Testing Majorana (LFV) and Dirac Neutrino ( $\Delta N_{eff}$ ) Mass Models

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#### $\nu$ oscillation

 $\ell_{\alpha}^{-}$ 

n

 $U_{\alpha j}$ 

In Standard Model: no  $\nu_R ! \Longrightarrow M_{\nu} = 0.$ 

 $\pi$ 

 $U^*_{ au j}$ 

$$|\nu_{\alpha}\rangle = \sum_{i=e,\mu,\tau} U_{i\alpha} |\nu_{i}\rangle \Longrightarrow M_{\nu} \neq 0$$

$$P(\nu_{\tau} \to \nu_{\alpha}) = \left| \sum_{j} U_{\tau j}^{*} U_{\alpha j} \exp\left(-i\frac{m_{j}^{2}L}{2E}\right) \right|^{2}$$

#### Distance L

 $exp(-ipx) \rightarrow exp(-im_i^2 L/2E)$ 

 $\nu_i$ 

NuFIT 5.2 (2022)
------------------

_		Normal Ore	lering (best fit)	Inverted Ordering $(\Delta \chi^2 = 2.3)$			
		bfp $\pm 1\sigma$	$3\sigma$ range	bfp $\pm 1\sigma$	$3\sigma$ range		
	$\sin^2 heta_{12}$	$0.303\substack{+0.012\\-0.011}$	$0.270 \rightarrow 0.341$	$0.303\substack{+0.012\\-0.011}$	$0.270 \rightarrow 0.341$		
	$ heta_{12}/^{\circ}$	$33.41_{-0.72}^{+0.75}$	$31.31 \rightarrow 35.74$	$33.41_{-0.72}^{+0.75}$	$31.31 \rightarrow 35.74$		
	$\sin^2 heta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \rightarrow 0.620$	$0.578\substack{+0.016\\-0.021}$	$0.412 \rightarrow 0.623$		
	$ heta_{23}/^{\circ}$	$49.1^{+1.0}_{-1.3}$	$39.6 \rightarrow 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \rightarrow 52.1$		
	$\sin^2  heta_{13}$	$0.02203\substack{+0.00056\\-0.00059}$	$0.02029 \rightarrow 0.02391$	$0.02219\substack{+0.00060\\-0.00057}$	$0.02047 \rightarrow 0.02396$		
	$ heta_{13}/^{\circ}$	$8.54_{-0.12}^{+0.11}$	$8.19 \rightarrow 8.89$	$8.57_{-0.11}^{+0.12}$	$8.23 \rightarrow 8.90$		
	$\delta_{ m CP}/^{\circ}$	$197^{+42}_{-25}$	$108 \rightarrow 404$	$286^{+27}_{-32}$	$192 \rightarrow 360$		
	$\frac{\Delta m_{21}^2}{10^{-5} \ \mathrm{eV}^2}$	$7.41_{-0.20}^{+0.21}$	$6.82 \rightarrow 8.03$	$7.41_{-0.20}^{+0.21}$	$6.82 \rightarrow 8.03$		
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \rightarrow +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \rightarrow -2.408$		

 $u_{\alpha} \leftrightarrow \nu_{\beta} \text{ prove that SM global symmetry}$   $U(1)_{L_{e}} \times U(1)_{L_{\mu}} \times U(1)_{L_{\tau}} \Rightarrow U(1)_{L_{\mu}-L_{\tau}} \times U(1)_{L_{\mu}+L_{\tau}-2L_{e}} \text{ is broken!}$ 

Lepton Flavor is definitely violated, so where is it?

Dirac vs Majorana

• Dirac neutrinos:

Introduce  $\nu_R$  to the SM  $(SU(3)_C \otimes SU(2)_L \otimes U(1)_Y)$  allowing  $\mathscr{L}_Y : y_\nu \bar{L} H \nu_R + h \cdot c$ .

- $\nu = \nu_L + \nu_R \neq \bar{\nu}$
- $U(1)_L$  conserved
- $m_{\nu} = y_{\nu} \langle H \rangle \approx 0.1 \text{eV}$ , this means Yukawa coupling  $y_{\nu} \sim 10^{-12}$ !!  $\implies$  difficult to measure
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 $\nu_R$  is a SM gauge singlet (1,1,0)

- Majorana neutrinos:
  - $\nu = \nu_L + \nu_L^c = \overline{\nu}$
  - $U(1)_L$  broken  $\implies$  neutrinoless double beta decay  $0\nu\beta\beta$
  - Allow mass term  $M \bar{\nu}_R^c \nu_R$  or add SU(2) triplet  $\Delta$

## Outline

• Majorana neutrinos test with lepton flavor violation

Prediction requires flavor structure ( $\nu$  oscillations) and new physics scale



Radiative  $\nu$ -models:

Zee Model, Extended Scotogenic Model, Flavor (LQ) Model

• Dirac neutrinos test with  $N_{\rm eff}$ 

#### Flavor violating decays

- $\mu \rightarrow e\gamma @ \text{MEG}, \mu \rightarrow 3e @ \text{Mu3e}$
- $\mu \leftrightarrow e$  conversion @ Mu2e
- $\tau \to \ell \gamma, \ \tau \to \mu \bar{\ell} \ell$  @ Belle II

