A New Understanding of Coherence in Bragg-Primakoff Scattering

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CETUP 2023 | June 23, 2023

Searches for Axion-like Particles

QCD Axions

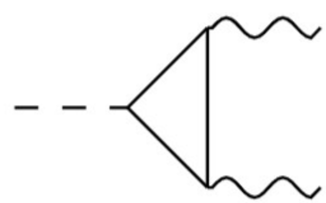
$$a\mathrm{Tr}[G_{\mu\nu}\tilde{G}^{\mu\nu}]$$

Dynamically relax CP violation in QCD

DM Axions

$$m_a \lesssim 0.1\,\mathrm{eV}$$

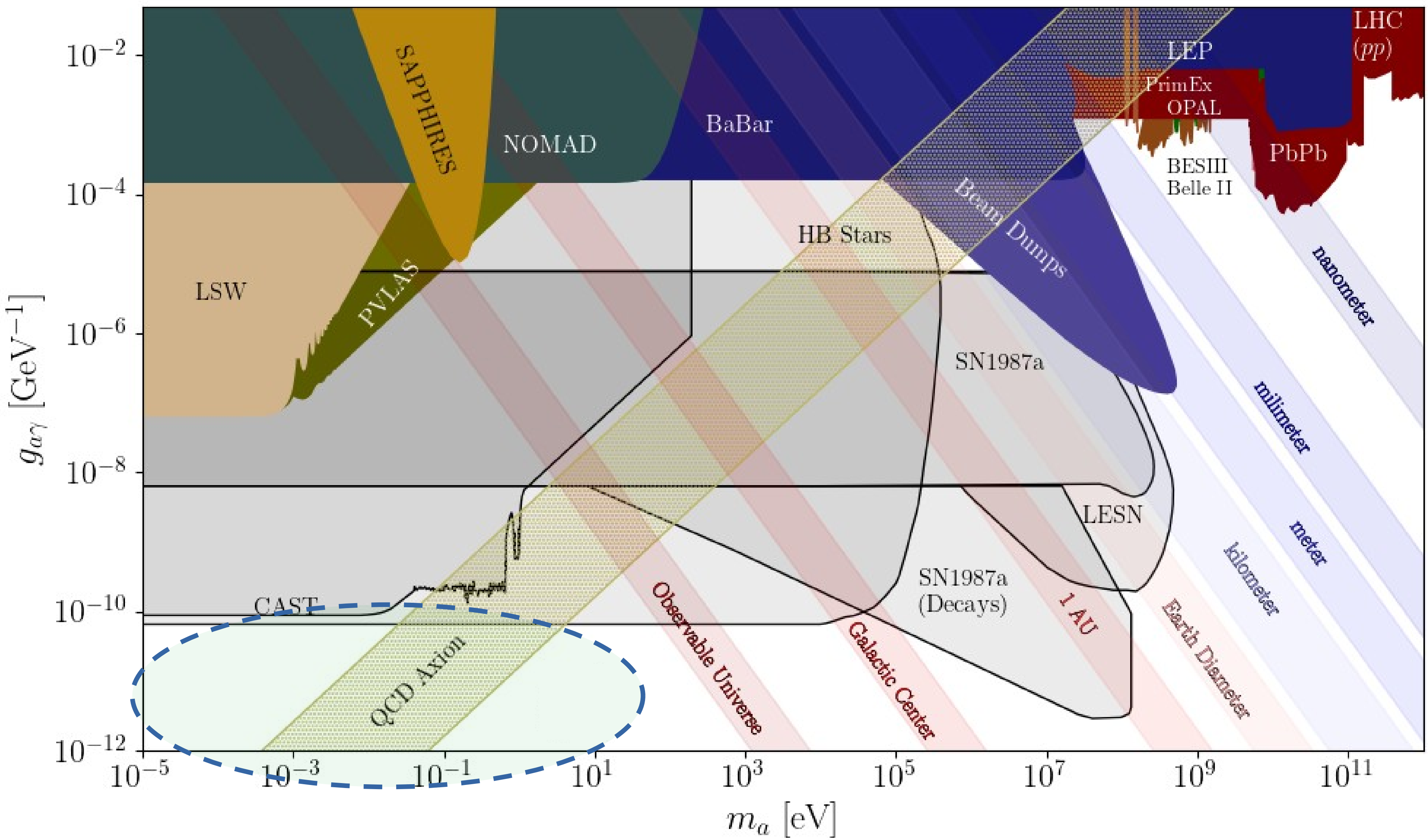
Long Lived ~ Age of Universe



$$aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

EW couplings arise easily and make for sensitive probes

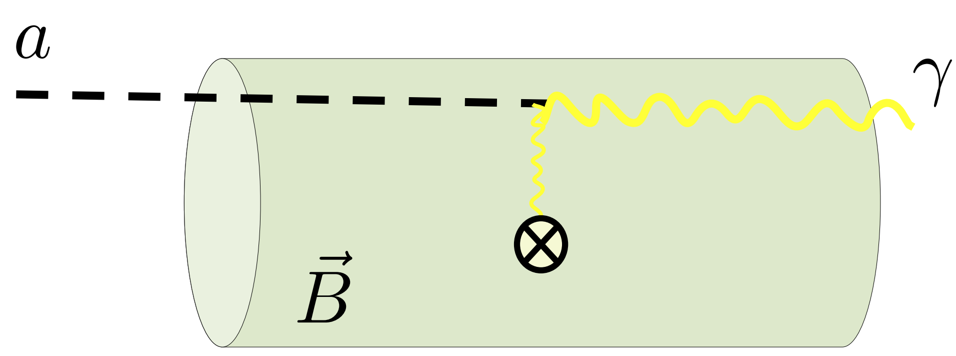
QCD Axions and Axion-like Particles



Canonical Axion-photon detection schemes

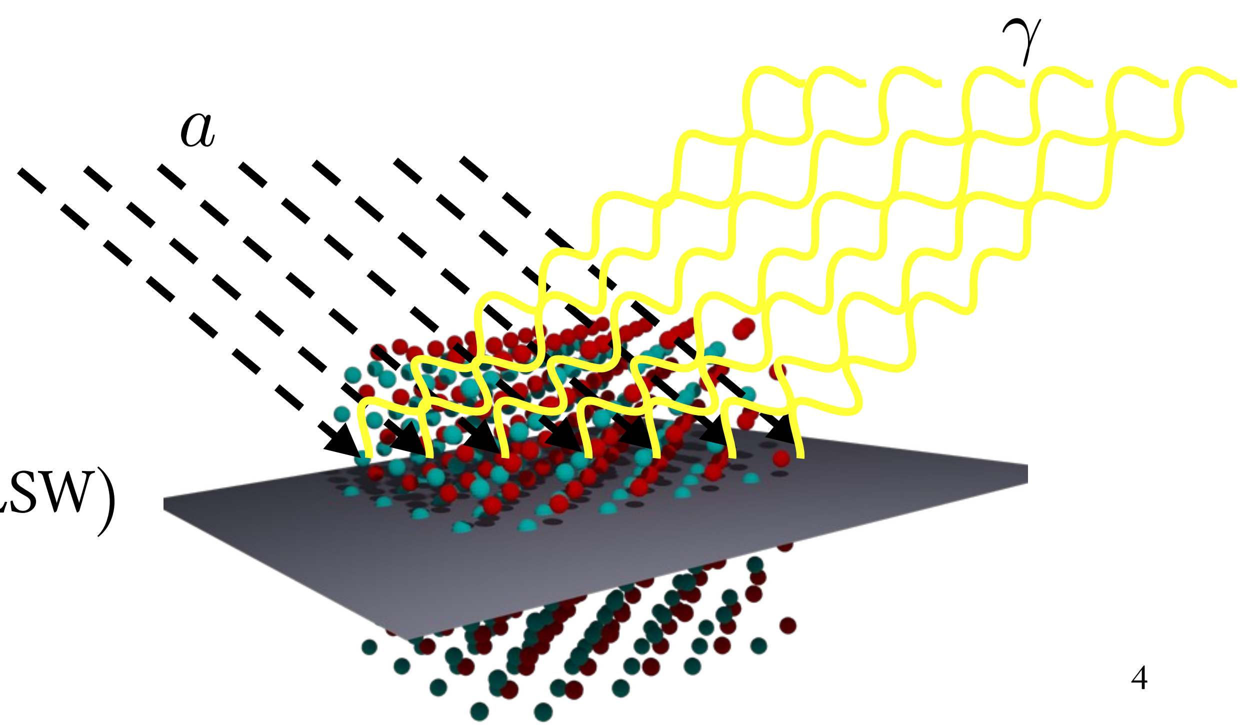
Sikivie, 1983

