Energy-dependent neutrino mixing parameters at oscillation experiments

Pedro Machado

CETUP* 2023

Based on

Babu, Brdar, de Gouvêa, M 2108.11961, 2209.00031

Outline

Quantum corrections

Neutrino masses

How can quantum corrections and the mechanism of neutrino masses leave an imprint on oscillation phenomenology?

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Quantum corrections

How do we predict phenomena?

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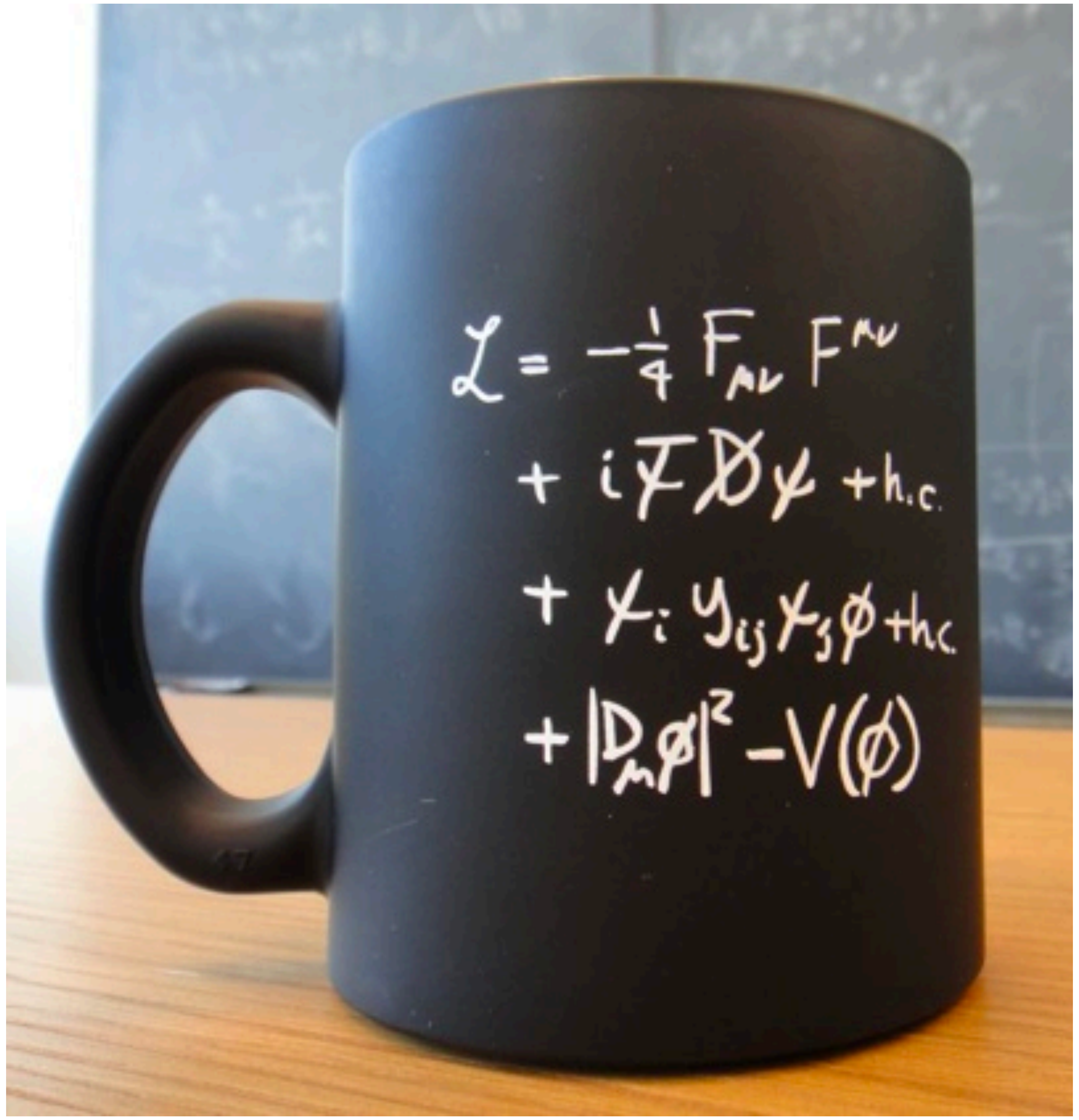
Quantum corrections

Start with a "master equation"



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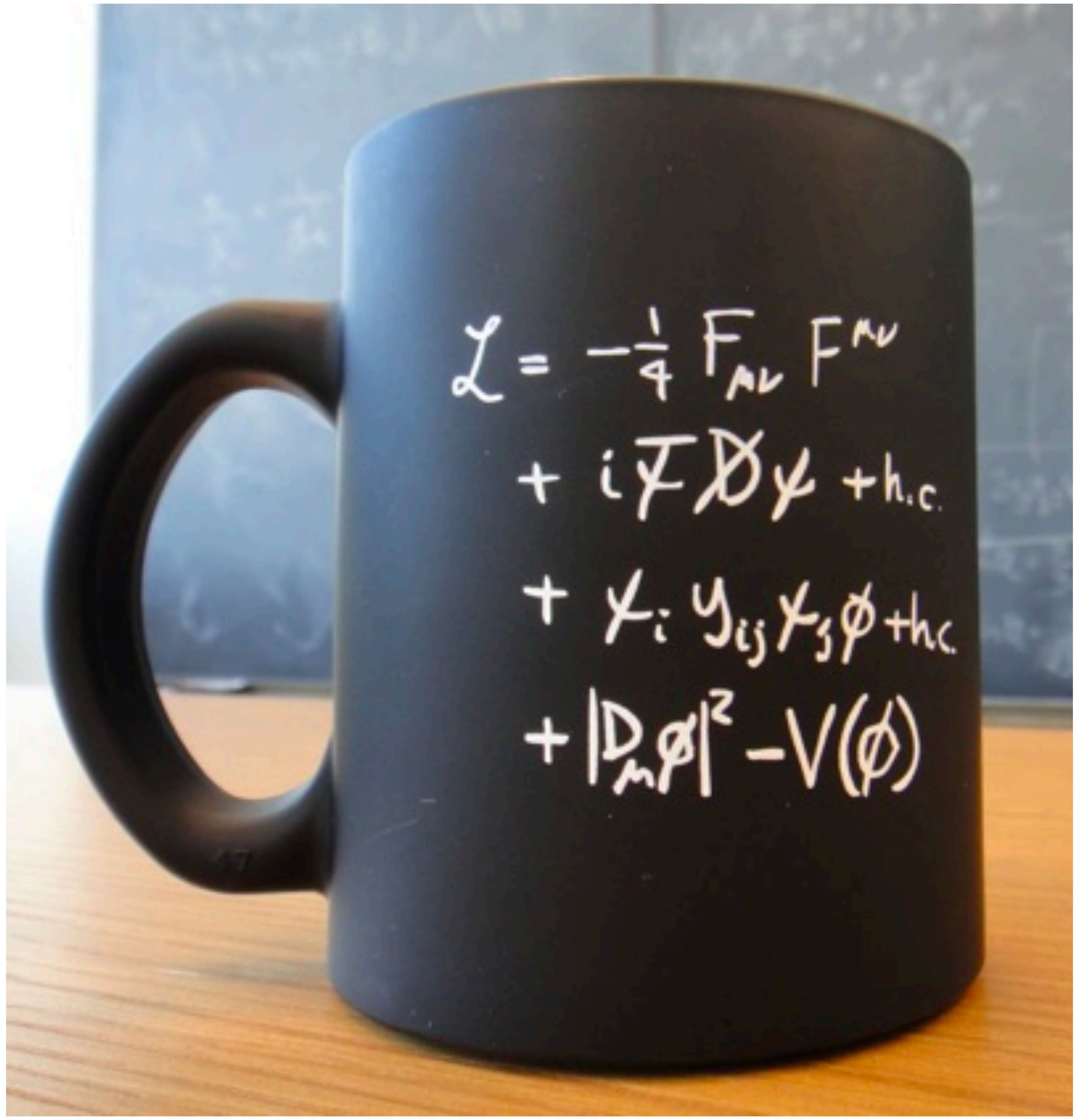

