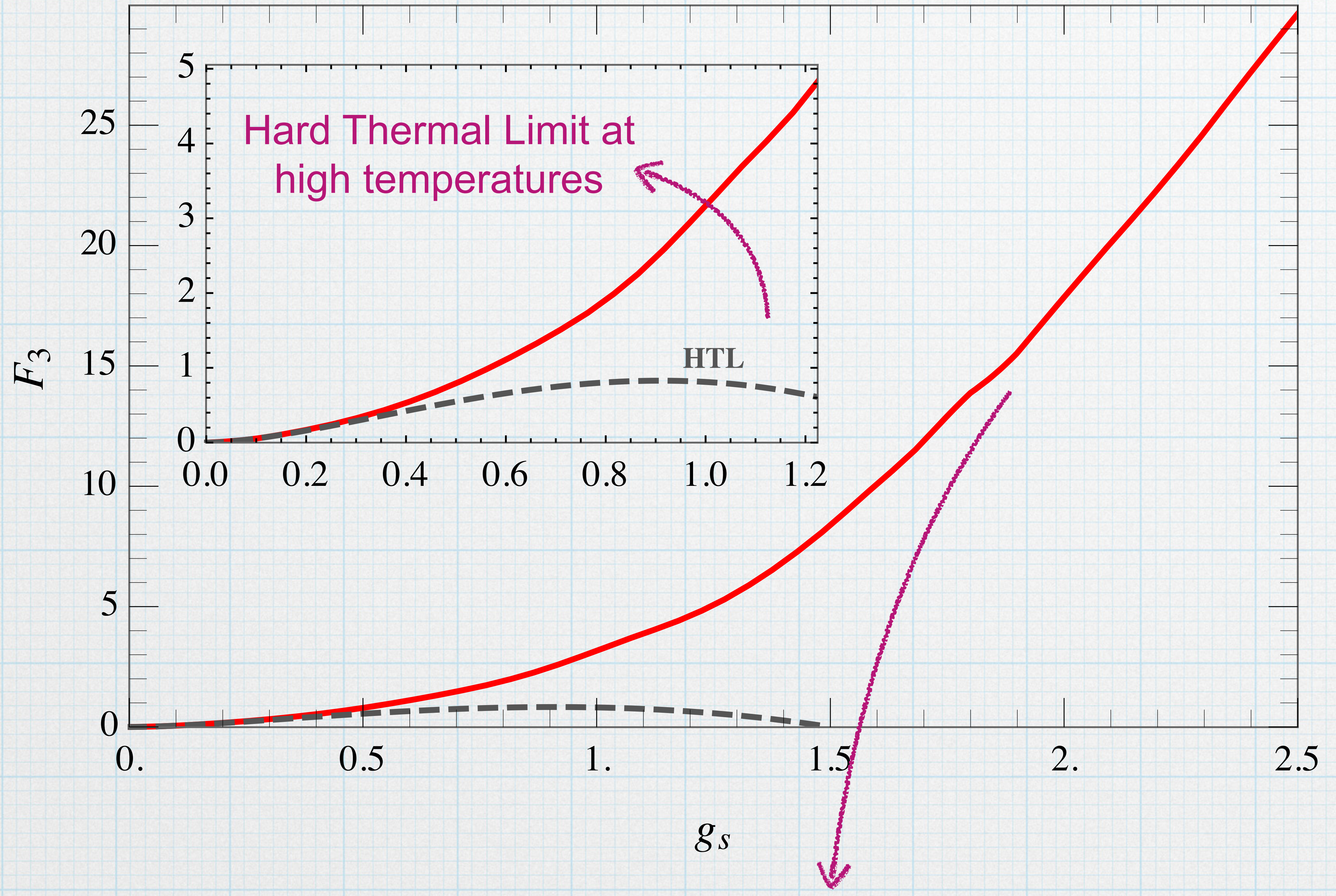
* Using accurate treatment of thermal bath at different temperatures to have a correct estimation of axion abundance and put a constraint on axion mass.