- Coherent conversion in EM Field
- Helioscopes, Haloscopes



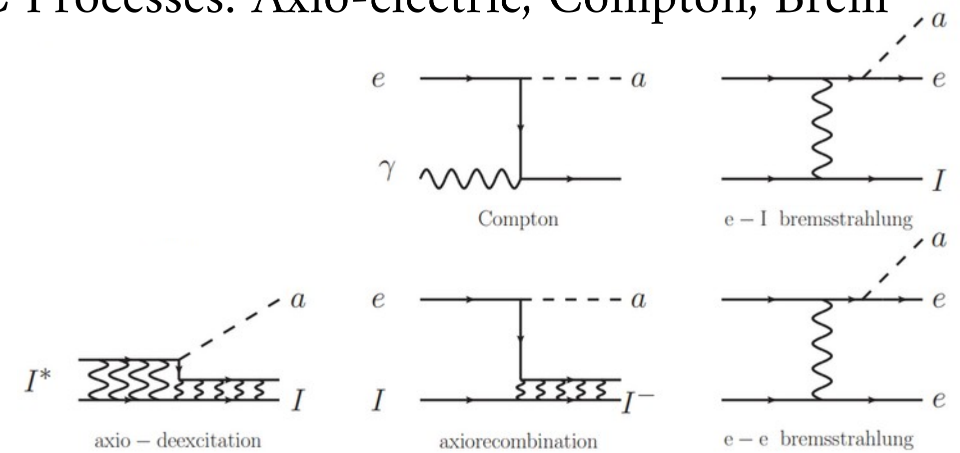
Buchmuller, Hoogeveen 1990

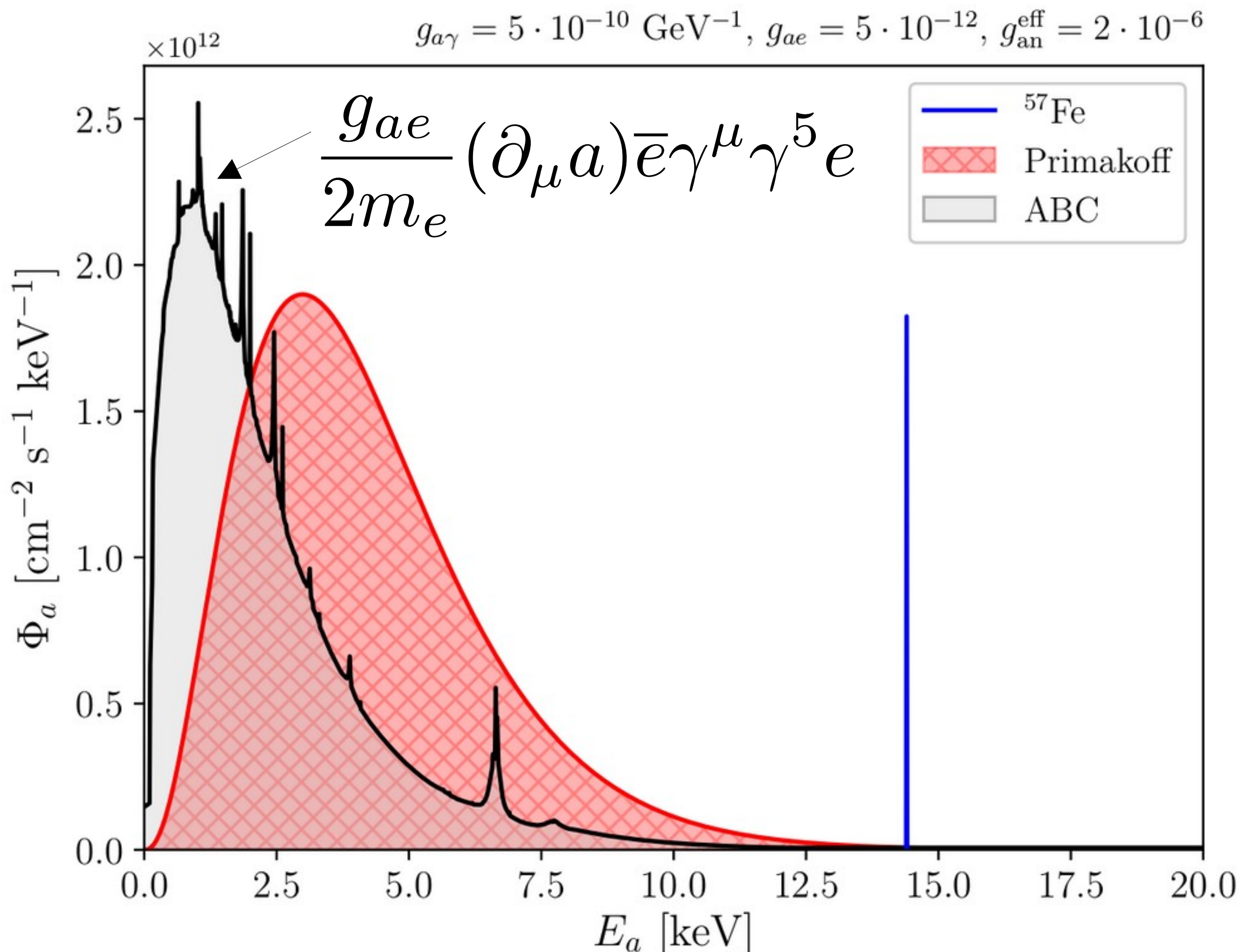
- Coherent Bragg-Primakoff conversion
- Light-shining-through-wall (LSW)
- Solar Axions



Solar Axions

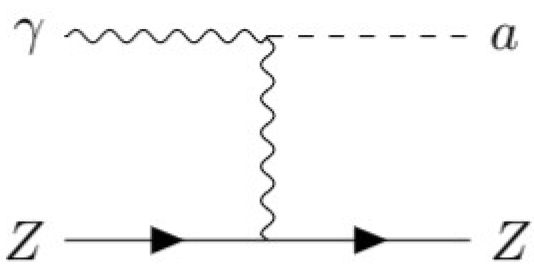
ABC Processes: Axio-electric, Compton, Brem





Solar Axions

Primakoff Scattering



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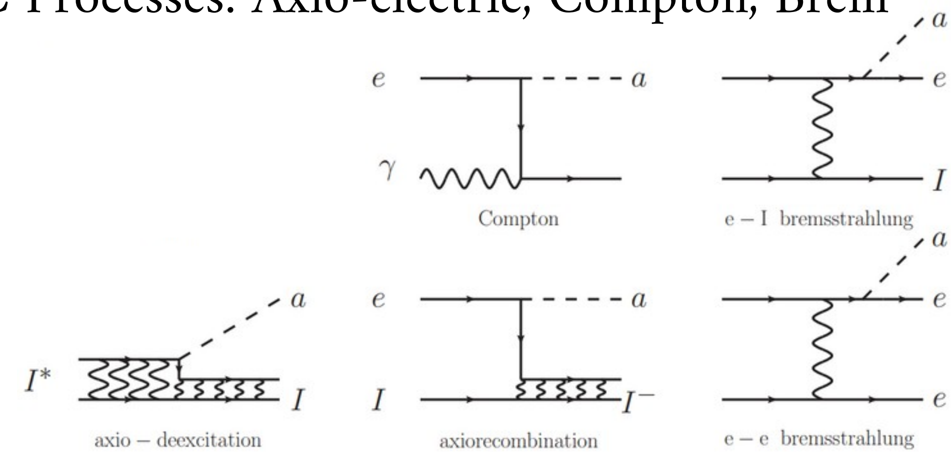
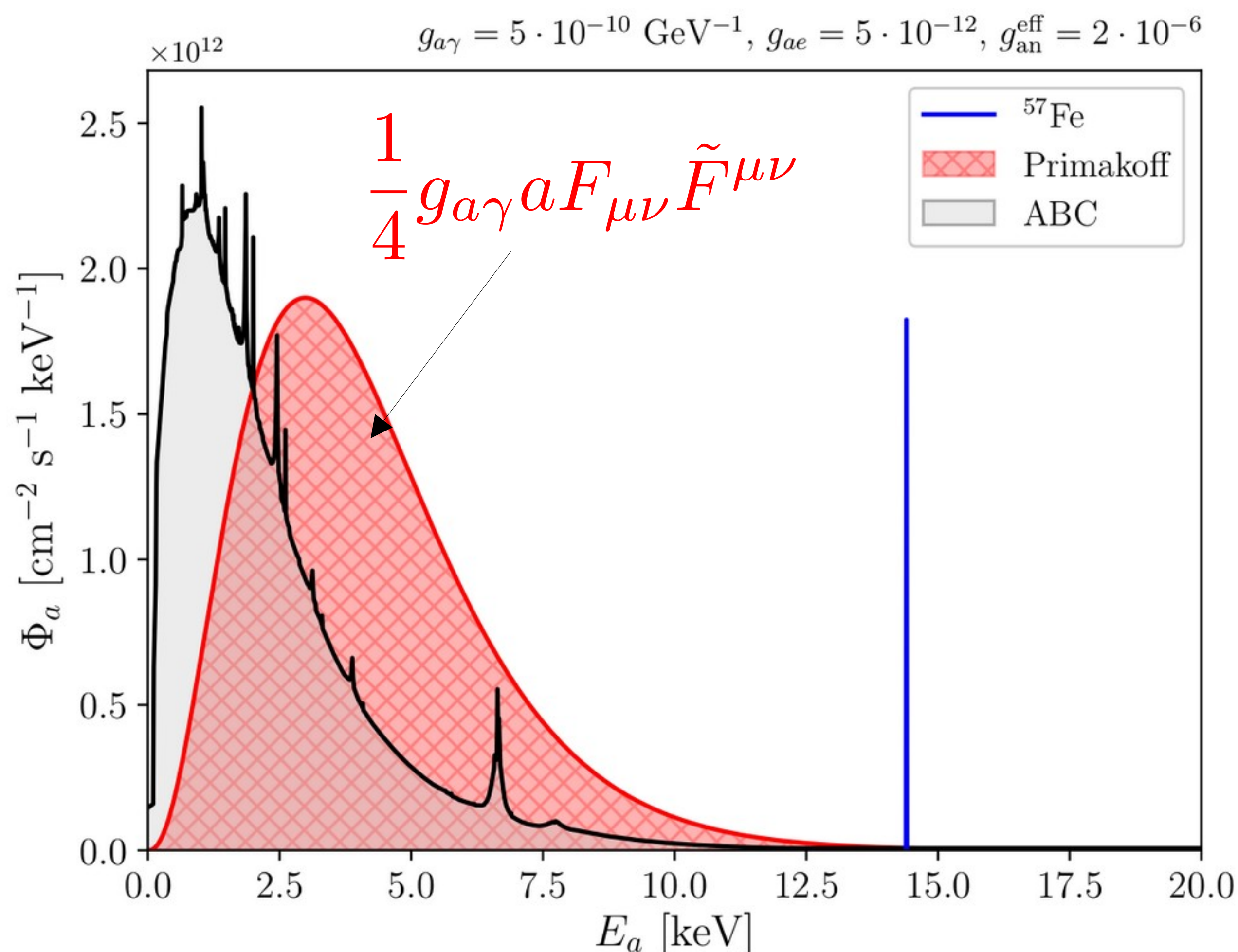
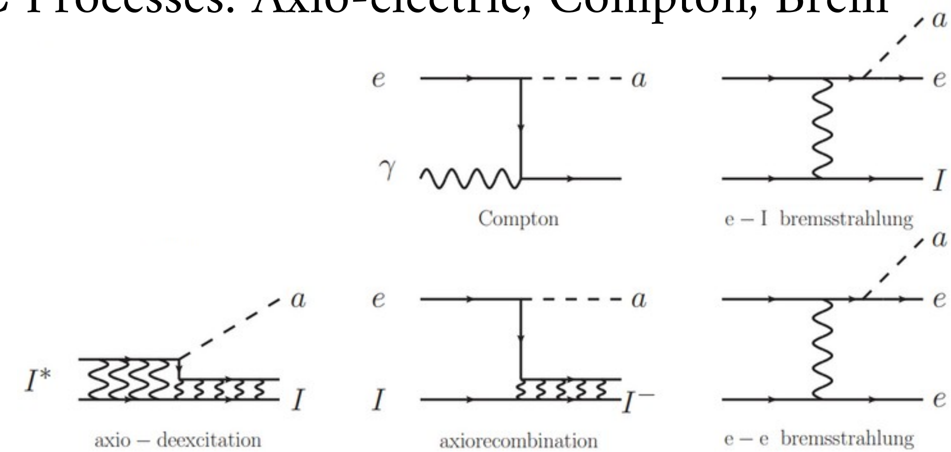


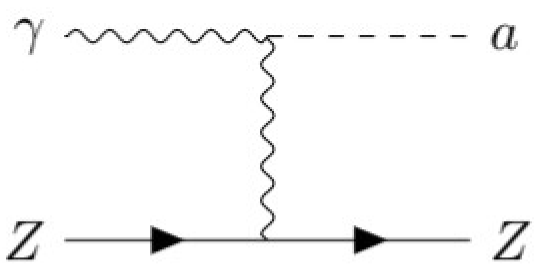


Solar Axions

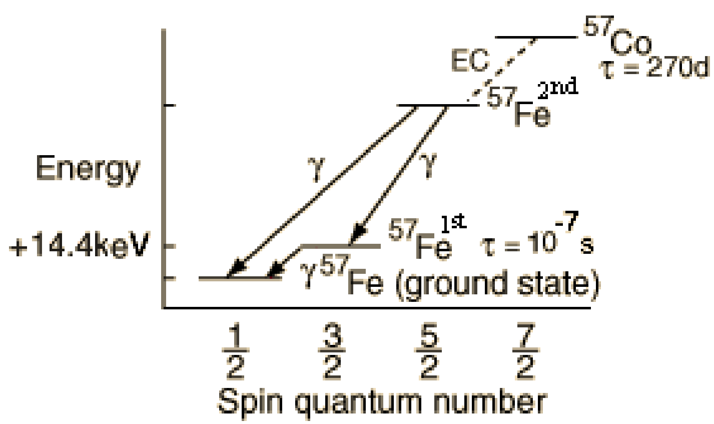
ABC Processes: Axio-electric, Compton, Brem