Now, solve it

Evolve $|a\rangle$ to $|b\rangle$:

$$\langle b | \exp(-iHt) | a \rangle = U(a, b; t)$$

$$U(a,b;t) = \int \mathcal{D}x(t) \exp\left(i \int dt' \mathcal{L}\right)$$

All possible paths

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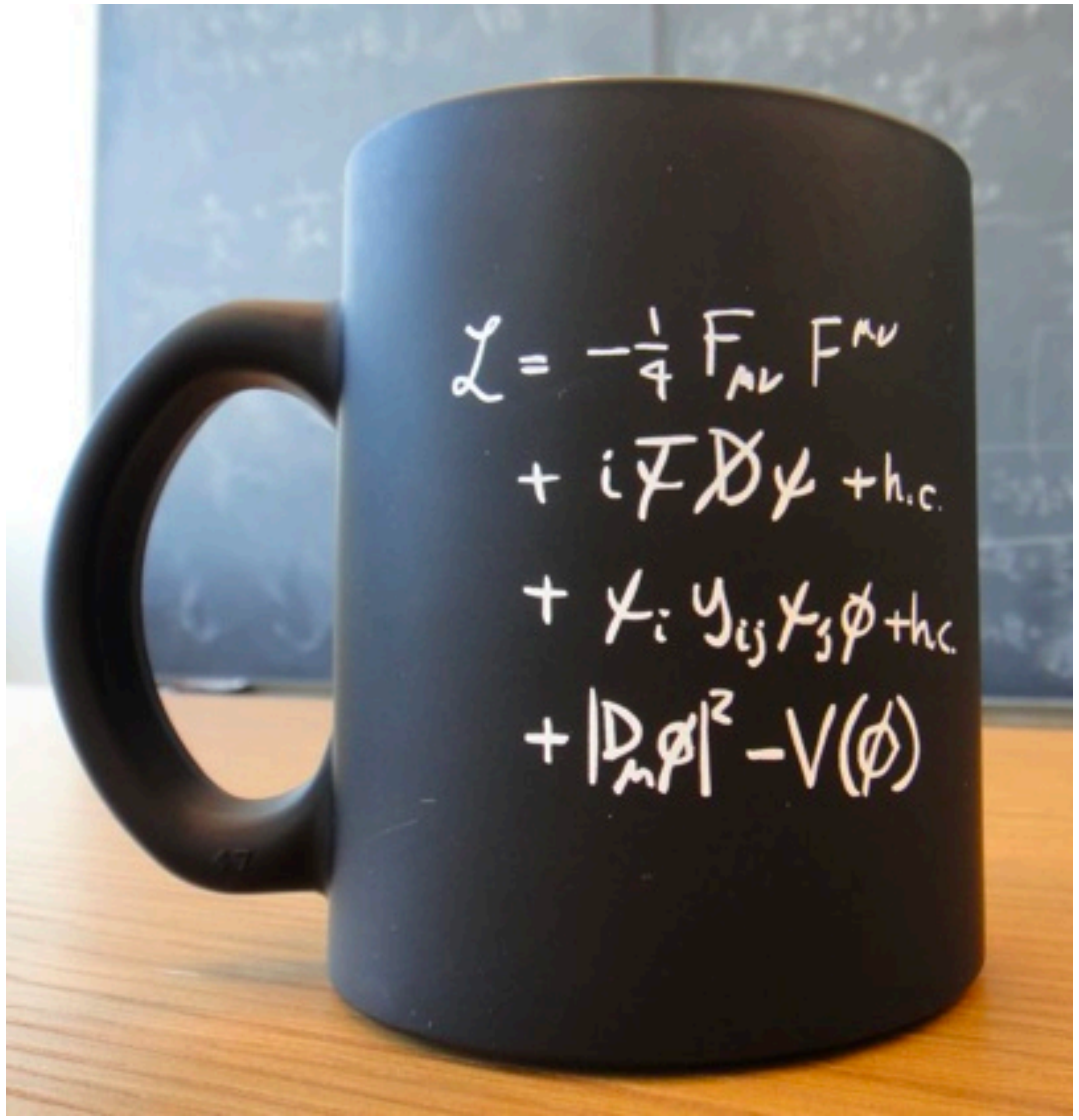

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All possible paths

We can’t do it exactly, we’ve got to rely on **perturbation theory**

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Quantum corrections

Take electron-electron scattering

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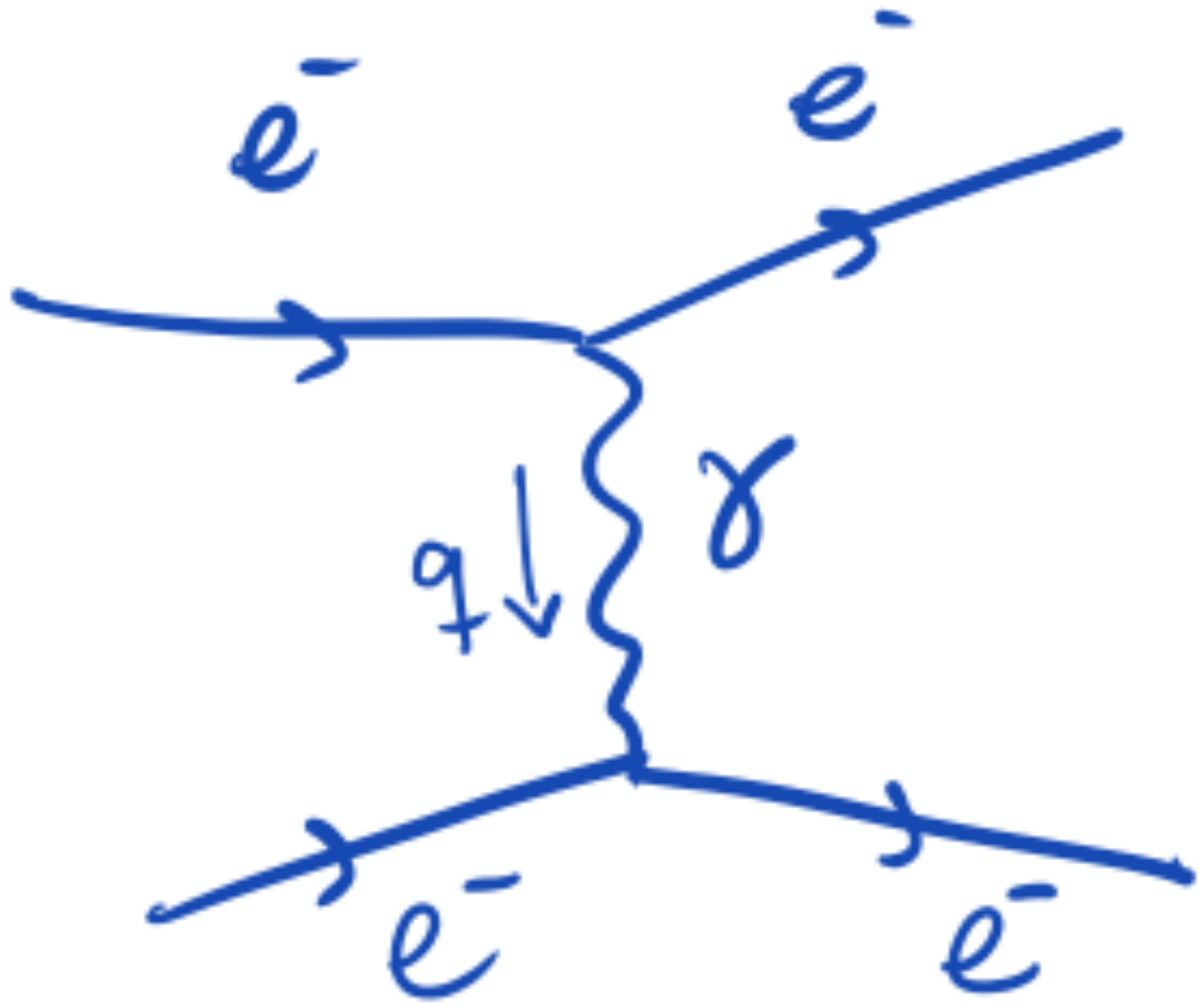
Quantum corrections