	Present bound	Future sensitivity
$\mu \to e\gamma$	$4.2 \times 10^{-13}$	$6 \times 10^{-14}$
$\tau \to e\gamma$	$3.3 \times 10^{-8}$	$9 \times 10^{-9}$
$\tau \to \mu \gamma$	$4.4 \times 10^{-8}$	$7 \times 10^{-9}$
$\mu \rightarrow eee$	$1.0 \times 10^{-12}$	$\sim 10^{-16}$
$\tau \to eee$	$2.7 \times 10^{-8}$	$5 \times 10^{-10}$
$ au  o \mu \mu \mu$	$2.1 \times 10^{-8}$	$3.5 \times 10^{-10}$
$\tau^- \to e^- \mu^+ \mu^-$	$2.7 \times 10^{-8}$	$4.5 \times 10^{-9}$
$\tau^- \to \mu^- e^+ e^-$	$1.8 \times 10^{-8}$	$3 \times 10^{-10}$
$\tau^- \to e^+ \mu^- \mu^-$	$1.7 \times 10^{-8}$	$2.5 \times 10^{-10}$
$\tau^- \to \mu^+ e^- e^-$	$1.5 \times 10^{-8}$	$2.2 \times 10^{-10}$
$e^{-}\mu^{+} \leftrightarrow e^{+}\mu^{-}$	$8.3 \times 10^{-11}$	$2 \times 10^{-14}$
$\mu \leftrightarrow e$ [Au]	$7 \times 10^{-13}$	
conv. [Al]	—	$6 \times 10^{-17}$



• LFV at colliders



Neutrino Oscillation  $\implies$  Flavor Violation

- Dirac neutrinos:  $\mathscr{L}_Y : y_{\nu} \bar{L} H \nu_R + h \cdot c$ .
  - $m_{\nu} = y_{\nu} \langle H \rangle \approx 0.1 \text{ eV}$
  - Suppressed by Dirac mass,  $m_{\nu}$



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• Seesaw mass:  $\nu$ -mass is induced via Weinberg's dim-5 operator  $\mathscr{L}_{Y}: 1/2 M_{R} \overline{N}_{R}^{c} N_{R} + m_{D} \overline{\nu}_{L} N_{R} + h \cdot c \cdot \frac{\langle \phi \rangle}{\mathbf{X}} \qquad \langle \phi \rangle \qquad \mathbf{X}$ 

**Type I / Type III :**  $m_{\nu} \sim m_D^2 / M_R$ 

 $\left|A(\ell_{\alpha} \to \ell_{\beta} \gamma) \propto (m_D M_R^{-2} m_D^{\dagger})_{\alpha\beta} \simeq m_{\nu} / M_R\right|$ Structure in  $m_D$  can give large effect

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Seesaw mass:  $\nu$ -mass is induced via Weinberg's dim-5 operator  $\mathscr{L}_{Y}: 1/2 M_{R}\overline{N}_{R}^{c}N_{R} + m_{D}\overline{\nu}_{L}N_{R} + h.c.$  $\mathscr{L}: y\bar{L}^c\Delta L + \mu H\Delta H + h.c.$  $\langle \phi \rangle$  $\mathbf{x}^{\langle \phi \rangle}$  $\langle \phi \rangle_{\mathbf{k}}$  $\langle \phi \rangle$ **Type II** :  $m_{\nu} \simeq y \langle \Delta \rangle$ **Type I / Type III :**  $m_{\nu} \sim m_D^2/M_R$  $\left|A(\ell_{\alpha} \to \ell_{\beta} \gamma) \propto (m_D M_R^{-2} m_D^{\dagger})_{\alpha\beta} \simeq m_{\nu} / M_R\right|$  $BR(\tau \to \mu \gamma) \simeq 23BR(\tau \to e \gamma) \simeq 3.5BR(\mu \to e \gamma)$ Structure in  $m_D$  can give large effect Prediction of LFV ratios via  $m_{\nu}$ 

What about radiative neutrino mass models?

- Each loop has  $1/(16\pi^2)$  suppression
- Can tie to explain anomalies

 $(g-2)_{\mu}$ , dark matter, *B* anomalies, ... that fixes new physics scale.

Prediction for LFV ?

Zee Model, Extended Scotogenic Model, Flavor (LQ) Model

Radiative  $\nu$  mass generation

- Neutrino masses are zero at tree level:  $\nu_R$  may be absent
- Small, finite masses are generated as quantum corrections
- Typically involves exchange of two scalars leading to lepton number violation
   Majorana Masses
- Simple realization: Zee Model, which has a second Higgs doublet and a



• Smallness of neutrino mass is explained via loop and chiral suppression

• New physics in this framework may lie at the TeV scale; if connected to  $(g-2)_{\mu} \Longrightarrow$  Prediction for LFV

Zee Model

- Gauge symmetry is same as the Standard Model
  - $-\mathscr{L}: \bar{L}^{c}fL\eta^{+} + \bar{\ell}\tilde{Y}L\tilde{H}_{1} + \bar{\ell}YL\tilde{H}_{2} \mu H_{1}H_{2}\eta^{-}$

$$f = \begin{pmatrix} 0 & f_{e\mu} & f_{e\tau} \\ -f_{e\mu} & 0 & f_{\mu\tau} \\ -f_{e\tau} & -f_{\mu\tau} & 0 \end{pmatrix} \qquad \qquad Y = \begin{pmatrix} Y_{ee} & Y_{e\mu} & Y_{e\tau} \\ Y_{\mu e} & Y_{\mu\mu} & Y_{\mu\tau} \\ Y_{e\tau} & Y_{\tau\mu} & Y_{\tau\tau} \end{pmatrix}$$



• If  $Y \propto M_{\ell}$ , which happens with a  $Z_2$ , then the model is ruled out [Wolfenstein '80]