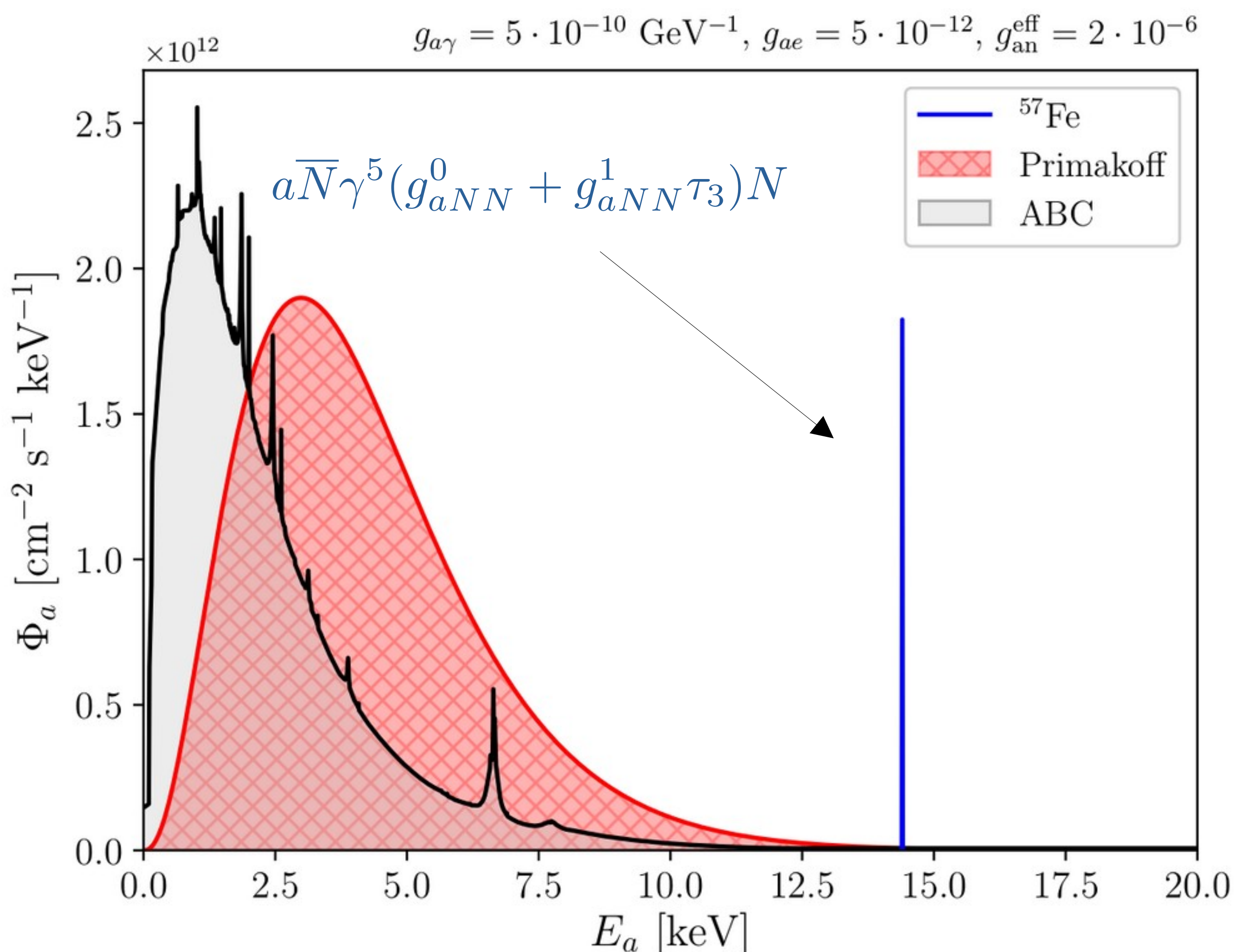


Primakoff Scattering

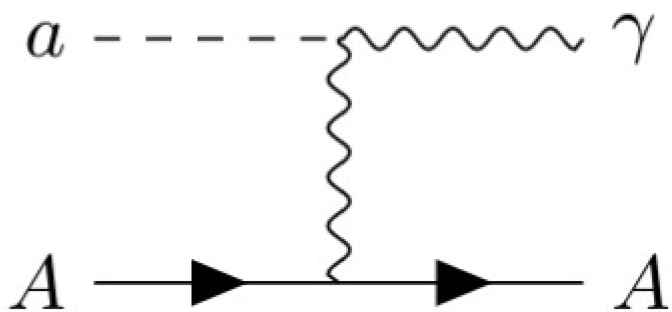


57Fe / **nuclear de-excitation**





Inverse Primakoff Scattering



- Atomic coherence:
  - $q << 1/R$
  - $\sigma \propto Z^2$
- Forward scattering, elastic:

$$E_\gamma \simeq E_a$$

$$\langle \mathcal{M}_P \rangle = \frac{4e^2 g_{a\gamma}^2}{t^2} E_\gamma^2 m_N^2 k^2 \sin^2\theta$$

$$\frac{d^2\sigma}{d\Omega dE_\gamma} = \frac{g_{a\gamma}^2}{16\pi^2}\frac{k_a^4}{q^4}|F(q)|^2 \sin^2(2\theta)\delta(E_a - E_\gamma)$$

Atomic Form Factor

Bragg-Primakoff Scattering: Crystal Structure

$$\vec{a}_1 = \frac{d}{2}(0,1,1)$$

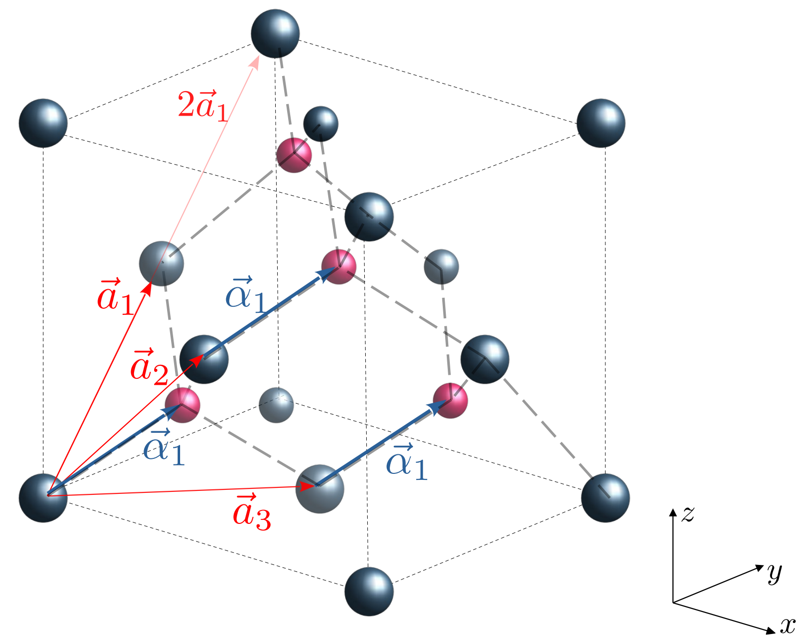
$$\vec{a}_2 = \frac{d}{2}(1,0,1)$$

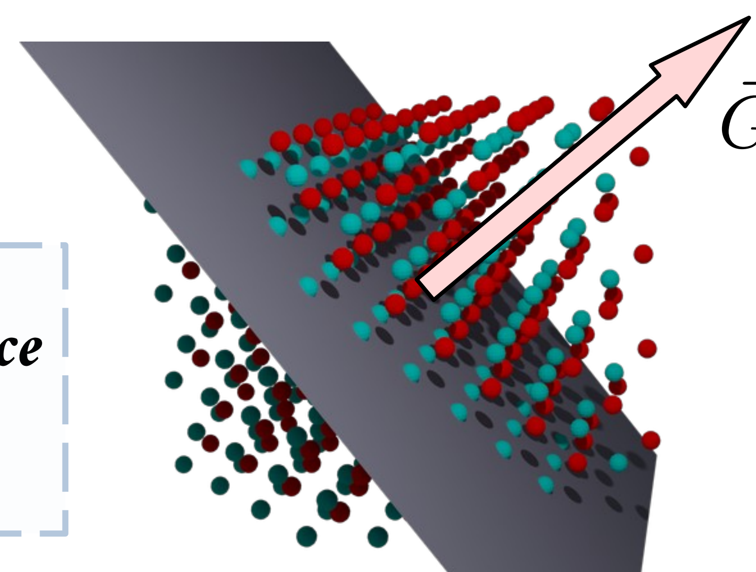
$$\vec{a}_3 = \frac{d}{2}(1,1,0)$$

Discrete

Fourier Transform

$$\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$$





$|\vec{G}| \sim \text{Å}^{-1}$ ***Reciprocal Interatomic Distance***

→ *Momentum Transfer Scale ~* ***keV***

Bragg-Primakoff Scattering

$$\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$$

Laue Condition

$$\vec{q} = \vec{k} - \vec{k}' = \vec{G}$$ ⇒ ***Coherence***

Bragg Condition (Elasticity)

$$E_a = \frac{|\vec{G}|^2}{2\hat{k}\cdot\vec{G}} \Longleftrightarrow n\lambda = 2d\sin\theta$$ ⇒ $E_a \simeq$ keV

astro-ph/9811359

$$R(E_1, E_2) = g_{a\gamma}^2 \frac{\pi\hbar c V}{v_a^2} \sum_{\vec{G}} \frac{d\Phi_a}{dE_a} \cdot \frac{1}{|\vec{G}|^2} \sum_j |F_j(\vec{G}) S_j(\vec{G})|^2 \sin^2(2\theta)\mathcal{W}$$