First order



$\sim \alpha = \frac{e^2}{q^2}$

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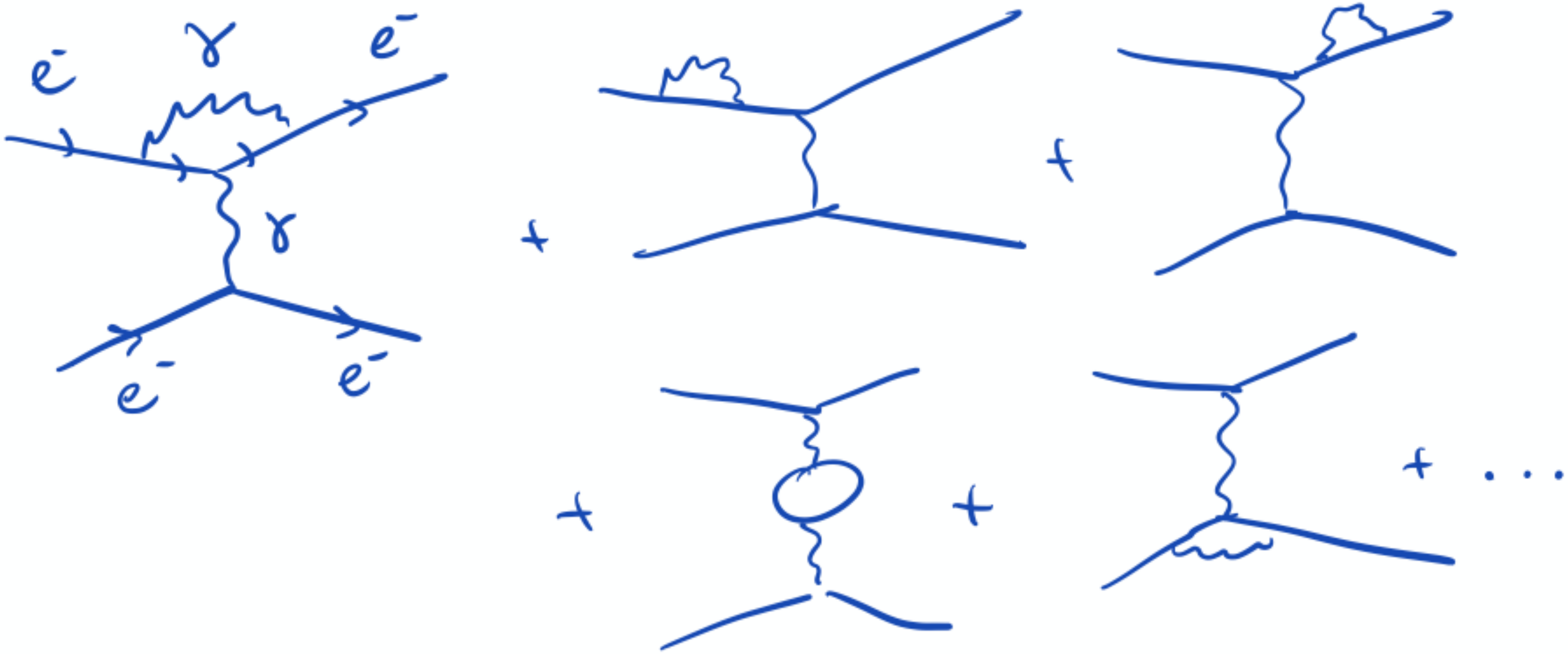
Quantum corrections

Second order



Quantum corrections

Now, it would be great to redefine our Lagrangian in a way that can easily account for quantum corrections

$$A(1\text{st} + 2\text{nd} + \ldots) \sim \frac{e^2}{q^2}$$

We can redefine constants to absorb the higher order effects, but quantum corrections are scale dependent

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Renormalization:

an organization principle to deal with quantum corrections

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Quantum corrections

By measuring observables at several different scales,
we have confirmed the *running* of constants

Here are three examples

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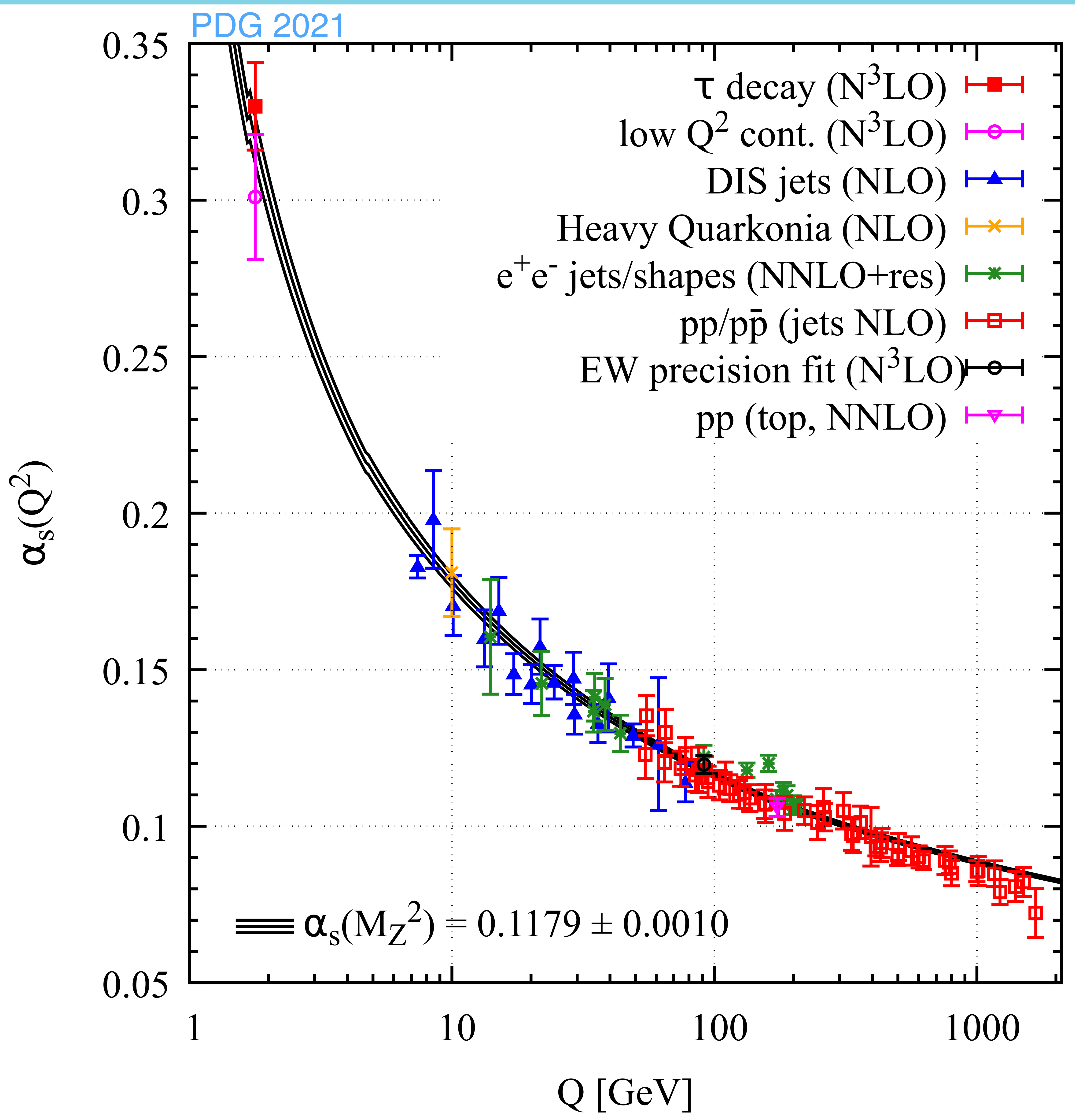
Quantum corrections

Strong interaction coupling

How do we see it?