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$$\eta^{+}$$

$$\eta^{+}$$

$$H_{2}^{+}$$

$$H_{2}^{+}$$

$$H_{2}^{+}$$

$$H_{2}^{+}$$

$$H_{2}^{+}$$

$$\mu_{j}$$

$$M_{\nu} = \kappa \left( fM_{\ell}Y + Y^{T}M_{\ell}f^{T} \right)$$

$$\kappa = \frac{1}{16\pi^{2}} \sin 2\phi \log \left( \frac{m_{h^{+}}^{2}}{m_{H^{+}}^{2}} \right)$$

• If  $Y \propto M_{\ell}$ , which happens with a  $Z_2$ , then the model is ruled out

[Wolfenstein '80]

• General Parameterization to solve for  $M_{\nu}$ :  $Y = \kappa^{-1} M_{\ell}^{-1} (Z + Q)$ 

$$Q = \begin{pmatrix} 2q_4 - \frac{f_{\mu\tau}}{f_{e\tau}}q_1 & \frac{f_{\mu\tau}}{f_{e\tau}}(q_4 - q_2) & -\frac{2f_{\mu\tau}}{f_{e\mu}}q_4 - \frac{f_{\mu\tau}}{f_{e\tau}}q_3 \\ q_1 & q_2 + q_4 & \frac{2f_{e\tau}}{f_{e\mu}}q_4 + q_3 \\ -\frac{f_{e\mu}}{f_{e\tau}}q_1 & \frac{f_{e\mu}}{f_{e\tau}}(q_4 - q_2) & -\frac{f_{e\mu}}{f_{e\tau}}q_3 \end{pmatrix} \qquad \qquad Z = \begin{pmatrix} -\frac{M_{e\tau}^{\nu}}{f_{e\tau}} & 0 & -\frac{M_{\tau\tau}^{\nu}}{2f_{e\tau}} \\ 0 & \frac{f_{e\mu}M_{\tau\tau}^{\nu} - 2f_{e\tau}M_{\mu\tau}^{\nu}}{2f_{e\tau}f_{\mu\tau}} & 0 \\ \frac{M_{ee}^{\nu}}{2f_{e\tau}} & \frac{M_{\mu\mu}^{\nu}}{2f_{e\tau}} & 0 \end{pmatrix}$$

[Pleitez, et al. '17]

 $\begin{aligned} |q_{1}| < \sqrt{4\pi} m_{\mu} \kappa & |q_{2}| < \sqrt{4\pi} |f_{e\tau}/f_{\mu\tau}| m_{e} \kappa + \sqrt{\pi} |f_{e\mu}/f_{e\tau}| m_{\mu} \kappa + \sqrt{\pi} m_{\tau} \kappa \\ |q_{3}| < \sqrt{4\pi} |f_{e\tau}/f_{e\mu}| m_{\tau} & |q_{4}| < \sqrt{\pi} |f_{e\mu}/f_{e\tau}| m_{\mu} \kappa + \sqrt{\pi} m_{\tau} \kappa \end{aligned}$ 

## Zee Model prediction for LFV

ν<sub>aL</sub> ↔ ν<sub>bL</sub> ⇒ e, μ, τ number are violated
 Second Higgs to explain (g - 2)<sub>μ</sub> ⇒ Prediction for LFV



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Minimal texture  $\implies$  concrete Prediction

$$Y = \begin{pmatrix} g - 2 \end{pmatrix}_{\mu} \\ 0 & \frac{-M_{\mu\mu}^{\nu}}{2f_{e\mu}m_{e}\kappa} & 0 \\ 0 & 0 & 0 \\ \frac{M_{ee}^{\nu}M_{\mu\mu}^{\nu}}{4f_{e\mu}m_{\tau}\kappa M_{\mu\tau}^{\nu}} & 0 & 0 \end{pmatrix}$$

• Leads to  $M_{e\tau}^{\nu} = M_{\tau\tau}^{\nu} = 0$ (texture-2 zero)

Minimal texture  $\implies$  concrete Prediction  $(g-2)_{\mu}$ 0.65<sub>[</sub> ■ NH 0.60 IH NH 1 $\sigma$  allowed  $\sin^2 \theta_{23}^{23}$ 0.50 IH 1 $\sigma$  allowed Belle Belle II (5 ab<sup>-1</sup>) Excluded @ • Leads to  $M_{e\tau}^{\nu} = M_{\tau\tau}^{\nu} = 0$ (texture-2 zero) 0.45 DUNE 336 kt-MW-years 90% CL NH :  $m_{\ell} > 3.8 \text{ meV}, \quad m_{\beta\beta} > 0.15 \text{ eV}$ IH  $3\sigma$  allowed  $\delta_{\rm CP} \simeq [266 - 269]$ 0.40 NH 3 $\sigma$  allowed  $\alpha_1 \simeq [182 - 187], \quad \alpha_2 \simeq [177 - 179]$ 0.5 0.25 1.5 0.1 1 IH:  $m_{\ell} > 5.0 \text{ meV}, \quad m_{\beta\beta} > 0.48 \text{ eV}$ BR( $\tau \to e\mu^+ e^-$ ) [10<sup>-8</sup>]  $\delta_{\rm CP} \simeq [270 - 271]$  $\alpha_1 \simeq [175 - 179], \quad \alpha_2 \simeq [180 - 182]$ 



• Flavor violating Yukawa coupling  $Y_{12} \equiv Y_{e\mu}$  can explain recent CMS excess (3.8 $\sigma$  local) in resonant  $e\mu$  channel [Afik, Dev, Thapa, '23]