Coherence

Momentum sum

Solar axion flux

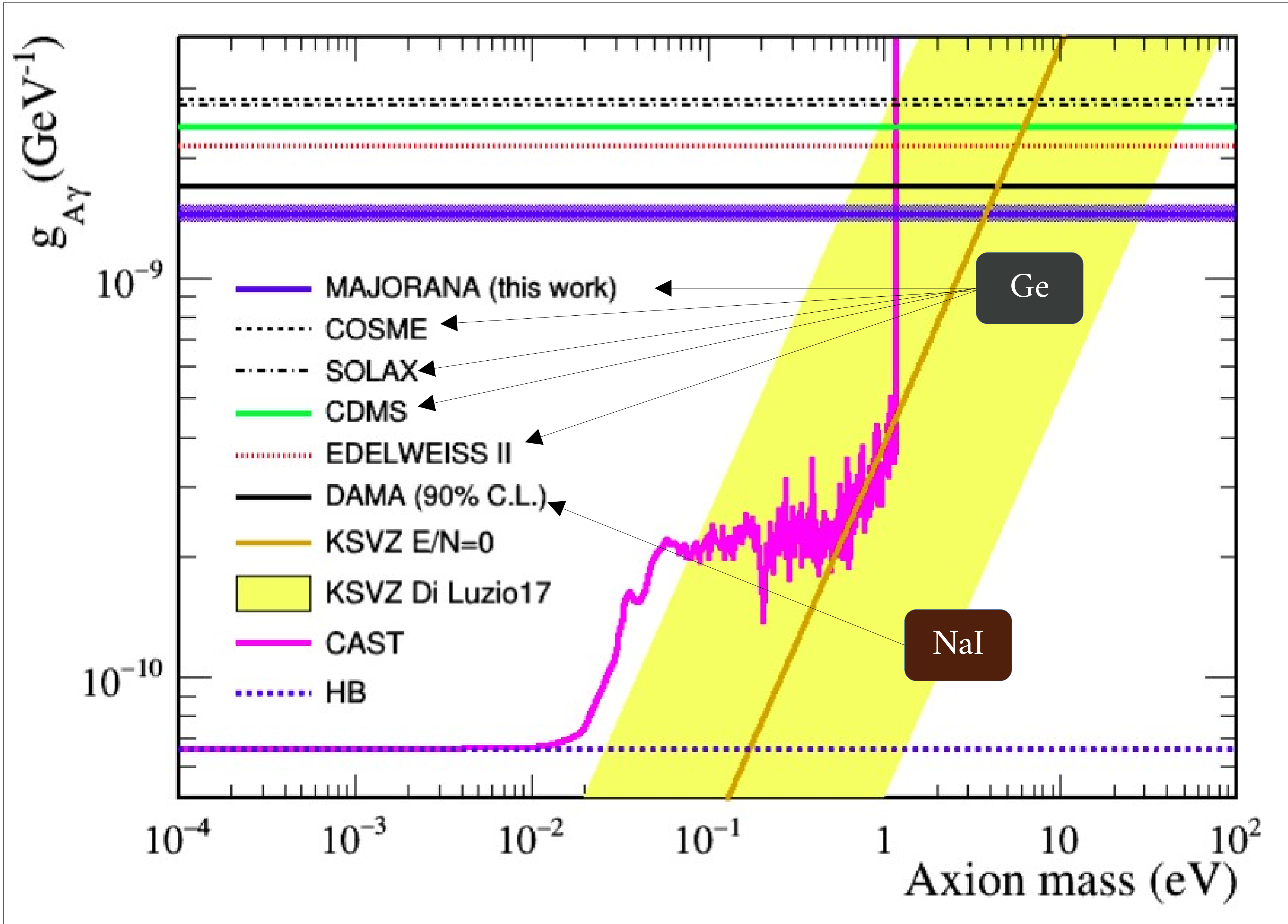
cross section

Structure factor

Detector response

Bragg-Primakoff Scattering: Solar ALP Limits

Phys. Rev. Lett. 129, 081803 (Majorana Demonstrator)



Wait a minute…something is broken

Light-shining-through-wall (LSW) theory

$$P_{a\to\gamma} \propto L_{att}^2 \to R \propto \frac{L_{att}}{L} \times V$$

Dependence *on attenuation depth*

$$L_{att} = L_{bragg} \sim 1\,\mu\mathrm{m}$$

COHERENT PRODUCTION OF LIGHT SCALAR OR PSEUDOSCALAR PARTICLES IN BRAGG SCATTERING

W. BUCHMÜLLER [a,b] and F. HOOGEVEEN [a]

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[b] *Deutsches Elektronen-Synchrotron DESY, D-2000 Hamburg, FRG*

Solar ALP Theory

$$R \propto V$$

No Dependence *on attenuation depth*

Prospects of solar axion searches with crystal detectors

S. Cebrián, E. García, D. González, I.G. Irastorza, A. Morales *, J. Morales, A. Ortiz de Solórzano, J. Puimedón, A. Salinas, M.L. Sarsa, S. Scopel, J.A. Villar
Laboratorio de Física Nuclear, Universidad de Zaragoza, 50009, Zaragoza, Spain

Received 23 November 1998

Buchmuller, Hoogeveen:

Instead of Bragg scattering one can of course also consider Laue scattering, where the penetration depth is much larger. For 100 keV photons and scattering angle $\Theta \sim 1°$ one can achieve $l \sim 1$ cm [13]. This would improve the lower bound (24) on the mass scale M by three orders of magnitude up to $\sim 10^6$ GeV. Such an experiment would clearly be very interesting. It remains to be seen, however, whether the effective electric field $\bar{E}$ which appears in eq. (23) is the same as for Bragg scattering. A detailed calculation will be published elsewhere.

Laue-type scattering considered in 2017:

Theoretical calculation of coherent Laue-case conversion between x-rays and ALPs for an x-ray light-shining-through-a-wall experiment

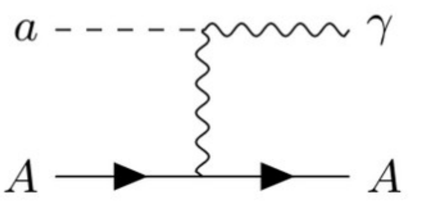
T. Yamaji,[1,*] T. Yamazaki,[2] K. Tamasaku,[3] and T. Namba[4]

What is coherence??

ALP momentum k

Photon momentum k'

$$f(k,k') = \langle \mathcal{M}_P \rangle \, F_A(q)$$



Sum over lattice centers

$$\mathcal{M}(\vec{k},\vec{k}') = \sum_{j=1}^{N} f(\vec{k},\vec{k}') e^{i(\vec{k}'-\vec{k})\cdot\vec{r}_j}$$

$$|\,\mathcal{M}(\vec{k},\vec{k}')\,|^2 = \sum_{i=1}^{N} |\,f\,|^2 + \sum_{j\neq i}^{N}\sum_{i=1}^{N} f^\dagger f e^{-i\vec{q}\cdot(\vec{r}_i-\vec{r}_j)}$$

Incoherent piece $\propto N$

Coherent piece $\propto N^2$

Coherence Possible If…

(a) $\vec{q}\cdot(\vec{r}_i-\vec{r}_k) \ll 1$

(b) $\vec{q}\cdot(\vec{r}_i-\vec{r}_k) = 2\pi n$

What is coherence??

$$\vec{q} = \vec{G} \rightarrow \vec{q} \cdot (\vec{r}_i - \vec{r}_j) = 2\pi n$$

1

$$| \mathcal{M}(\vec{k}, \vec{k}') |^2 = \sum_{i=1}^{N} | f |^2 + \sum_{j \neq i}^{N} \sum_{i=1}^{N} f^\dagger f e^{-i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)}$$

Math math math

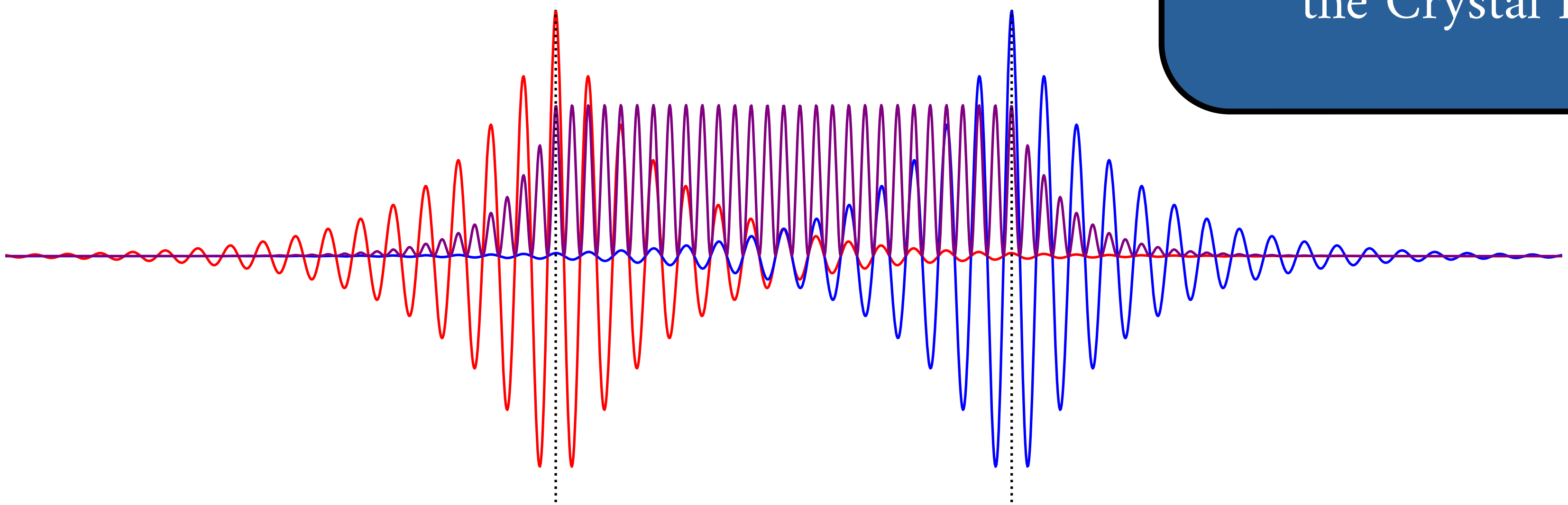
math math...

$$R(E_1, E_2) = g_{a\gamma}^2 \frac{\pi \hbar c V}{v_a^2} \sum_{\vec{G}} \frac{d\Phi_a}{dE_a} \cdot \frac{1}{|\vec{G}|^2} \sum_j |F_j(\vec{G}) S_j(\vec{G})|^2 \sin^2(2\theta) \mathcal{W}$$

With this treatment of coherence, we recover the canonical solar axion Bragg-Primakoff Event Rate

Ok, now with absorption

Place Atomic Target in Background Dielectric of the Crystal Bulk

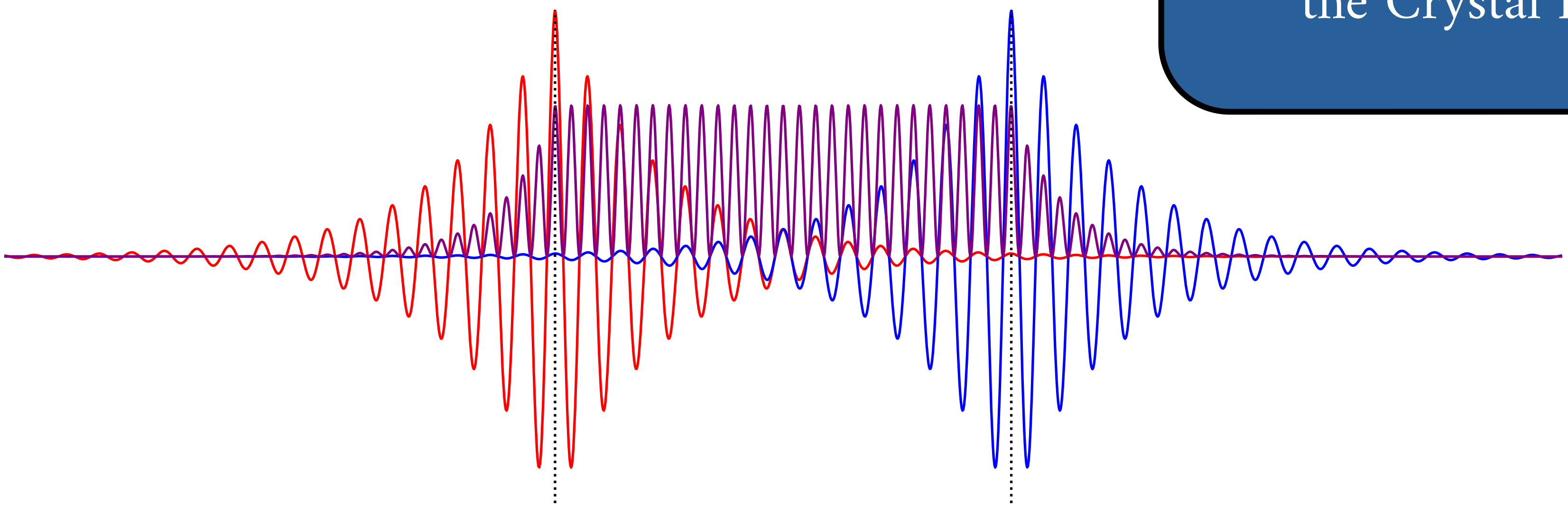


$$\vec{k}' \to \bar{n}\vec{k}', \quad \bar{n} = n - i\kappa$$

$$e^{i\bar{n}\vec{k}'\cdot(\vec{r}_i - \vec{r}_j)} \to \underbrace{e^{in\vec{k}'\cdot(\vec{r}_i - \vec{r}_j)}}_{\to 1} \underbrace{e^{-\frac{\mu}{2}|\hat{k}'\cdot(\vec{r}_j - \vec{r}_i)|}}_{\text{Attenuation Factor}}$$