- QCD production at e^+e^- colliders
- Deep inelastic scattering observables
- QCD jet production at hadron colliders
- …



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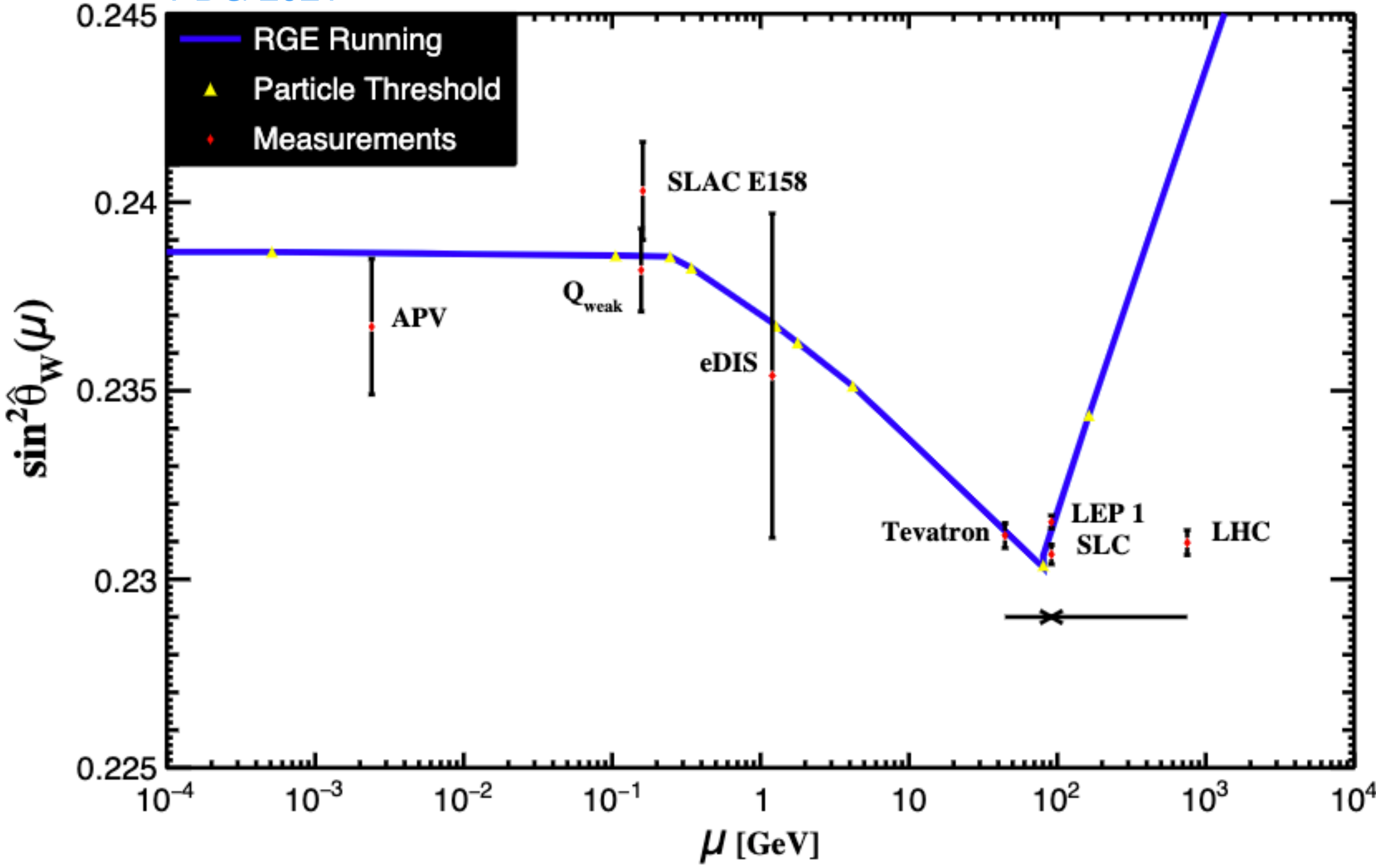
Weak mixing angle

How do we see it?
- Weak interaction cross sections
- Ratio between Z and W boson masses
- Parity violation observables
- …

PDG 2021



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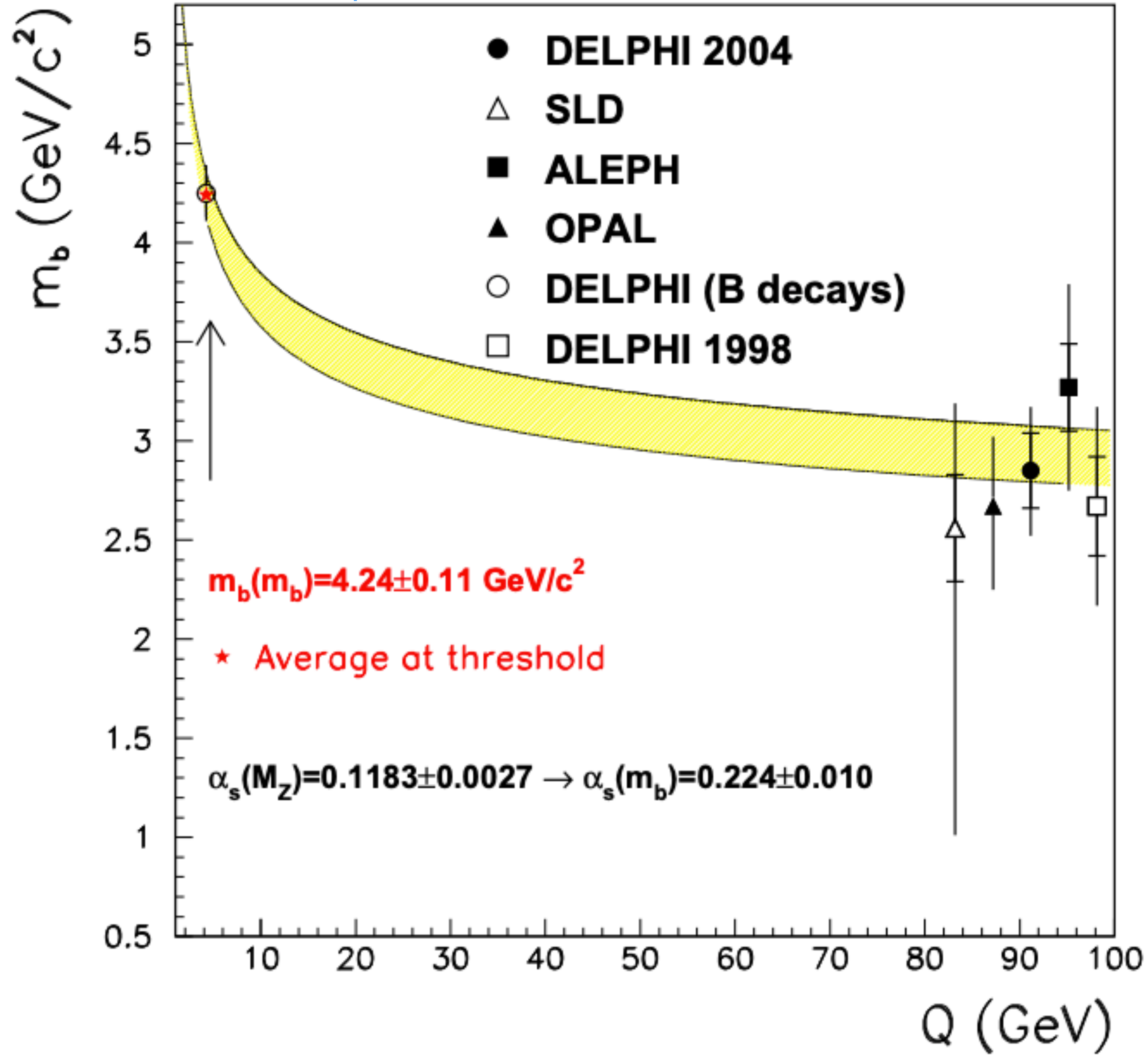
Mass of the b quark

How do we see it?

- b-jet observables near the Z mass scale

DELPHI hep-ex/0603046



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Quantum corrections

"Constants" actually run

Their values depend on the scale at which we are measuring some physical process

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Where do neutrino masses come from?

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Where do neutrino masses come from?

Simply put: we don’t know.