- CMS reported  $3.8\sigma$  local ( $2.8\sigma$  global) excess in the resonant  $e\mu$  search around 146 GeV, with a preferred cross-section of  $3.89^{+1.25}_{-1.13}$  fb
- Use the lepton (PDF) of the proton to explain CMS excess! [Bertone et.al '15, Buonocore '20, Dreiner '21]



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- Leptophilic neutral (pseudo) scalars with Yukawa coupling  $Y_{e\mu} \sim 0.55 - 0.81$ gives right cross-section
- Same parameter space can explain  $(g 2)_{\mu}$  as well as CDF *W*-boson mass anomaly

#### Radiative neutrino mass models

- Each loop has  $1/(16\pi^2)$  suppression
- Can tie to explain anomalies

 $(g-2)_{\mu}$ , dark matter, *B* anomalies, ... that fixes new physics scale.

Prediction for LFV ?

Zee Model, Extended Scotogenic Model, Flavor (LQ) Model

#### Scotogenic Model: Dark Matter

- No Standard Model particle inside the loop
- Neutrino mass has no chiral suppression; new scale can be large
- Z<sub>2</sub> symmetry forbids tree-level contribution
   to Dirac neutrino mass and gives rise to dark
   matter candidate. DM require TeV scale new
   physics



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• Most parameters are probed but still difficult to have firm prediction!





Can we do more!

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• Extend scotogenic model with charged singlet  $S^{-}(1,1, -1, -)$ that allows  $f_{ij} \ \bar{\ell}_{R_i} S^{-} \bar{N}_{R_j}$ 

[Dcruz, Thapa, '22]



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Neutrino mass, AMM, Scalar DM, Fermionic DM, W-mass correction, LFV prediction • The lightest of the right-handed neutrinos is the fermionic DM candidate.





Prediction for LFV

$$Y = U_{\text{PMNS}} \sqrt{M_{\nu}^{\text{diag}}} R^{\dagger} \sqrt{\Lambda^{-1}}^{\dagger}$$

$$\Lambda_{k} = \frac{M_{N_{k}}}{16\pi^{2}} \left[ \frac{m_{H}^{2}}{m_{H}^{2} - M_{N_{k}}^{2}} \log \frac{m_{H}^{2}}{M_{N_{k}}^{2}} - (m_{H} \to m_{A}) \right]$$

[Casas, Ibarra '01]

## **Fermionic DM**



Requires  $m_H \simeq m_A$  for neutrino mass fit

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- SM is flavor universal ⇒ any deviation is key signature of physics beyond the SM
- Hints of deviations from SM in semileptonic B decays



 $R_{K^{(*)}}$  anomaly is gone !



Tension with the SM at ~  $3\sigma$  level

 $LFUV \Longrightarrow$  Neutrino mass  $\Longrightarrow LFV$ 

 $\ell_R$ 

 $u_L$ 

 $u_R$ 

- Neutrino mass model to resolve *B*-anomalies [ $R_2(3,2,7/6)$  or  $S_1(\overline{3},1,1/3)$ ]
- The same  $R_2$  and  $S_1$  LQ also induce muon  $(g 2)_{\mu}$
- Flavor structure is very constrained
- Framework can be tested at LHC as well as in processes such as  $\tau \rightarrow e\gamma$

 $LFUV \implies$  Neutrino mass  $\implies LFV$ 

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• Majorana neutrinos test with lepton flavor violation

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• Dirac neutrinos test with  $N_{\rm eff}$ 

- Neutrinos may well be Dirac particles  $\implies \Delta L = 0$
- Oscillation experiments cannot distinguish Dirac neutrinos from Majorana neutrinos
- If Dirac nature ⇒ important to understand the smallness of their masses
- Dirac leptogenesis to explain observed baryon asymmetry is an attractive feature [Dick, Lindner, Ratz, Wrig, '99]
- Dirac seesaw can be achieved in Mirror Models

[Lee, Yang '56; Foot, Volkas '95; Berezhiani, Mohapatra '95, Silagadze '97]

Dirac Neutrinos from Left-Right Symmetry  $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$  Dirac Neutrinos from Left-Right Symmetry  $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ 

• Fermion Representation:

$$Q_{L}(3,2,1,1/3) = \begin{pmatrix} u \\ d \end{pmatrix}_{L} \qquad Q_{R}(3,1,2,1/3) = \begin{pmatrix} u \\ d \end{pmatrix}_{R}$$
$$\psi_{L}(1,2,1,-1) = \begin{pmatrix} \nu \\ e \end{pmatrix}_{L} \qquad \psi_{R}(1,1,2,-1) = \begin{pmatrix} \nu \\ e \end{pmatrix}_{R}$$

Dirac Neutrinos from Left-Right Symmetry

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- Vector-like fermion introduced to realize "universal seesaw" for charged fermion masses
   [Davidson, Wali '87]

P(3,1,1,4/3), N(3,1,1,-2/3), E(1,1,1,-2)

$$\begin{array}{ccc} M_{u} = \begin{pmatrix} 0 & y_{u} \kappa_{L} \\ y_{u}^{\dagger} \kappa_{R} & M_{P^{0}} \end{pmatrix} & M_{d} = \begin{pmatrix} 0 & y_{d} \kappa_{L} \\ y_{d}^{\dagger} \kappa_{R} & M_{N^{0}} \end{pmatrix} & M_{\ell} = \begin{pmatrix} 0 & y_{\ell} \kappa_{L} \\ y_{\ell}^{\dagger} \kappa_{R} & M_{E^{0}} \end{pmatrix} \\ & m_{u_{i}} \approx \frac{y_{u_{i}}^{2} \kappa_{L} \kappa_{R}}{M_{P_{i}^{0}}}, & m_{d_{i}} \approx \frac{y_{d_{i}}^{2} \kappa_{L} \kappa_{R}}{M_{N_{i}^{0}}}, & m_{\ell_{i}} \approx \frac{y_{\ell_{i}}^{2} \kappa_{L} \kappa_{R}}{M_{E_{i}^{0}}} \end{array}$$