Ok, now with absorption

Place Atomic Target in Background Dielectric of the Crystal Bulk



New attenuation term

$$| \mathcal{M}(\vec{k},\vec{k}') |^2 = \sum_{i=1}^{N} | f_i |^2 + \sum_{j\neq i}^{N} \sum_{i=1}^{N} f_j^\dagger f_j e^{-i\vec{q}\cdot(\vec{r}_i - \vec{r}_j)} e^{-\frac{1}{2\lambda}|\hat{k}'\cdot(\vec{r}_j - \vec{r}_i)|}$$

Borrmann Effect of Anomalous Absorption

$$\mu_{\text{eff}} = \mu_0 \left[1 - \frac{F''(hkl)}{F''(000)} \right]$$

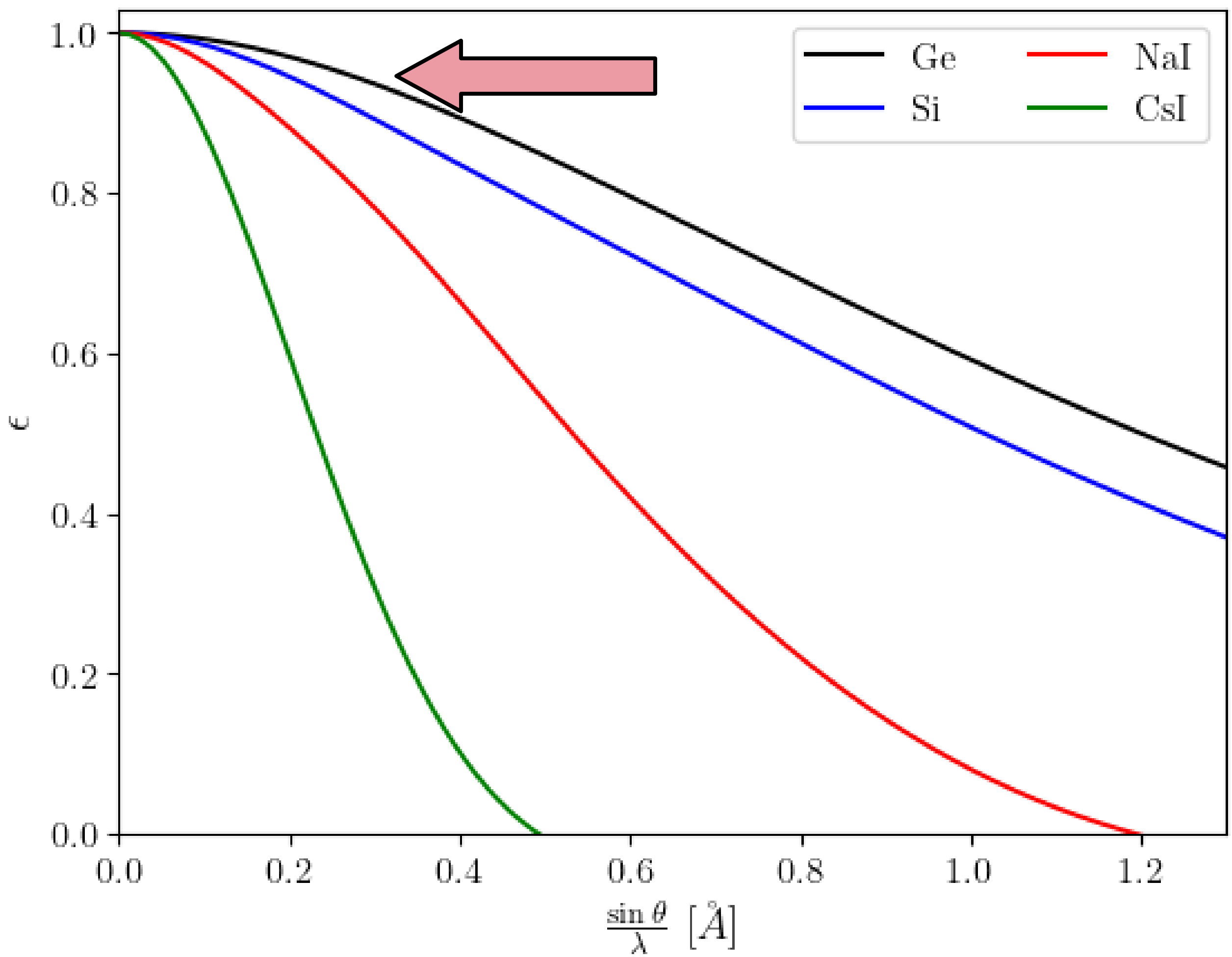
- Borrmann, Batterman (1961), Wagenfield (1987), Biagini (1990)…
- An anomalous decrease of the absorption coefficient → increase in the mean free path / attenuation length
- Depends on imaginary form factor

$$F''(hkl) = |S(hkl)| \Delta f''(\vec{q} = \vec{G}(hkl))$$

- **Lifts some of the coherence suppression!**

Borrmann Effect of Anomalous Absorption

$$\mu_{\text{eff}} = \mu_0 \left[1 - \frac{F''(hkl)}{F''(000)}\right] \quad F''(hkl) = |S(hkl)|\Delta f''(\vec{q} = \vec{G}(hkl))$$



Suppression lifted for

$$\epsilon = \frac{F''(hkl)}{F''(000)} \to 1$$

Modified Event Rate

Numerically compute this on a lattice

$$I(\vec{k},\vec{G}) \equiv \sum_{j\neq i}^{N}\sum_{i=1}^{N} e^{-|\vec{k}'\cdot(\vec{r}_i-\vec{r}_j)|/(2\lambda)}$$

$$\frac{dN}{dt} = \frac{(2\pi)^3 e^2 g_{a\gamma}^2}{8\pi^2}\frac{V}{v_{\text{cell}}^2}\sum_{\vec{G}} I(\vec{k},\vec{G})\frac{d\Phi_a}{dE}\frac{k^2\sin^2(2\theta)}{|\vec{G}|^4}|F_A(\vec{G})S(\vec{G})|^2\mathcal{W}(E_1,E_2,E)$$

The effect is that our event rate gets attenuated by I

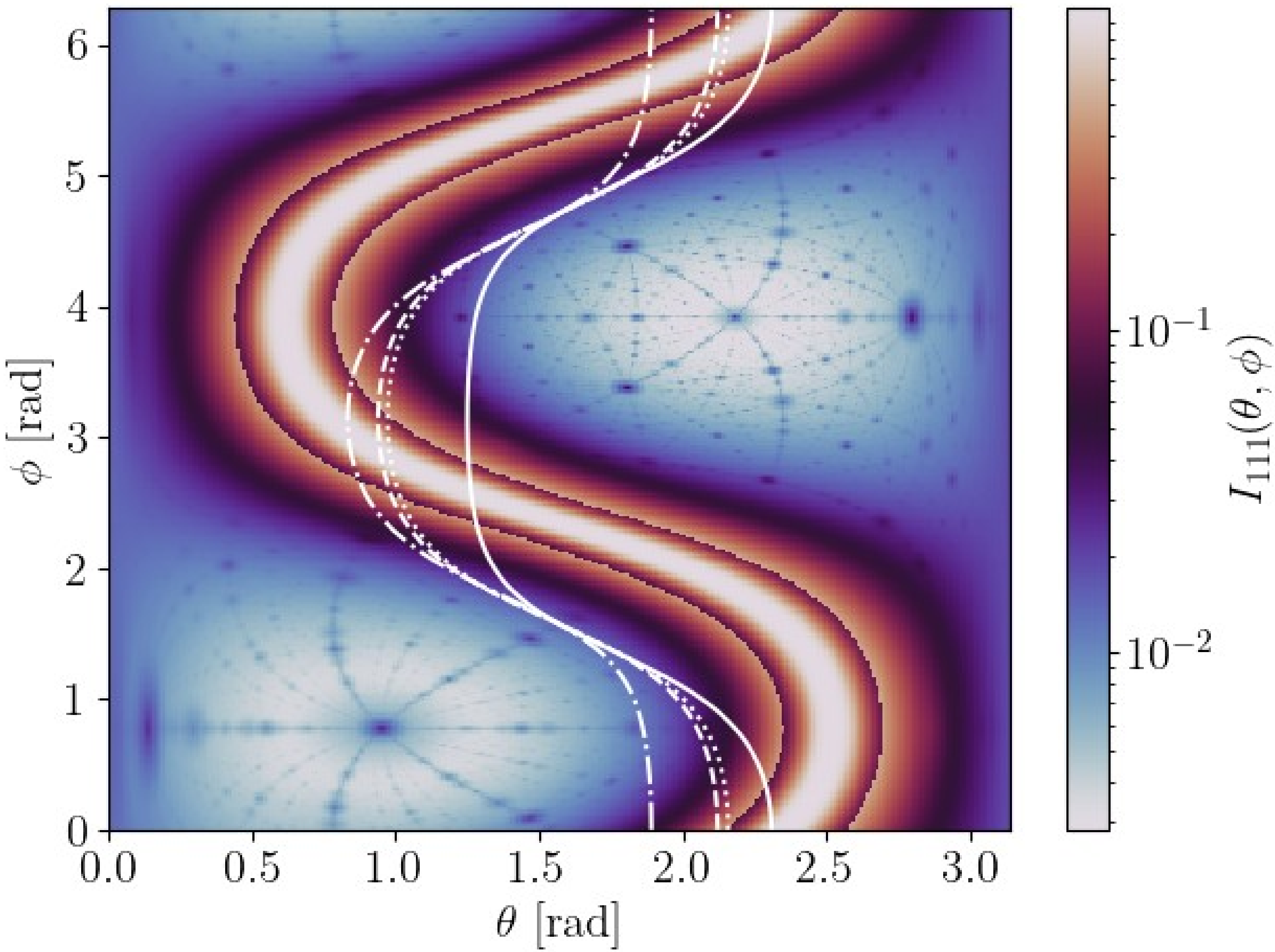
Absorption leads to a loss of coherence

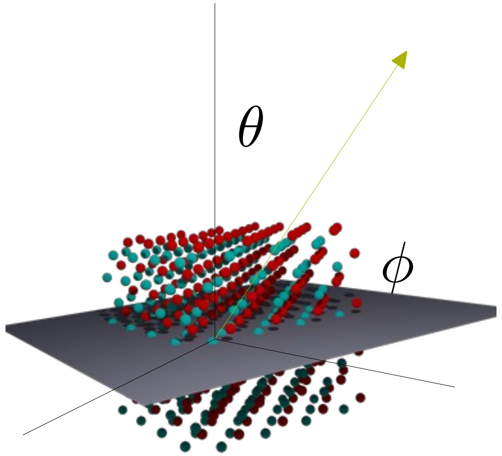
Absorption Sum: Magic Angles?