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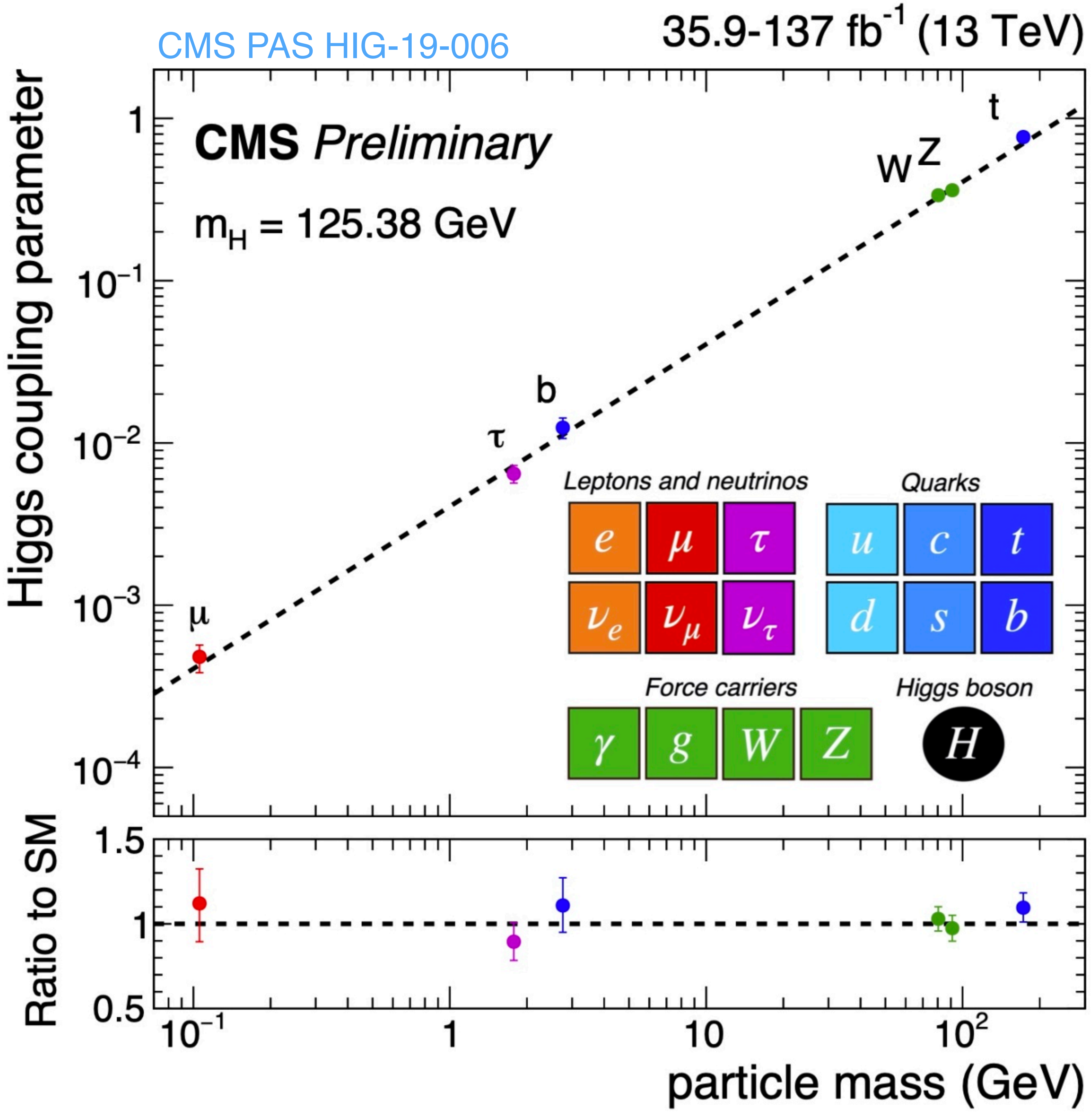
Where do neutrino masses come from?

It seems that the Higgs mechanism gives mass to the gauge bosons and third family charged fermions

Same mechanism can be present for all other charged fermions

But for neutrinos…

CMS PAS HIG-19-006



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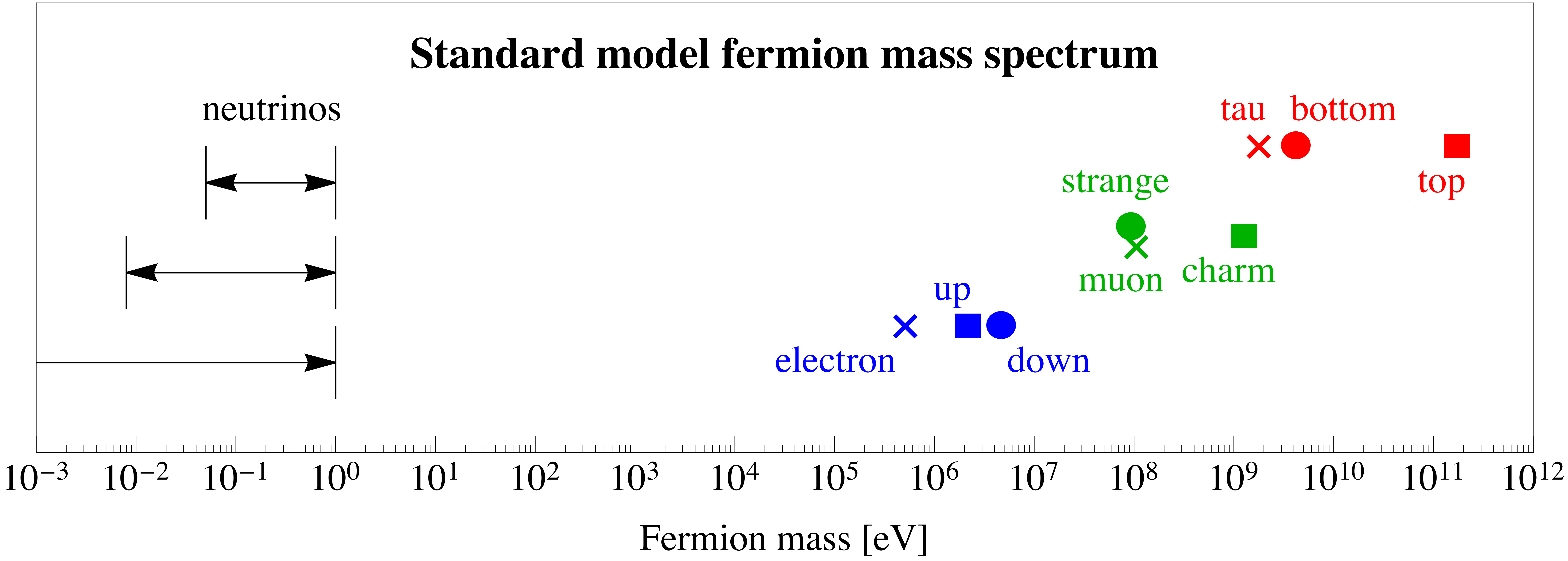

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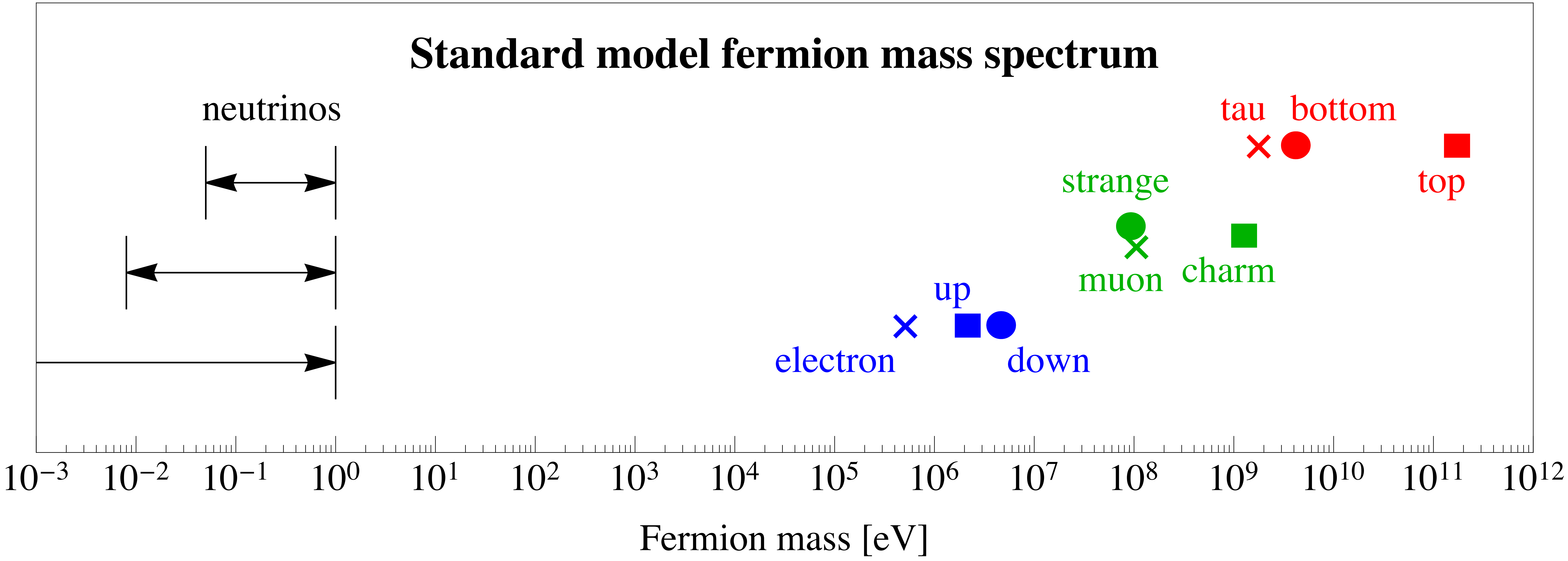