Dirac Neutrinos from Left-Right Symmetry

 $SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ 

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P(3,1,1,4/3), N(3,1,1,-2/3), E(1,1,1,-2)

$$M_{u} = \begin{pmatrix} 0 & y_{u}\kappa_{L} \\ y_{u}^{\dagger}\kappa_{R} & M_{P^{0}} \end{pmatrix} \qquad M_{d} = \begin{pmatrix} 0 & y_{d}\kappa_{L} \\ y_{d}^{\dagger}\kappa_{R} & M_{N^{0}} \end{pmatrix} \qquad M_{\ell} = \begin{pmatrix} 0 & y_{\ell}\kappa_{L} \\ y_{\ell}^{\dagger}\kappa_{R} & M_{E^{0}} \end{pmatrix}$$
$$m_{u_{i}} \approx \frac{y_{u_{i}}^{2}\kappa_{L}\kappa_{R}}{M_{P_{i}^{0}}}, \qquad m_{d_{i}} \approx \frac{y_{d_{i}}^{2}\kappa_{L}\kappa_{R}}{M_{N_{i}^{0}}}, \qquad m_{\ell_{i}} \approx \frac{y_{\ell_{i}}^{2}\kappa_{L}\kappa_{R}}{M_{E_{i}^{0}}}$$
$$\text{Higgs Representation:} \qquad \left[ \chi_{L}(1,2,1,1) = \begin{pmatrix} \chi_{L}^{+} \\ \chi_{L}^{0} \end{pmatrix}_{L} \qquad \chi_{R}(1,2,1,1) = \begin{pmatrix} \chi_{R}^{+} \\ \chi_{R}^{0} \end{pmatrix}_{R} \right]$$
$$SU(2)_{L} \times SU(2)_{R} \times U(1)_{X} \qquad \frac{\langle \chi_{R}^{0} \rangle}{M_{R}} \qquad SU(2)_{L} \times U(1)_{Y} \qquad \frac{\langle \chi_{L}^{0} \rangle}{M_{L}} \qquad U(1)_{\text{EM}}$$

- Higgs sector is very simple:  $\chi_L(1,2,1,1) + \chi_R(1,1,2,1)$
- $W_L^+ \leftrightarrow W_R^+$  mixing is absent at tree-level
- $W_L^+ \leftrightarrow W_R^+$  mixing is induced at the loop level, which in turn induces two-loop Dirac masses for neutrino [Babu, He '89]

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$$M_{\nu D} = \frac{-g^4}{2} y_t^2 y_b^2 y_\ell^2 \kappa_L^3 \kappa_R^3 \frac{r \ M_P M_N M_{E_\ell}}{M_{W_L}^2 M_{W_R}^2} \ I_{E_\ell}$$

 $I_{E_{\ell}} = \int \int \frac{d^4k d^4p}{(2\pi)^8} \frac{3M_{W_L}^2 M_{W_R}^2 + (p^2 - M_{W_L}^2)(p^2 - M_{W_R}^2)}{k^2(p+k)^2(k^2 - M_N^2)((p+k)^2 - M_p^2)p^2(p^2 - M_{E_{\ell}}^2)(p^2 - M_{W_L}^2)(p^2 - M_{W_R}^2)}$ [Babu, He, Su, **Thapa** '22]

## Testing Dirac Neutrinos with $N_{\rm eff}$

- CMB is sensitive to extra radiation density arising from new extra degrees of freedom that were in thermal equilibrium with the SM plasma
- $\nu_R$  (ultra-light new particles, new degrees of freedom) couples to other particles and are produced in the early universe and contribute to additional radiation density in early universe !
- The effect of such light particles is parameterized as  $\Delta N_{\text{eff}}$  and is measured in units of extra neutrino degrees of freedom
- Dirac neutrino modes of this type will modify  $N_{\text{eff}}$  by about 0.14

$$\Delta N_{\rm eff} \simeq 0.027 \left( \frac{106.75}{g_{\star} (T_{\rm dec})} \right)^{4/3} g_{\rm eff}$$
$$g_{\rm eff} = (7/8) \times (2) \times (3) = 21/4$$

## Dirac Neutrino in cosmology

- In SM  $N_{\rm eff} \simeq 3$
- Improvement on  $\Delta N_{\text{eff}}$  in CMB-S4
- Valid for 3  $\nu_R$  in thermal equilibrium with SM
- This gives strong constraint for any (eg. LR model)
   Dirac neutrino mass model



[Heeck, Abazajian '19; Babu, He, Su, Thapa '22]

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[Heeck, Abazajian '19; Babu, He, Su, Thapa '22]

Can we embed LR model into GUT ?