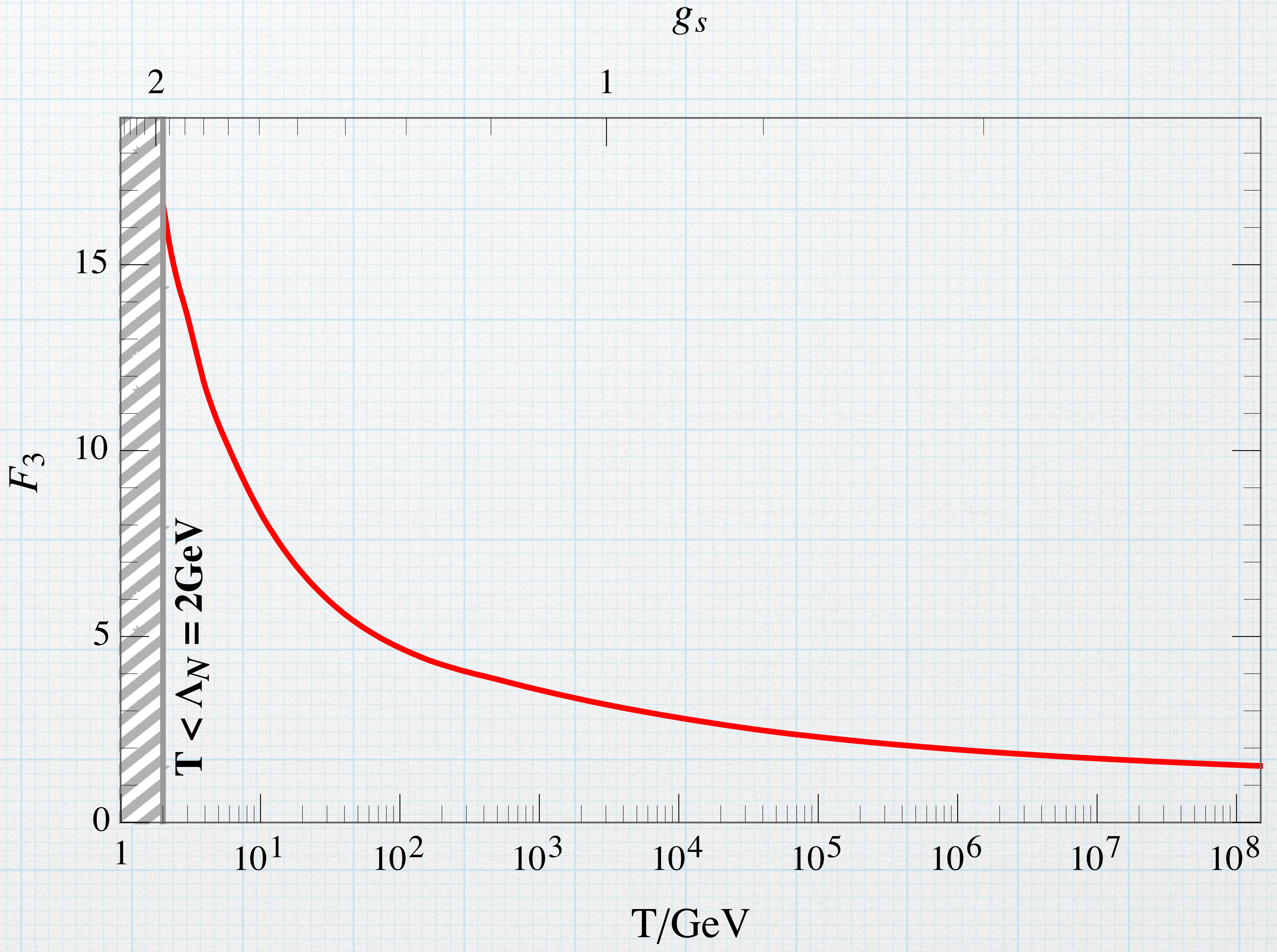
* Predicting the precise value of ΔN_{eff} for different axion models is theoretically motivated for future probes by experiments like CMB-S4 or any future mission.