$$I(\vec{k}, \vec{G}) \equiv \sum_{j \neq i}^{N} \sum_{i=1}^{N} e^{-|\vec{k}' \cdot (\vec{r}_i - \vec{r}_j)|/(2\lambda)}$$

$$\lambda = 1/\mu = \left(\sigma n[1-\epsilon]\right)^{-1}$$

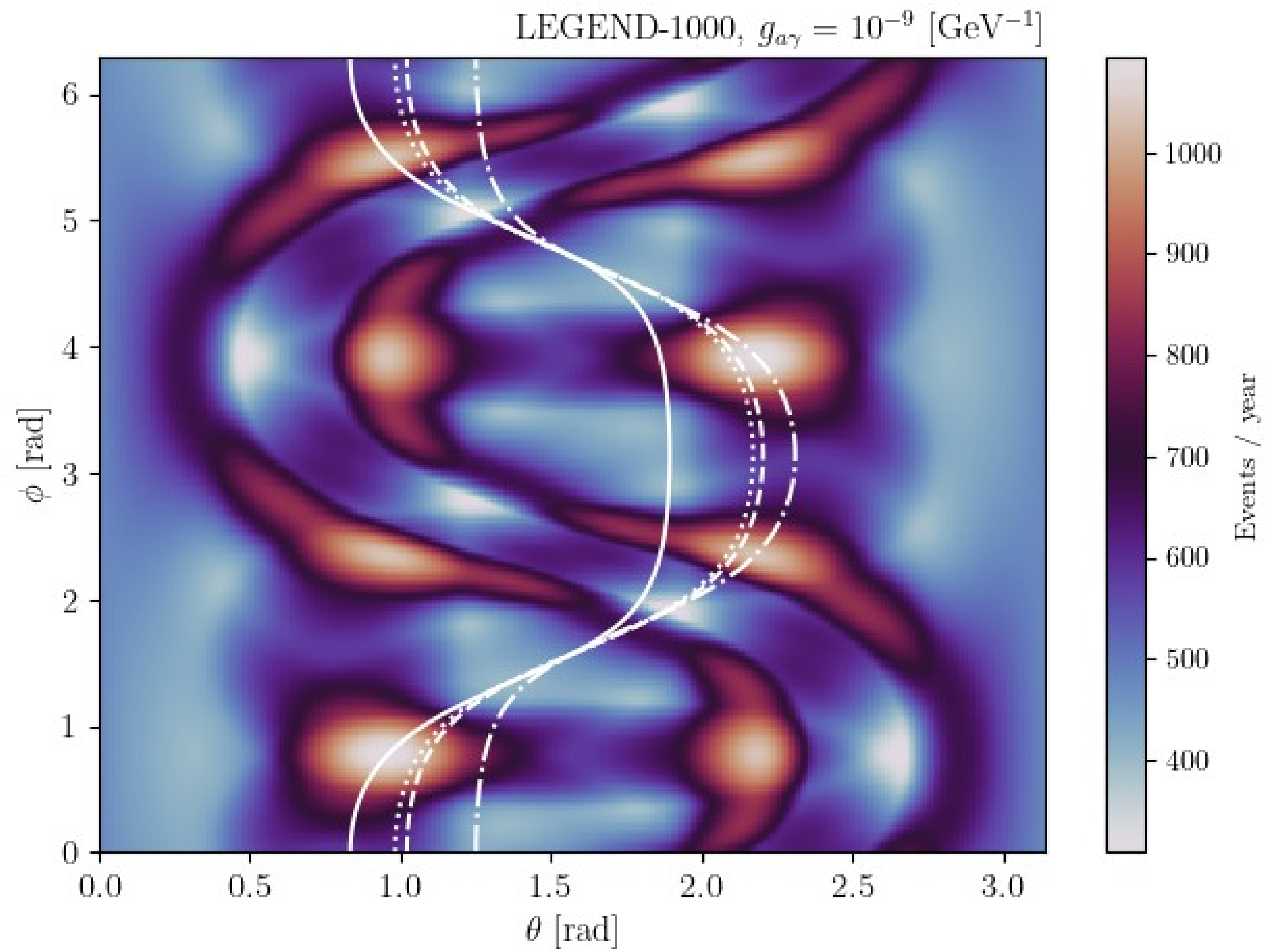
$$\vec{k}' = \vec{k} - \vec{G}$$





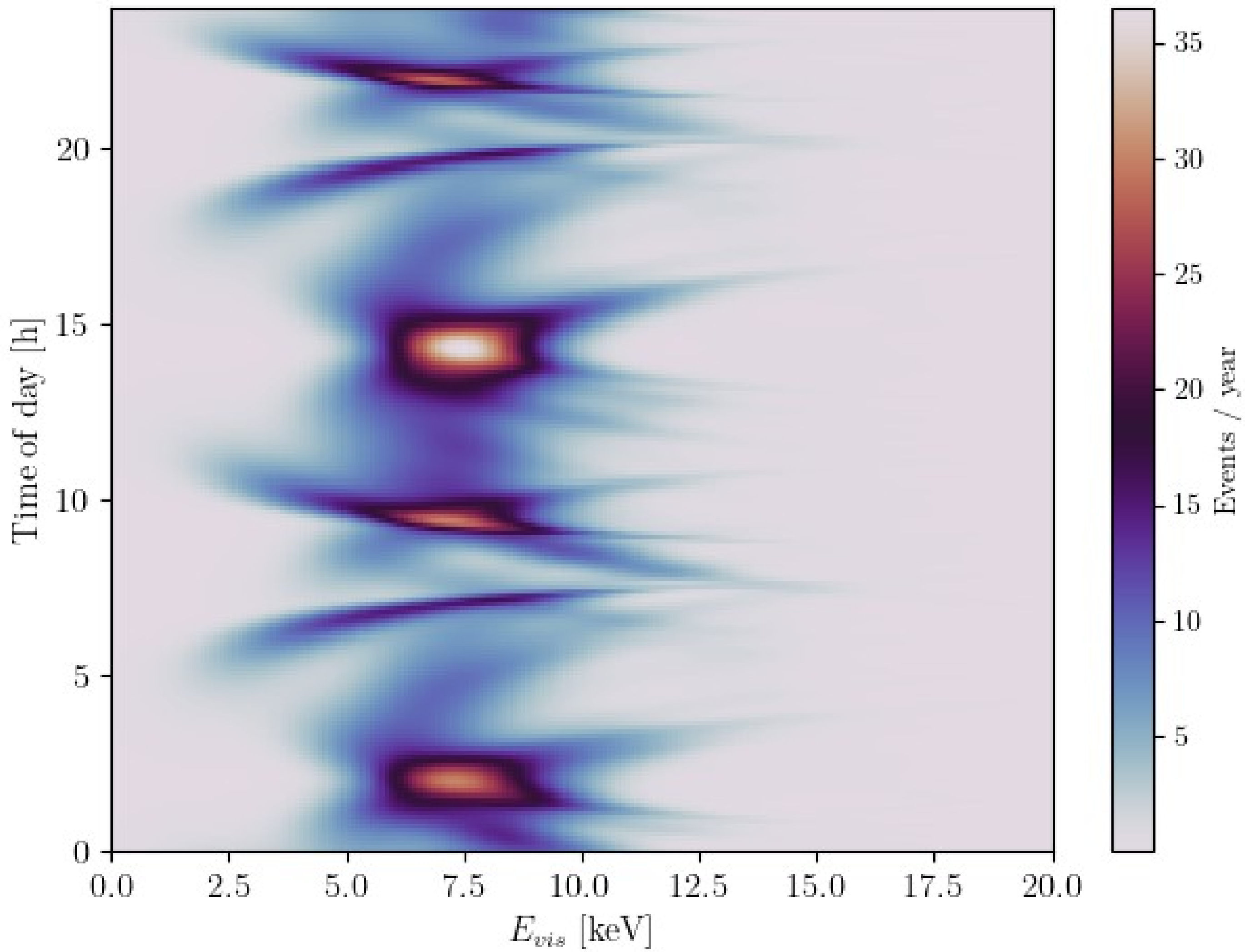
Modified Event Rate: By Angles

- Solar angle traces a trajectory throughout the day
- Seasonal + daily modulations
- Time dependence is a good discriminator + a **potential lever to minimize the absorption suppression**



Modified Event Rate: Energy / Time

- Solar angle traces a trajectory throughout the day
- Seasonal + daily modulations
- Time dependence is a good discriminator + a **potential lever to minimize the absorption suppression**