Embedding in  $SU(5)_L \times SU(5)_R$ 

- The fermion spectrum of the model has a natural embedding in  $SU(5)_L \times SU(5)_R$  unification
- All left-handed (right-handed) fermions of the SM fit into  $10 + \overline{5}$  of  $SU(5)_L (SU(5)_R)$
- The remaining vector-like quarks and leptons fill rest of the multiples

$$F_{L,R} = \begin{pmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e \\ -\nu \end{pmatrix}_{L,R} \qquad T_{L,R} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & U_3^c & -U_2^c & u_1 & d_1 \\ -U_3^c & 0 & U_1^c & u_2 & d_2 \\ U_2^c & -U_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & E^c \\ -d_1 & -d_2 & -d_3 & -E^c & 0 \end{pmatrix}_{L,R}$$

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• Parity can be imposed under which  $F_L \leftrightarrow F_R$  and  $T_L \leftrightarrow T_R$ 

Gauge coupling Unification

• The evolution of the gauge couplings constants at one-loop level are governed by the following RGEs  $\alpha_i = g_i^2/4\pi$ 

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) - \frac{b_i}{2\pi} \ln(\mu/\mu_0)$$

At  $m_t$  (top quark mass):

 $g_1 = 0.3583$ ,  $g_2 = 0.64779$ ,  $g_3 = 1.1666$ 

• With the SM particles, we obtain following beta function coefficients with properly normalized gauge couplings:

$$b_1 = \frac{41}{26}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -\frac{7}{2}$$

•  $SU(5) \times SU(5)$  group can directly break to the SM gauge group, where  $g_1, g_2, g_3$  meet at a single value

$$\alpha_{\rm GUT} = 2 \ \alpha_3 = \alpha_2 = \frac{13}{3}\alpha_1$$

$$\Rightarrow \sin^2 \theta_W = 3/16$$

⇒ Cannot reconcile value measured at electroweak scale





Not the only breaking chain; some have many attractive features ⇒ predicts Dirac neutrinos, firm prediction on oscillation parameters, and can solve strong CP problem. [Babu, Mohapatra, Thapa, in preparation]

#### More on Dirac Neutrinos: Dirac Leptogenesis

• Dirac neutrinos:  $y_{\nu} \bar{L} H \nu_R \Longrightarrow$  Higgs coupling strength  $y_{\nu} \sim 10^{-12}!!$  $\Longrightarrow$  too feeble to ever thermalize  $\nu_R$  in the early universe  $\Longrightarrow$  Dirac Leptogenesis  $\Longrightarrow$  matter/antimatter asymmetry [Dick, Lindner, Ratz, Wrig, '009]

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Idea: Take a new heavy particle X that decay out of equilibrium into a non-thermal  $\nu_R$  and a SM particle.

## Simple models:

Case	$SU(3) \times SU(2) \times U(1)$	spin	$g_X$	(B-L)(X)	Relevant Lagrangian terms that induce $X$ decay	$\varepsilon_{\rm wave}$	$\varepsilon_{ m vertex}$	$\Delta B$
a	$({f 1},{f 1},-1)$	0	1	-2	$ u_R e_R ar{X}, \ LL ar{X}$	$\checkmark$	X	0
b	(1, 2, 1/2)	0	2	0	$\bar{H}X, \ \bar{\nu}_R L X, \ \bar{L}e_R X, \ \bar{Q}_L d_R X, \ \bar{u}_R Q_L X, \ X^{\dagger} H^{\dagger} H H$	<b>√</b>	✓or X	0
С	(3, 1, -1/3)	0	3	-2/3	$d_R \nu_R X^{\dagger}, \ u_R e_R X^{\dagger}, \ Q_L L X^{\dagger}, u_R d_R X, \ Q_L Q_L X$	<b>\</b>	✓or X	0 or 1
d	$({f 3},{f 1},2/3)$	0	3	-2/3	$u_R \nu_R X^{\dagger}, \ d_R d_R X$	<b>\</b>	X	1
e	$({f 3},{f 2},1/6)$	0	6	4/3	$\bar{Q}_L \nu_R X, \ \bar{d}_R L X$	$\checkmark$	X	0
f	(1, 2, -1/2)	1/2	2	-1	$\bar{X}L, \ \bar{\nu}_R XH, \ \bar{X}e_R H$	$\checkmark$	1	0

[Heeck, Heisig, Thapa, '23]

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c	(3, 1, -1/3)	0	3	-2/3	$d_R \nu_R X^{\dagger}, \ u_R e_R X^{\dagger}, \ Q_L L X^{\dagger}, u_R d_R X, \ Q_L Q_L X$	1	✓or X	0  or  1
d	$({f 3},{f 1},2/3)$	0	3	-2/3	$u_R \nu_R X^{\dagger}, \ d_R d_R X$	1	×	1
e	(3, 2, 1/6)	0	6	4/3	$\bar{Q}_L \nu_R X, \ \bar{d}_R L X$	1	×	0
f	(1, 2, -1/2)	1/2	2	-1	$\bar{X}L, \ \bar{\nu}_R XH, \ \bar{X}e_R H$	1	1	0

[Heeck, Heisig, Thapa, '23]

## **Dirac Leptogenesis**



• *CP* asymmetry: opposite lepton asymmetries for left- and righthanded neutrinos with  $\Delta L = 0$ 

 $\Delta\nu=\nu_L-\bar\nu_L=-\left(\nu_R-\bar\nu_R\right)\neq 0$ 

•  $\nu_R$  are out of equilibrium after X decays and are invisible to the sphalerons, only left handed asymmetry is converted into

baryons.

$$Y_{\Delta B} \simeq 10^{-3} \epsilon \eta \simeq 10^{-10}$$

$$\Sigma_A \equiv Y_A + Y_{\bar{A}} \quad \Delta_A \equiv Y_A - Y_{\bar{A}}$$



## **Dirac Leptogenesis**



• *CP* asymmetry: opposite lepton asymmetries for left- and righthanded neutrinos with  $\Delta L = 0$ 