* Computing Boltzmann equation in the phase space for dark radiation and using correct quantum statistics is important for the precise estimation of dark radiation!

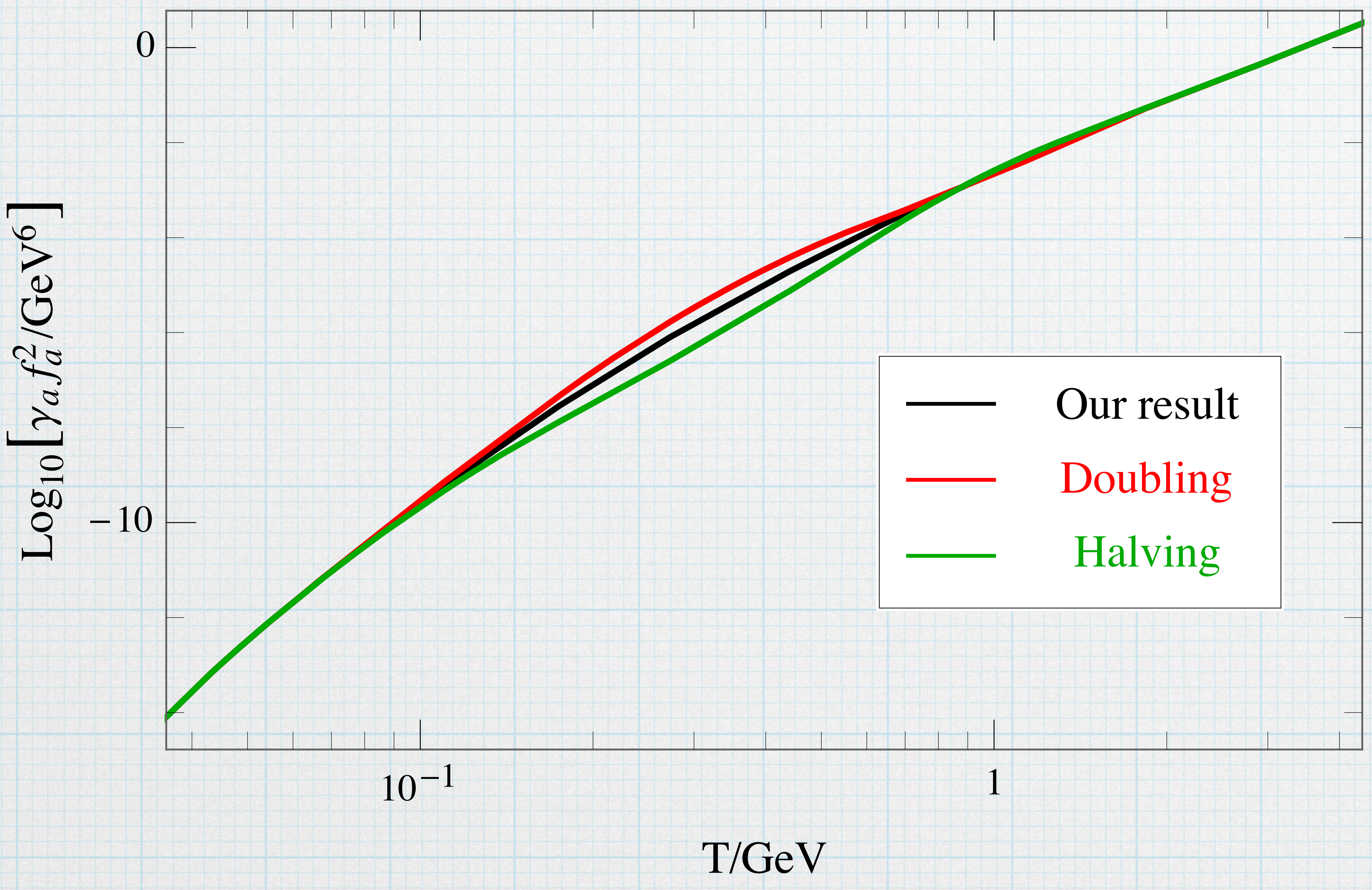
Thank you for your attention!