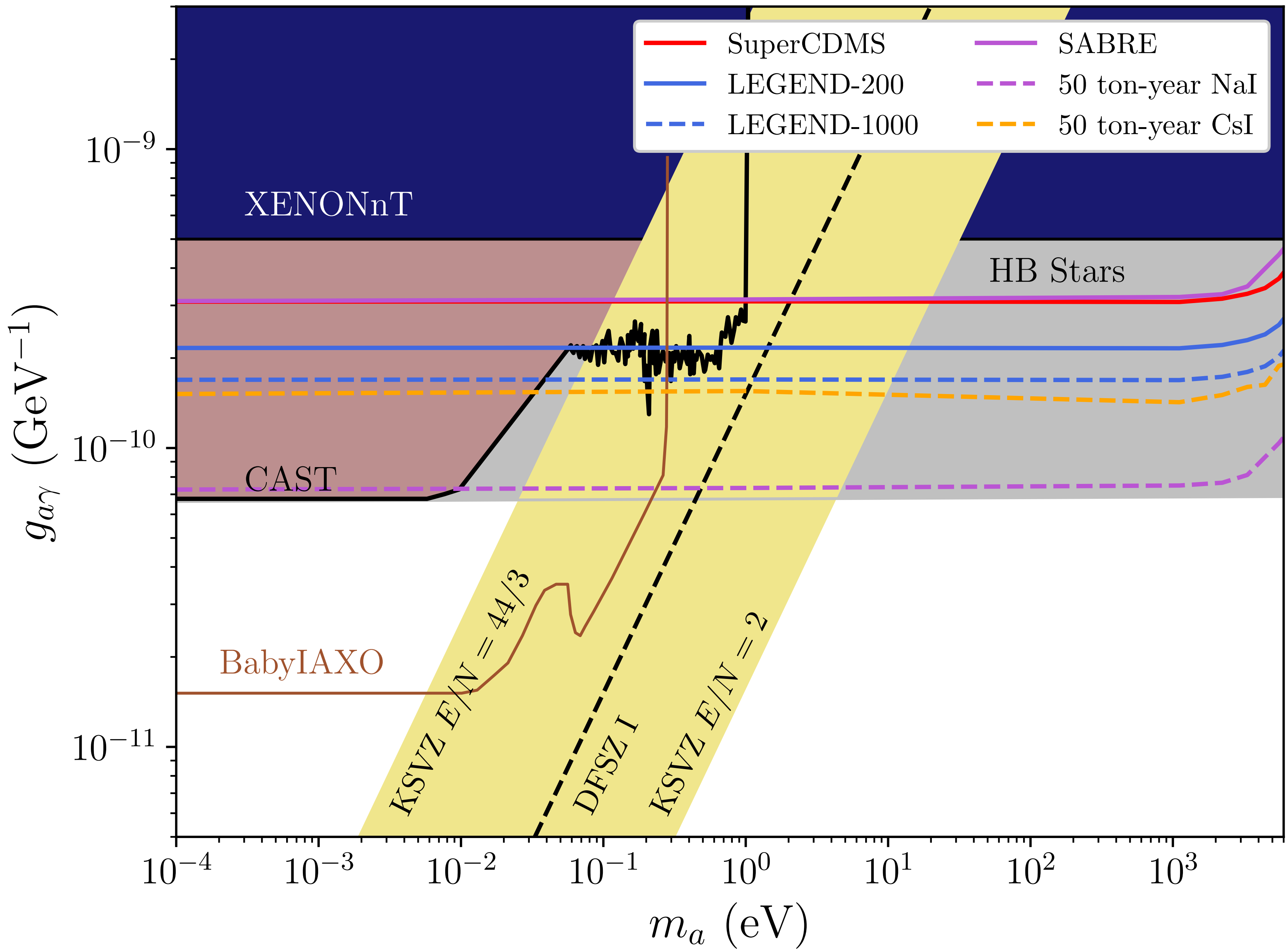


Experimental Prospects

Experiment	Total Mass	Threshold	Exposure (tonne-years)
SuperCDMS (Ge)	25.2 kg	1 keV	0.1
SuperCDMS (Si)	3.6 kg	1 keV	0.0144
LEGEND-200 (Ge)	195 kg	1 keV	0.78
LEGEND-1000 (Ge)	1 tonne	1 keV	4.16
SABRE (NaI)	50 kg	1 keV	0.15
tonne-scale NaI	5 tonne	1 keV	50
tonne-scale CsI	5 tonne	1 keV	50

Question: How big do we have to get to probe beyond the HB stars constraints?

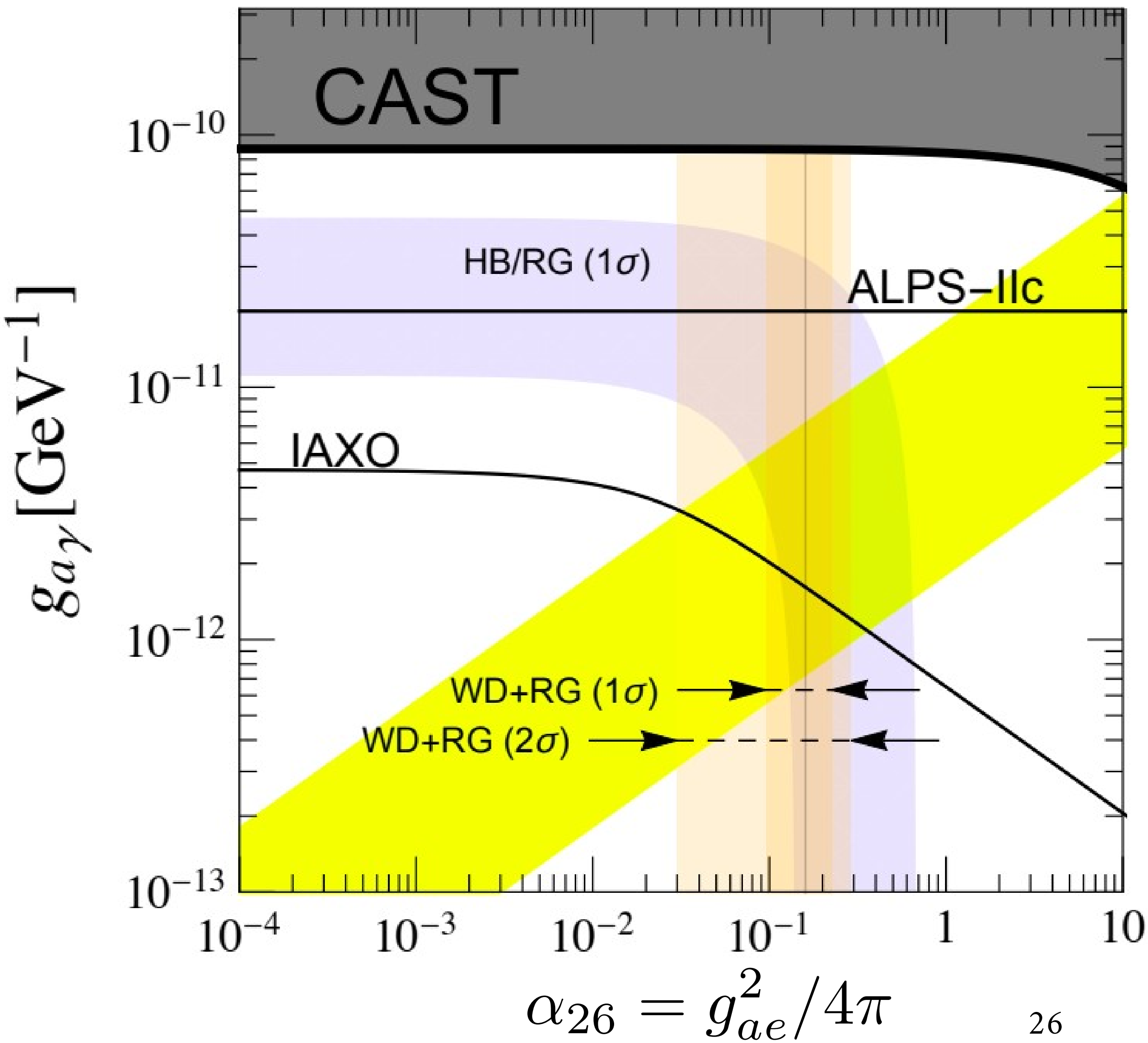
Sensitivities: With Absorption



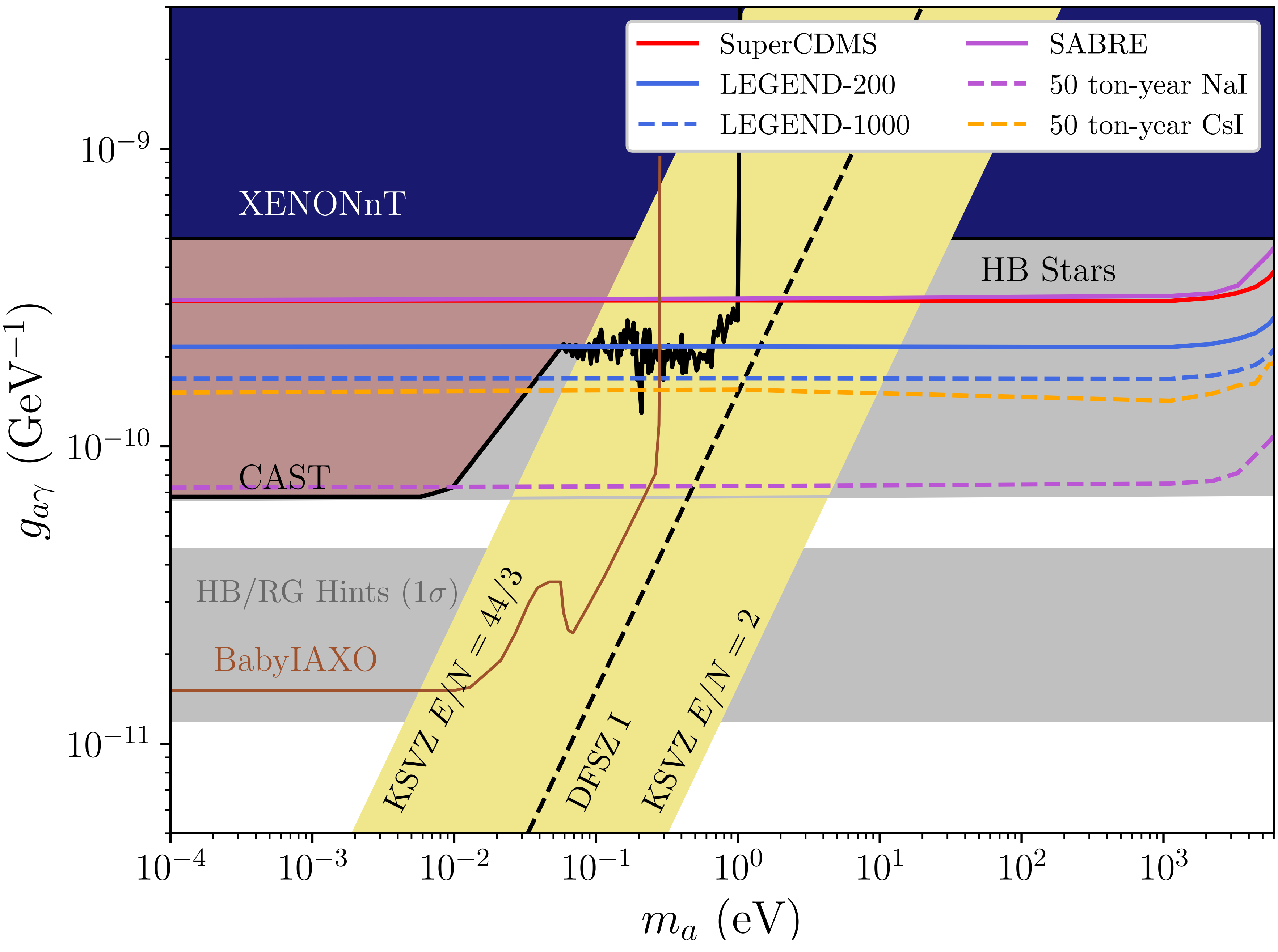
Cooling Hints as a Bonus

Observable	Stellar system	Significance
rate of period change $\dot{P}/P$ of WD variables	G117-B15A	4 σ
	R548	2 σ
	L19-2 (113)	1.5 σ
	L19-2 (192)	0.4 σ
	PG 1351+489	1.1 σ
Shape of WDLF	WD	2.3 σ
Lum. RGB-tip	Globular cluster M5	1.24 σ
	Globular cluster ω-Centauri	0.5-0.7 σ
HB stars ($R=N_{HB}/N_{RGB}$)	Globular cluster	2 σ