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•  $\nu_R$  are out of equilibrium after X decays and are invisible to the sphalerons, only left handed asymmetry is converted into baryons.  $Y_{\Lambda R} = \simeq 10^{-3} \epsilon \eta \simeq 10^{-10}$ 

$$\mathcal{L}_{A} \equiv Y_{A} + Y_{\bar{A}} \qquad \Delta_{A} \equiv Y_{A} - Y_{\bar{A}}$$

$$\eta \approx 2.1 \times 10^{-4}, \ \Delta N_{\text{eff}} \approx 0.082$$

$$\begin{array}{c} 0.01 \\ M_{X} = 10^{9} \text{ GeV}, \ \Gamma_{X} / \mathcal{H}(M_{X}) = 10^{-9}, \ B_{R} = 0.99 \\ 10^{-4} \\ 10^{-4} \\ 10^{-4} \\ 10^{-6} \\ 10^{-6} \\ 10^{-10} \\ 10^{-12} \\ 0.1 \\ 10 \\ 100 \\ 10^{5} \\ x = M_{X} / T$$

[Heeck, Heisig, **Thapa**, '23]

**T**7

When X decays so late into  $\nu_R \Longrightarrow$  shoots extremely highly relativistic  $\nu_R$  with energy  $\approx M_X/2 \Longrightarrow$  arbitrarily large  $N_{\text{eff}}$ 

## Dirac $\nu$ in the CMB

SI

- X decays into high-energy  $\nu_R$
- Testable  $\Delta N_{\rm eff}!$  $\implies$  Large parameter space is already excluded (Red) and can be probed (lightRed)



[Heeck, Heisig, **Thapa**, '23]

## Dirac $\nu$ in the CMB

- X decays into high-energy  $\nu_R$  !
- Testable  $\Delta N_{\rm eff}!$  $\implies$  Large parameter space is already excluded (Red) and can be probed (lightRed)

directly with Leptoquarks



 $\implies$  Predicts proton decay  $p \rightarrow K^+ \bar{\nu}_R$ 

#### Conclusion

- Neutrino oscillations require extension of the SM.
- Models for both Majorana and Dirac neutrinos were discussed with some models to incorporate Dark Matter, Dirac leptogenesis, and various anomalies.
- Most of the models discussed can be probed through
  - LFV experiments  $(\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma)$ ,
  - N<sub>eff</sub>
- Hope that anomalies are confirmed CDF,  $(g-2)_{\mu}$ ,  $R_D R_{D^*}$ !

Thank you



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## $SU(3)_C \times SU(2)_L \times U(1)_Y$ with an extended scalar sector

Category	Model	Fields	Loop?	Ref.
Class-I	Model-I	$egin{array}{c} {m S_1(\overline{3},1,1/3)}\ \omega(\overline{6},1,2/3) \end{array}$	two-loop	[Babu, Leung, '01] [Kohda, Sachdeva, Waite, '19]
	Model-II	$egin{array}{c} {m S_3(\overline{3},3,1/3)}\ \omega(\overline{6},1,2/3) \end{array}$	two-loop	[Babu, Leung, '01]
Class-II	Model-III	$egin{array}{c} S_1(\overline{3},1,1/3)\ \widetilde{R}_2(3,2,1/6) \end{array}$	one-loop two-loop	[Dorsner, Fajfer, Košnik, '17] [Catà, Mannel, '19] [Babu, Julio, '10]
	Model-IV	$egin{array}{c} S_3(\overline{3},3,1/3)\ \widetilde{R}_2(3,2,1/6) \end{array}$	one-loop	[Dorsner, Fajfer, Košnik, '17]
Class-III	Model-V	$egin{array}{c} R_2(3,2,7/6)\ S_3(\overline{3},3,1/3)\ \chi(3,1,2/3) \end{array}$	one-loop	[Saad, <b>AT</b> , '20]
	Model-VI	$egin{array}{c} R_2(3,2,7/6)\ S_3(\overline{3},3,1/3)\ \Delta(1,4,3/2) \end{array}$	one-loop	[Popov, Schmidt, White, '19] [Babu, Dev, Jana, <b>AT</b> , '20]
	Model-VII	$egin{array}{l} S_1(\overline{3},1,1/3)\ R_2(3,2,7/6)\ \xi(3,3,2/3) \end{array}$	two-loop	[Julio, Saad, $\mathbf{AT}$ , '22]

Charged Current Anomaly:  $R_{D^{(*)}}$ :  $R_2 \sim (3, 2, 7/6)$ 

 $\mathscr{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[ (1+g_V)(\bar{\tau}_L \gamma^\mu \nu_L) \left( \bar{c}_L \gamma_\mu b_L \right) \right] + g_s \left( \bar{\tau}_R \nu_L \right) (\bar{c}_R b_L) + g_T \left( \bar{\tau}_R \sigma^{\mu\nu} \nu_L \right) \left( \bar{c}_R \sigma_{\mu\nu} b_L \right)$ 



 $\operatorname{Re}\left(g_{S}^{\tau}\right)$ 

 $\gamma$ 

#### **Anomalous Magnetic Moment**

$$f^{R} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & f_{32}^{R} & 0 \end{pmatrix} \qquad f^{L} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & f_{32}^{R} & 0 \end{pmatrix} \qquad \overset{u_{R}}{\underset{\ell_{L}}{\overset{u_{R}}{\underset{\omega^{5/3}}{\overset{u_{L}}{\overset{$$