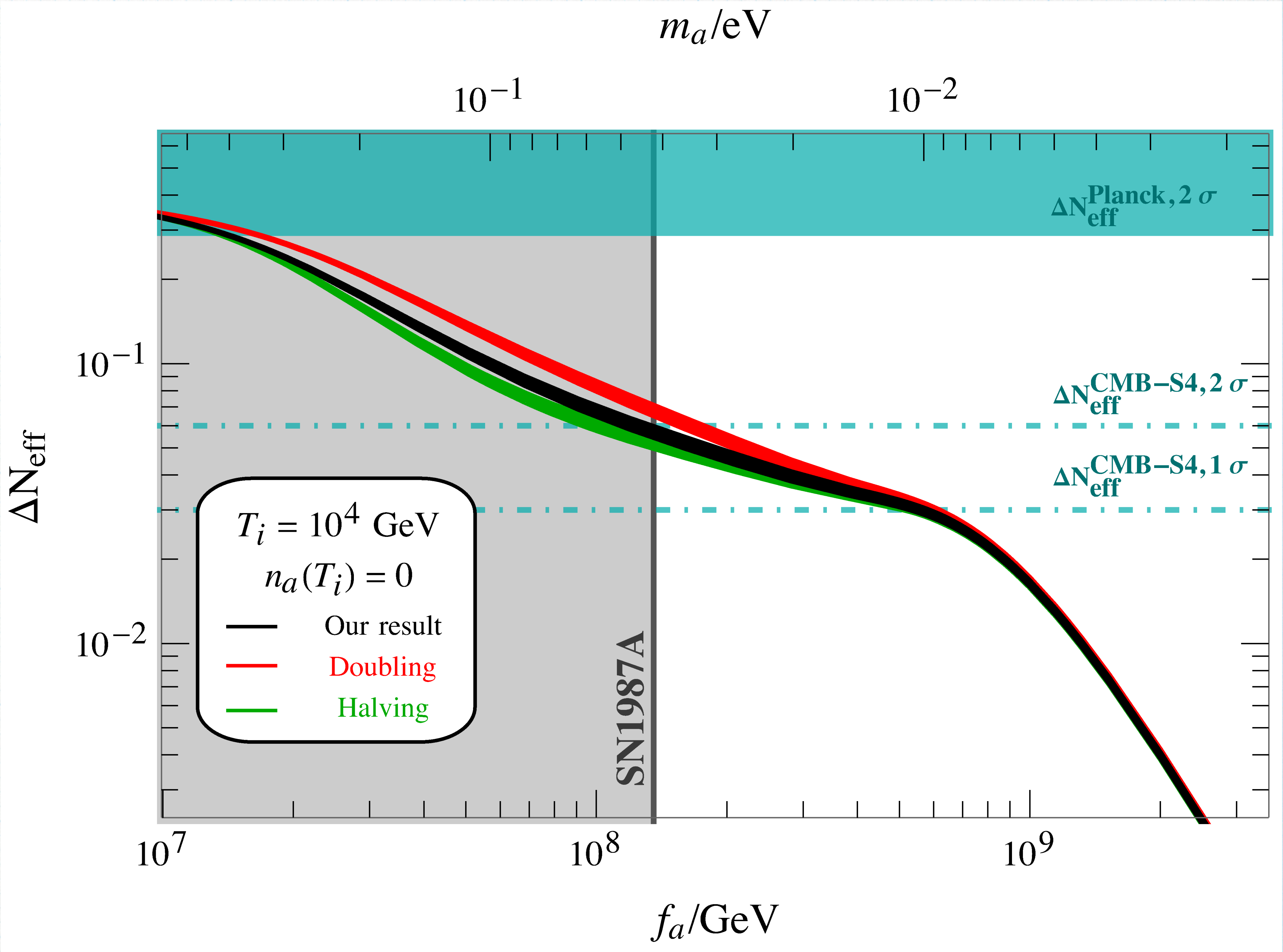
Thermal Effects on Gluon and Quark Scatterings