Maybe neutrino masses are small because they are suppressed by a large scale

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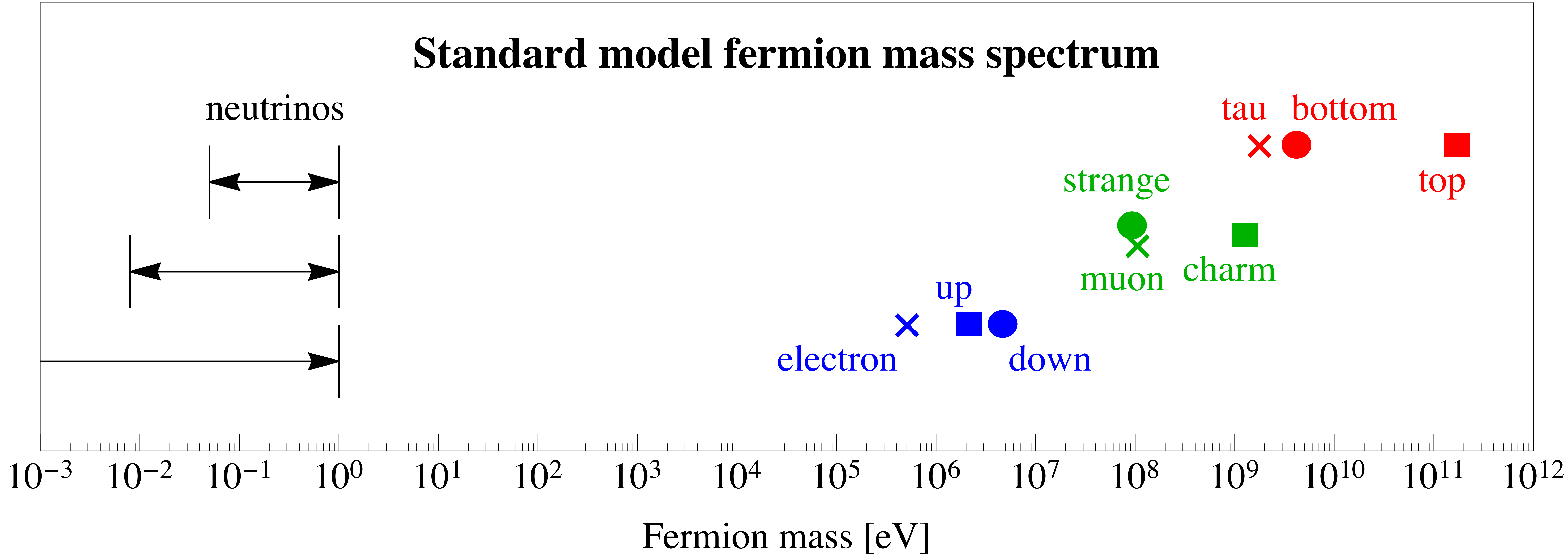

Maybe neutrino masses are small because they are suppressed by a large scale

or maybe neutrino masses are small because the scale of the mass mechanism is low!

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How can quantum corrections and the mechanism of neutrino masses leave an imprint on oscillation phenomenology?

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If the neutrino mass mechanism takes place at low scales, there could be significant running of the PMNS matrix