• For 1TeV LQ mass, the required product of Yukawa

$$(g-2)_{\mu}: f_{32}^{L}f_{32}^{R} = -0.0019$$

## **Experimental Constraints**

- $\ell_i \to \ell_j \gamma$
- $\mu e$  conversion
- $Z \rightarrow \tau \tau$  decay
- Rare D-meson decay

- $D^0 \overline{D}^0$  mixing
- Bounds from kaons
- Collider constraints
  - Pair-production Bounds
  - Dilepton Bounds



## **Bounds from kaons**

Process	Constraints
$K_L \rightarrow e^+ e^-$	$ \hat{f}_{de}^R \hat{f}_{se}^{R*}  \le 2.0 \times 10^{-3} \left(\frac{M_{R_2}}{\text{TeV}}\right)^2$
$K_L^0 \to e^{\pm} \mu^{\mp}$	$ \hat{f}_{d\mu}^R \hat{f}_{se}^{R*} + \hat{f}_{s\mu}^R \hat{f}_{de}^{R*}  \le 1.9 \times 10^{-5} \left(\frac{M_{R_2}}{\text{TeV}}\right)^2$
$K^0_L \to \pi^0 e^\pm \mu^\mp$	$ \hat{f}_{d\mu}^{R}\hat{f}_{se}^{R*} - f_{s\mu}^{R}f_{de}^{R*}  \le 2.9 \times 10^{-4} \left(\frac{M_{R_2}}{\text{TeV}}\right)^2$
$K^+ \to \pi^+ e^+ e^-$	$ \hat{f}_{de}^{R}\hat{f}_{s\mu}^{R*}  \le 2.3 \times 10^{-2} \left(\frac{M_{R_2}}{\text{TeV}}\right)^2$
$K^+ \to \pi^+ e^- \mu^+$	$\left  \hat{f}_{d\mu}^{R} \hat{f}_{se}^{R*}  ,  \hat{f}_{de}^{R} \hat{f}_{s\mu}^{R*}   \le 1.9 \times 10^{-4} \left( \frac{M_{R_2}}{\text{TeV}} \right)^2 \right $
$K-\bar{K}$	$ \hat{f}_{d\alpha}^{R*}\hat{f}_{s\alpha}^{R}  \le 0.0266 \left(\frac{M_{R_2}}{\text{TeV}}\right)$
$K^+ \to \pi^+ \nu \nu$	$\operatorname{Re}[\hat{y}_{de}^{L}\hat{y}_{se}^{L}] = [-3.7, 8.3] \times 10^{-4} \left(\frac{M_{S_1}}{\text{TeV}}\right)^2$
	$\left[\sum_{m \neq n}  \hat{y}_{dm}^L \hat{y}_{sn}^{L*} ^2\right]^{1/2} < 6.0 \times 10^{-4} \left(\frac{M_{S_1}}{\text{TeV}}\right)^2$
$B \to K^{(*)} \nu \nu$	$ \hat{y}_{b\alpha}^L \hat{y}_{s\beta}^L = [-0.036, 0.076] \left(\frac{M_{S_1}}{\text{TeV}}\right)^2, \left[R_{K^*}^{\nu\bar{\nu}} < 2.7\right] $
	$ \hat{y}_{b\alpha}^L \hat{y}_{s\beta}^L = [-0.047, 0.087] \left(\frac{M_{S_1}}{\text{TeV}}\right)^2, \left[R_K^{\nu\bar{\nu}} < 3.9\right] $



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#### Fit to Oscillation Data

 $M_{\nu^D} = y_{\ell} M_E I_E y_{\ell}^{\dagger}$ 

Oscillation	$3\sigma$ range	Model prediction				
parameters	NuFit5.1 <b>[51]</b>	BP I (NH)	BP II (NH)	BP III (IH)	BP IV (IH)	
$\Delta m_{21}^2 (10^{-5} \text{ eV}^2)$	6.82 - 8.04	7.42	7.38	7.35	7.35	
$\Delta m_{23}^2 (10^{-3} \text{ eV}^2) (\text{IH})$	2.410 - 2.574	-	-	2.48	2.52	
$\Delta m_{31}^2 (10^{-3} \text{ eV}^2) (\text{NH})$	2.43 - 2.593	2.49	2.51	-	-	
$\sin^2 \theta_{12}$	0.269 - 0.343	0.324	0.301	0.306	0.310	
$\sin^2 \theta_{23}$ (IH)	0.410 - 0.613	-	-	0.510	0.550	
$\sin^2 \theta_{23}$ (NH)	0.408 - 0.603	0.491	0.533	-	-	
$\sin^2 \theta_{13}$ (IH)	0.02055 - 0.02457	-	-	0.0219	0.0213	
$\sin^2 \theta_{13}(\mathrm{NH})$	0.02060 - 0.02435	0.0234	0.0213	-	-	
$\delta_{\rm CP}$ (IH)	192 - 361	-	-	$236^{\circ}$	$279^{\circ}$	
$\delta_{ m CP}$ (NH)	105 - 405	$199^{\circ}$	$280^{\circ}$	-	-	
$m_{\text{light}} (10^{-1})$	$^{-3}) eV$	0.66	2.04	14.1	8.50	
$M_{E_1}/M$	W <sub>R</sub>	917	45.5	1936	1990	
$M_{E_2}/M$	W <sub>R</sub>	0.650	0.43	0.12	0.11	
$M_{E_3}/M$	$W_R$	0.019	0.029	0.015	0.012	

[Babu, He, Su, Thapa '22]