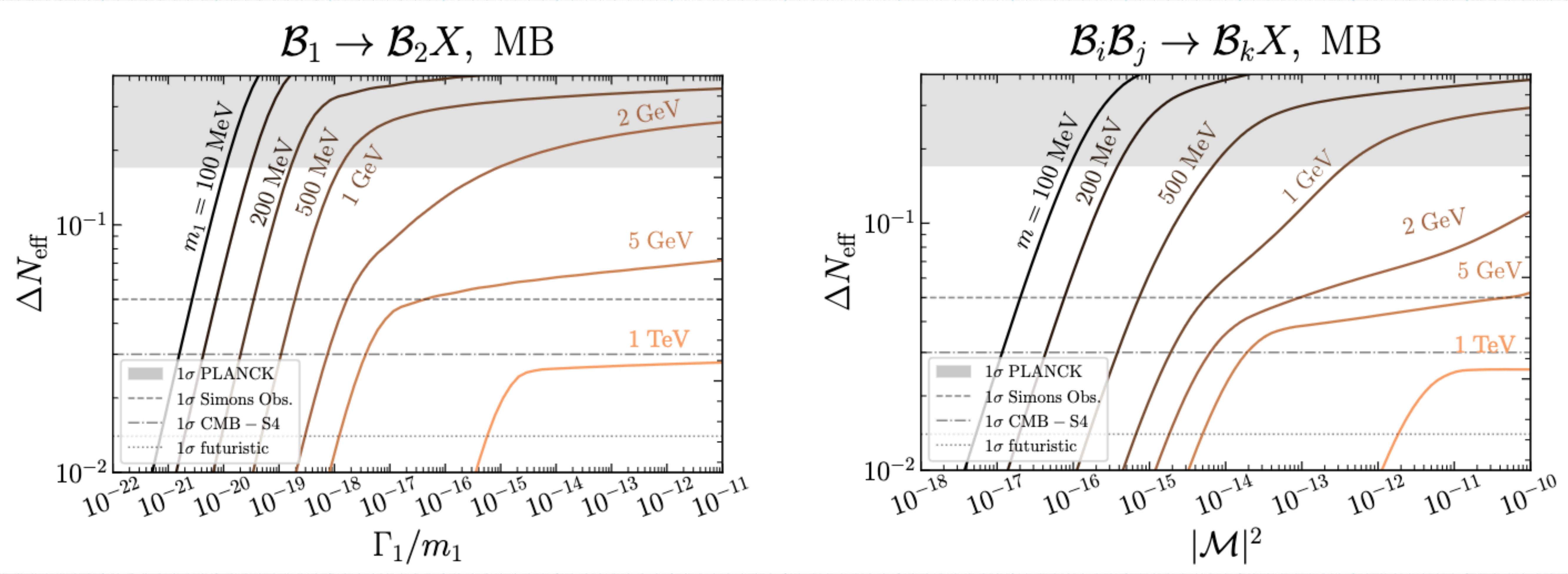
Uncertainties in the QCD Transition Regime