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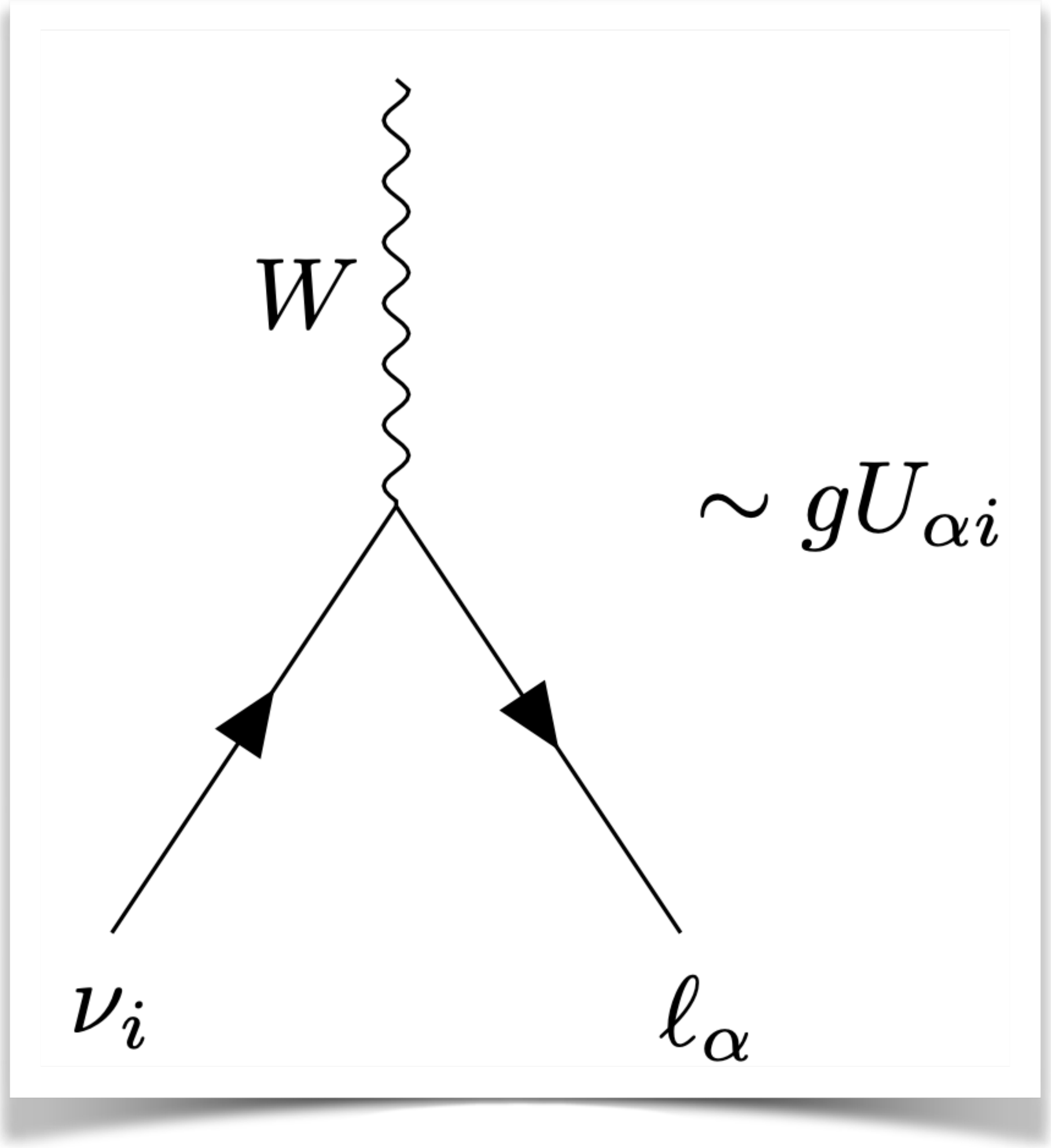
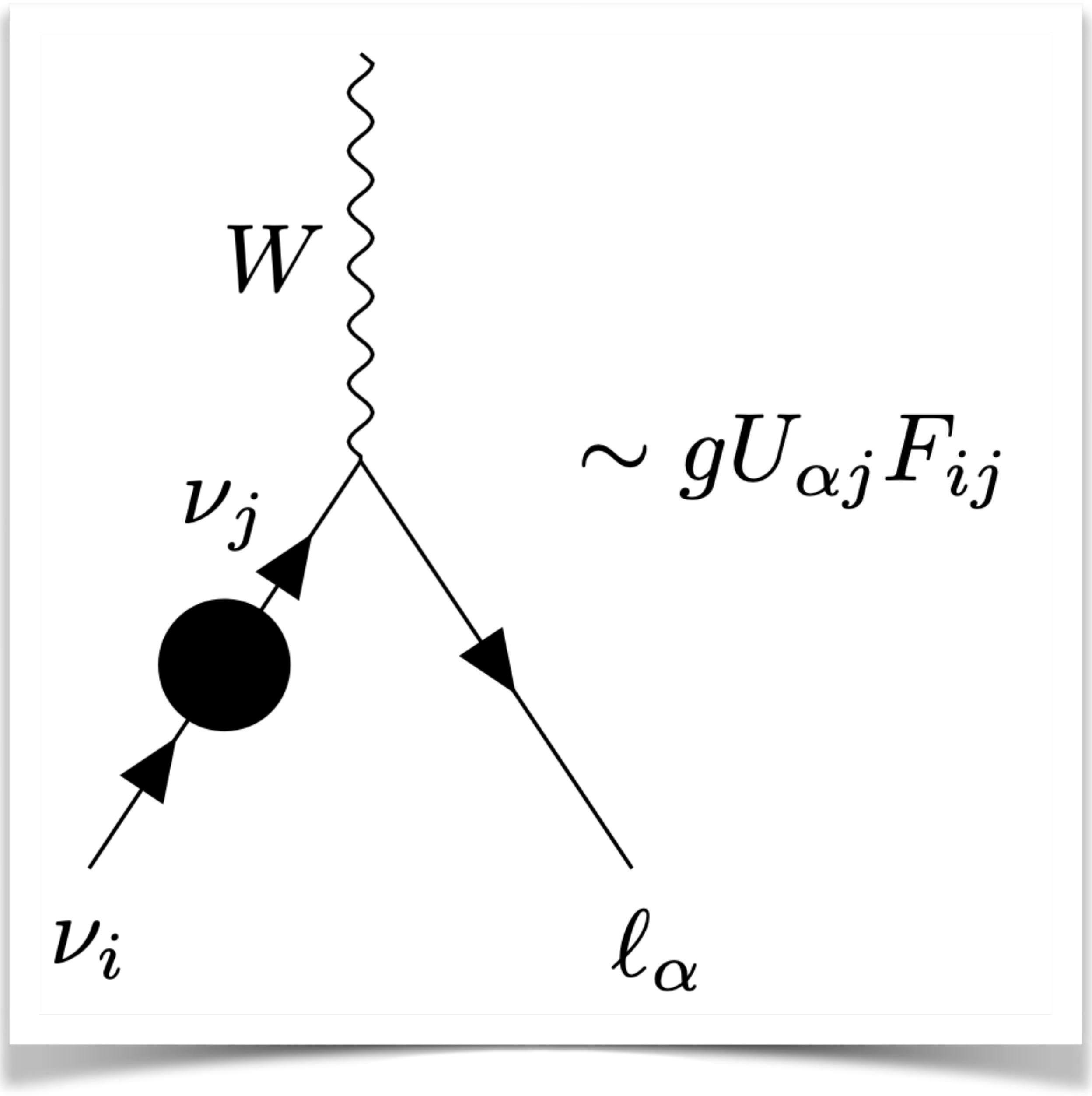
Energy dependent neutrino mixing





If there are significant quantum corrections to the neutrino mass matrix at low scales, the PMNS matrix becomes scale dependent.

This means that production and detection of neutrinos may not go via the same PMNS matrices.

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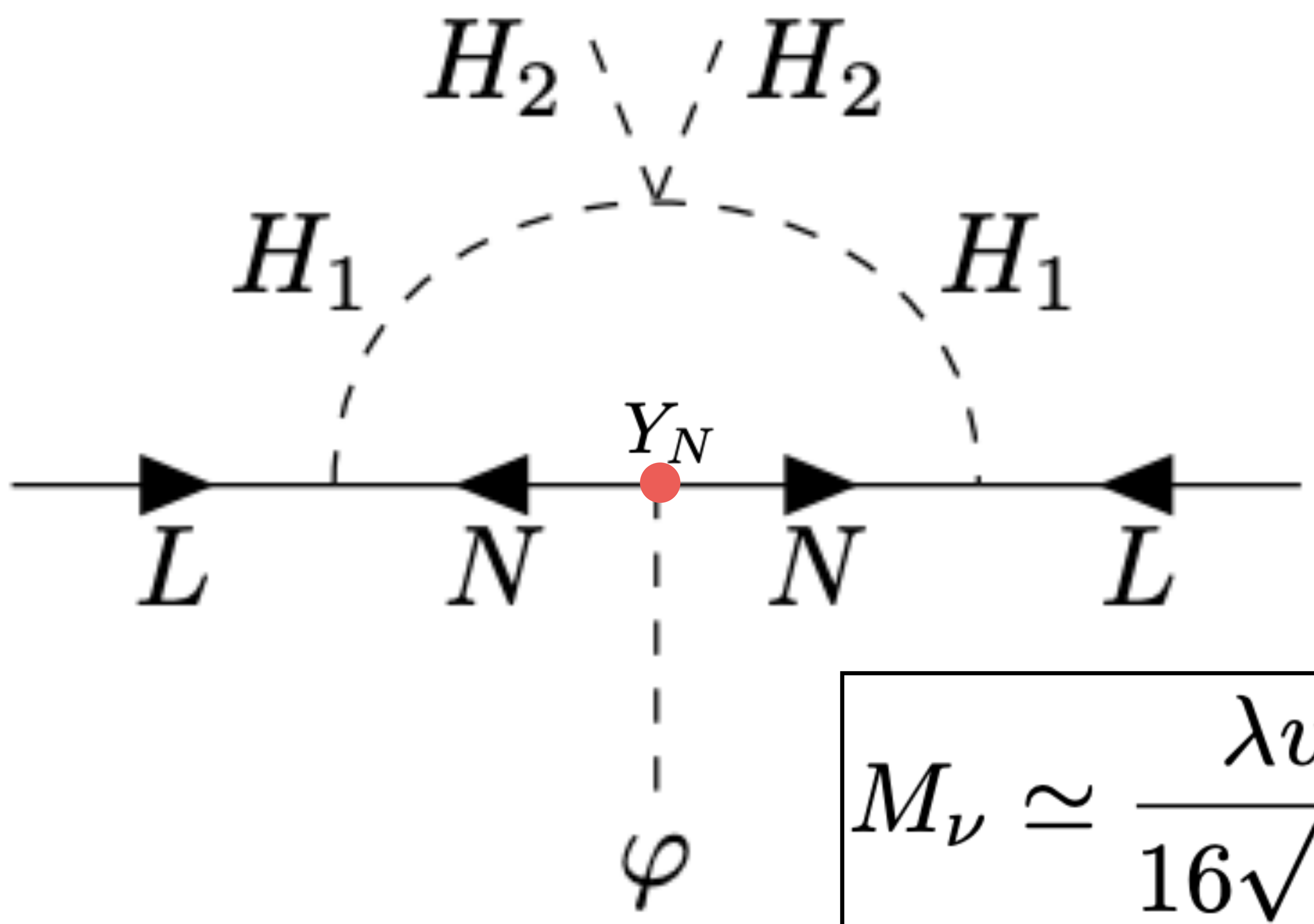
Energy dependent neutrino mixing