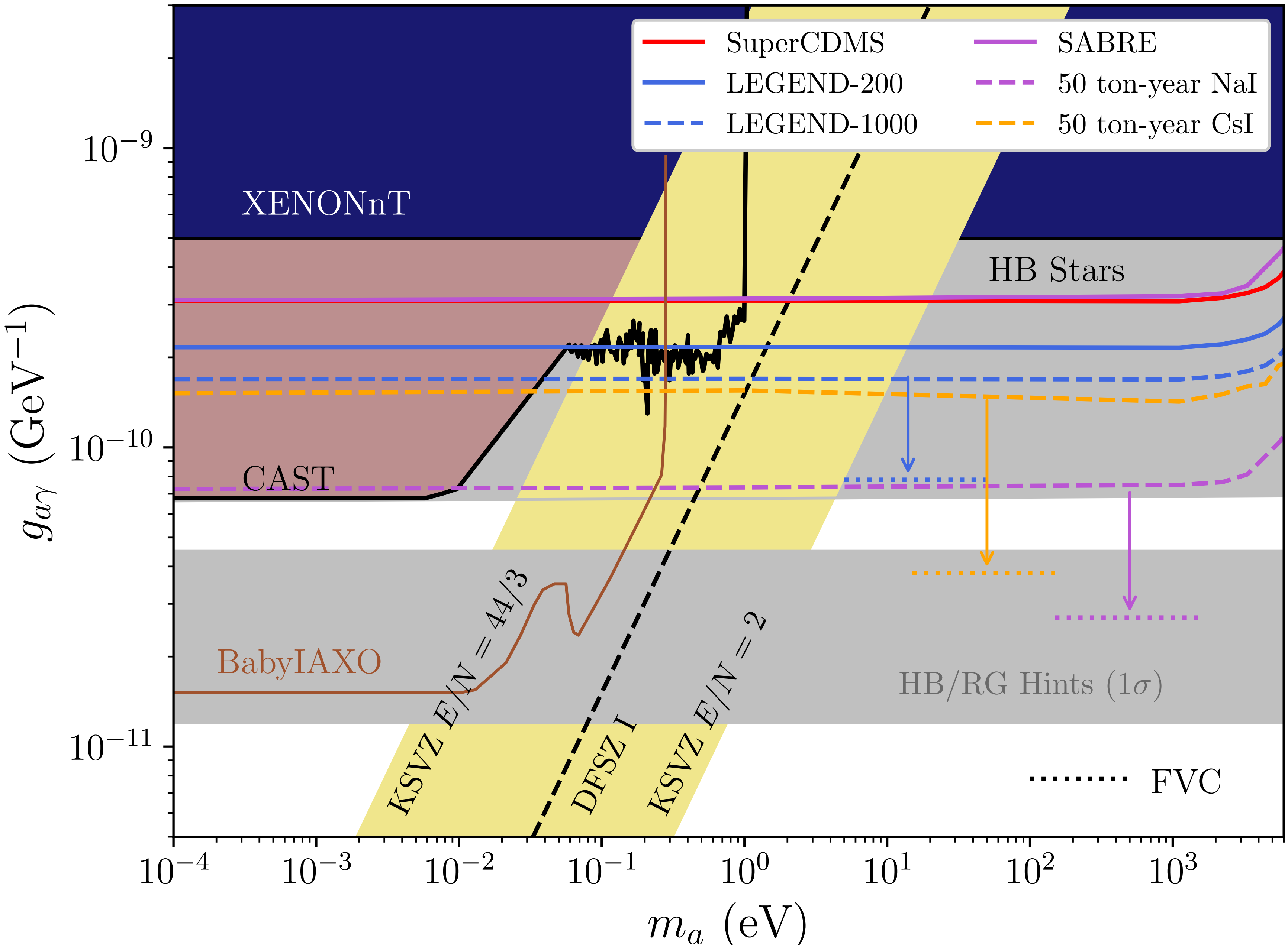
Gianotti, Irastorza, Redondo, Ringwald 1512.08108



Sensitivities: With Cooling Hints



Sensitivities: Can we get there?



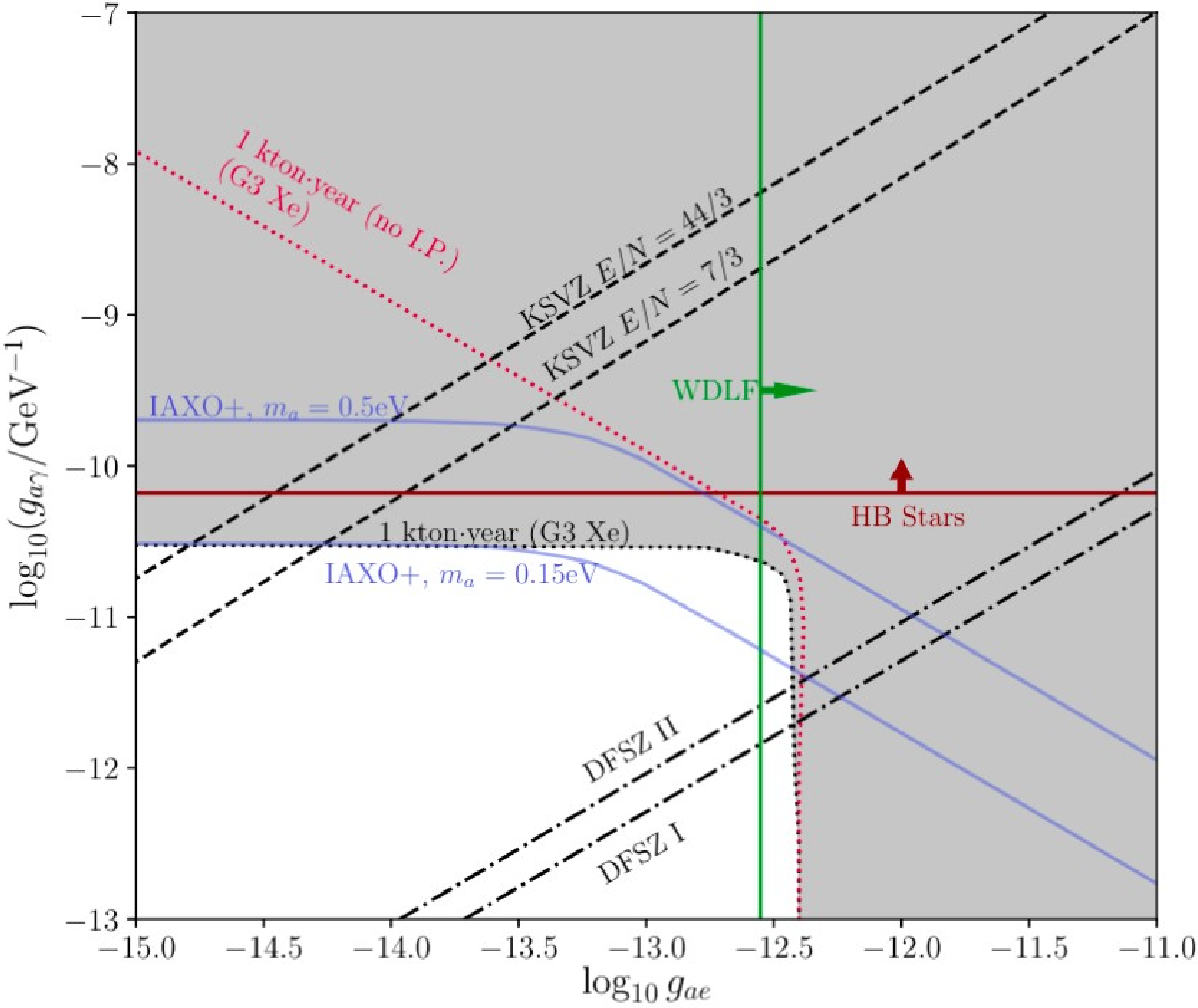
Takeaway

- **Coherent scattering** relies on **vanishing phases** in the matrix element sum
- These terms depend on the wave functions of the in and out states – **photon out states get attenuated** in a dielectric
- **This hasn't been accounted for** in solar axion Bragg-Primakoff searches!
- We want sensitivity to QCD axions beyond the HB stars constraints, but **we'll need clever thinking to properly utilize the technology we have to get there**

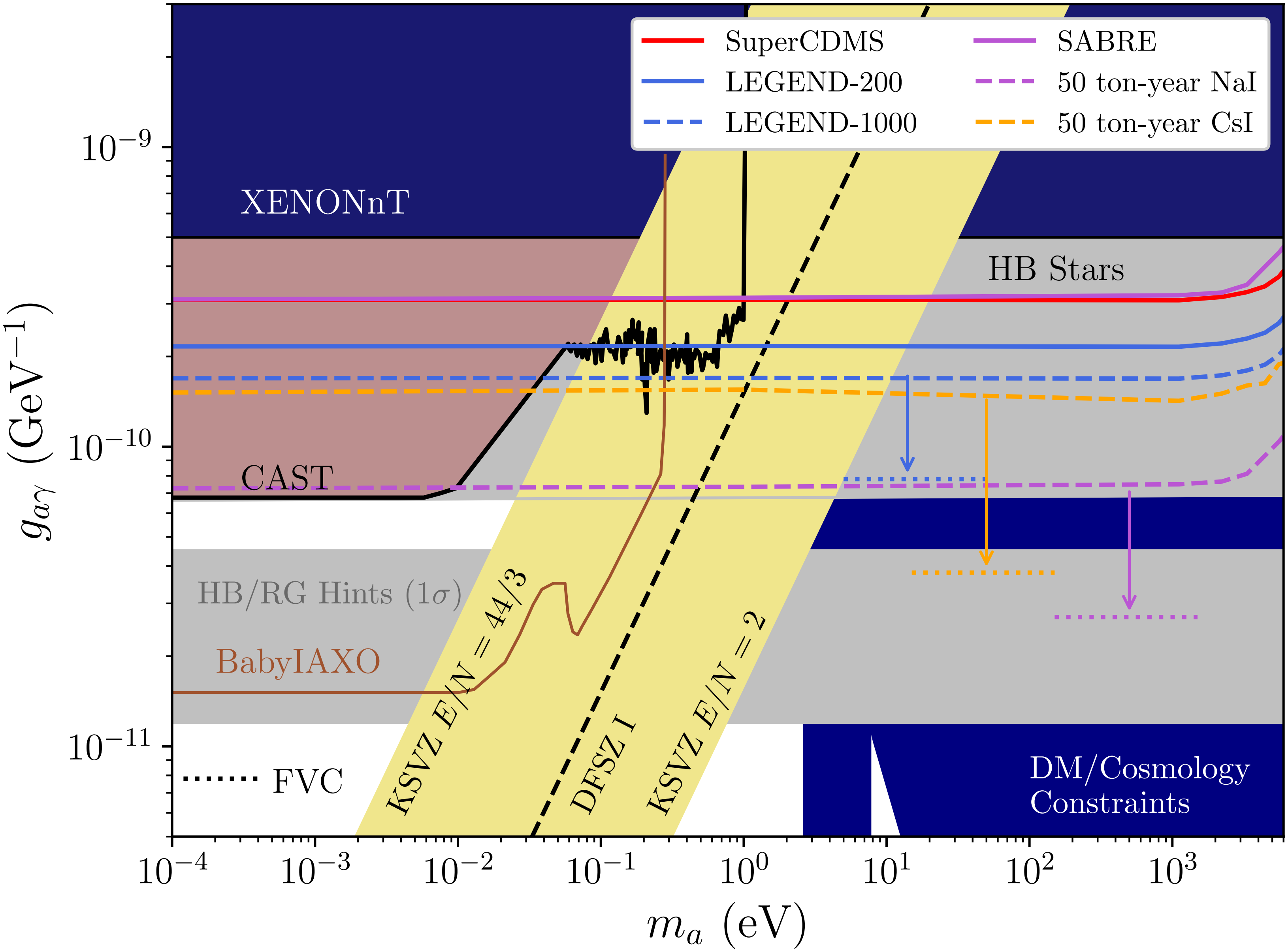
Backup Deck

Solar Axion Searches at LXe

Dent, Dutta, Newstead, Thompson Phys.Rev.Lett. 125 (2020) 13, 131805



Sensitivities: With DM Bounds



Solar Fluxes for Massive ALPs

- Primakoff: $\gamma + Z \to a + Z$
- Photon Coalescence (more important for > keV ALPs): $\gamma\gamma \to a$