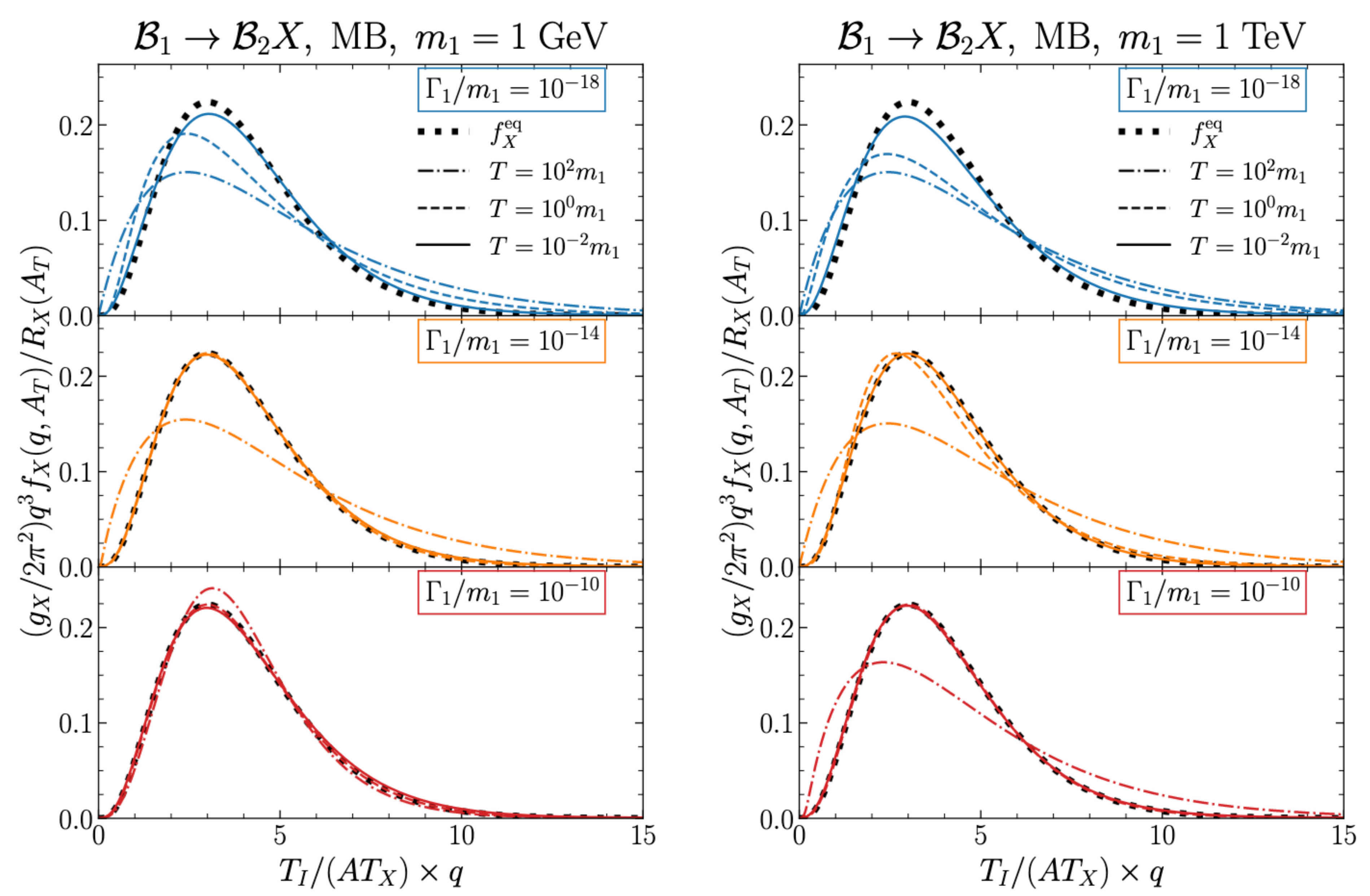


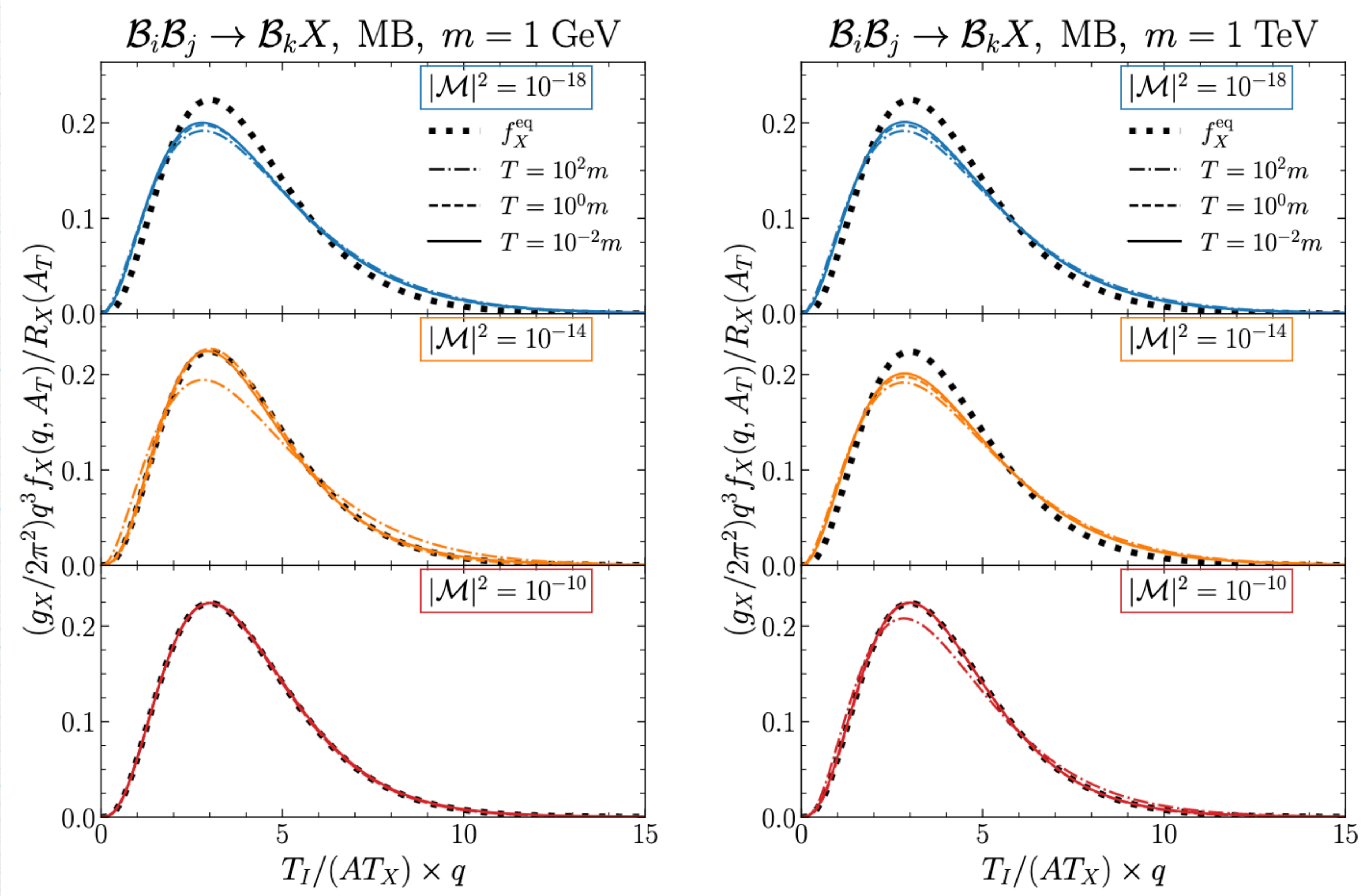


arXiv:2108.04259 arXiv:2108.05371

Dark Radiation in the Phase Space







arXiv:2311.04974