Two simple examples of neutrino mass mechanisms that can lead to significant running of the PMNS matrix



$$M_\nu \simeq \frac{\lambda v_\varphi}{16\sqrt{2}\pi^2} Y_\nu Y_N Y_\nu^T \ln \frac{M_H^2}{M_A^2}$$

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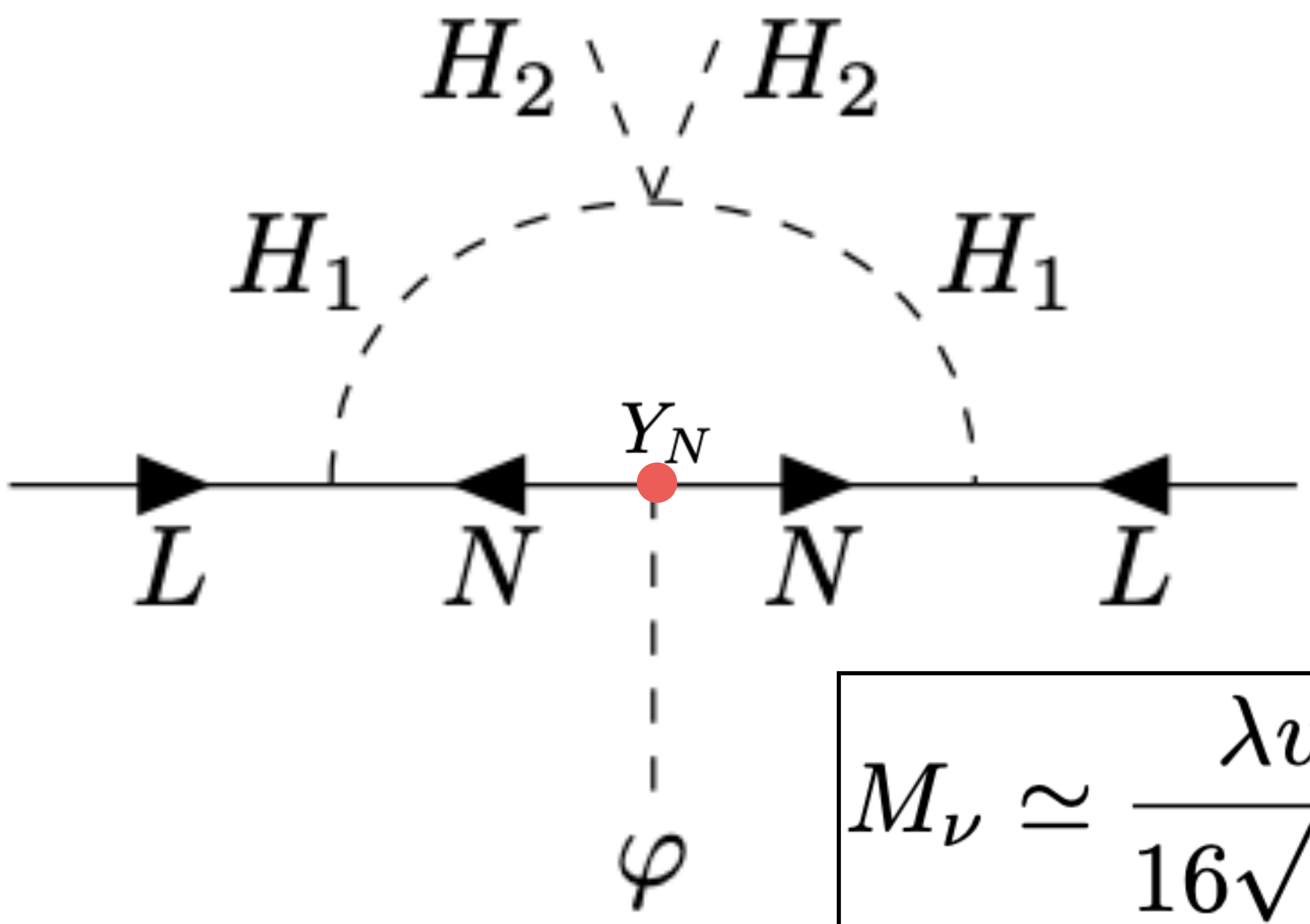

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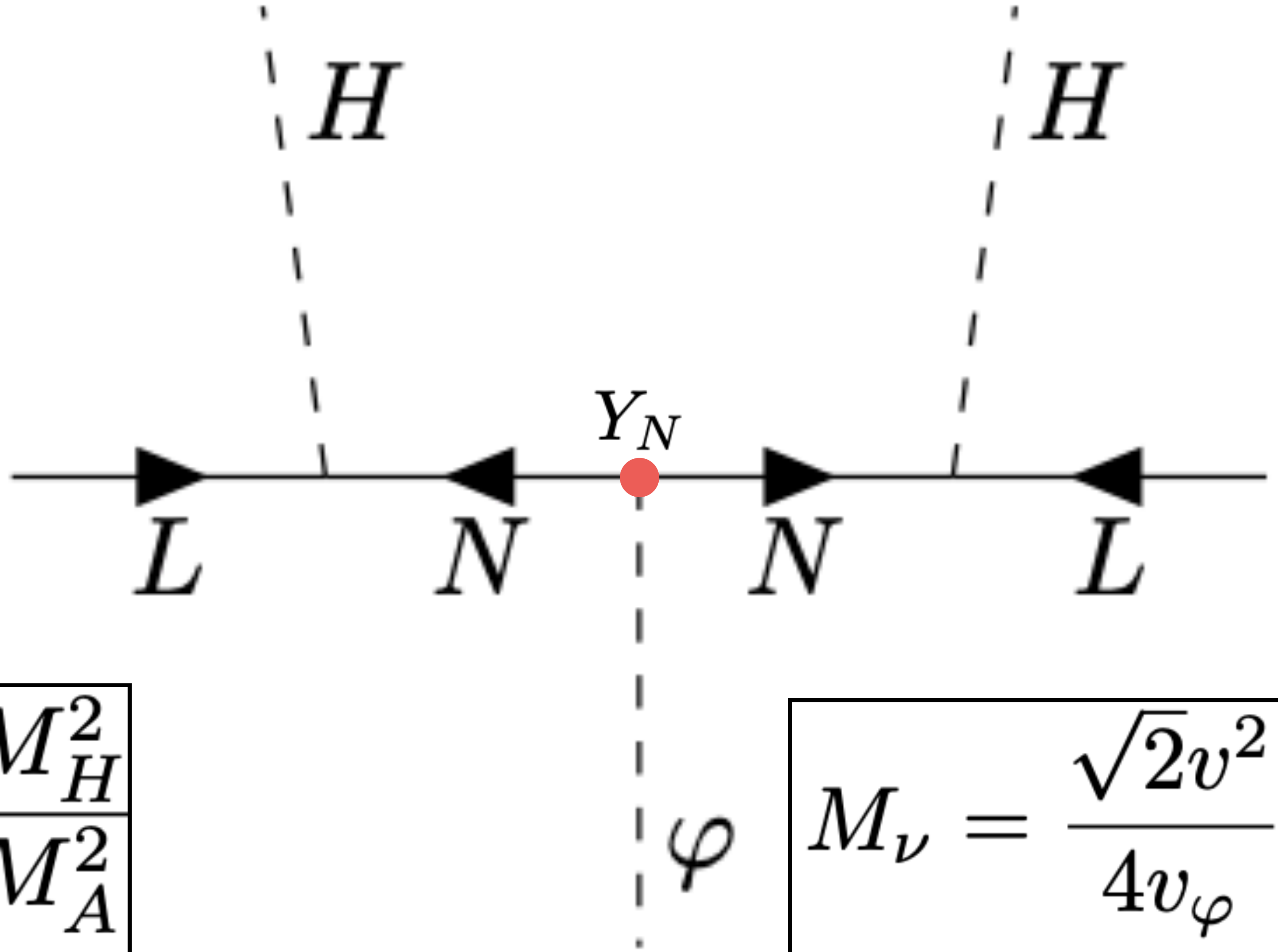

$$M_\nu = \frac{\sqrt{2}v^2}{4v_\varphi} Y_\nu (Y_N)^{-1} Y_\nu^T$$

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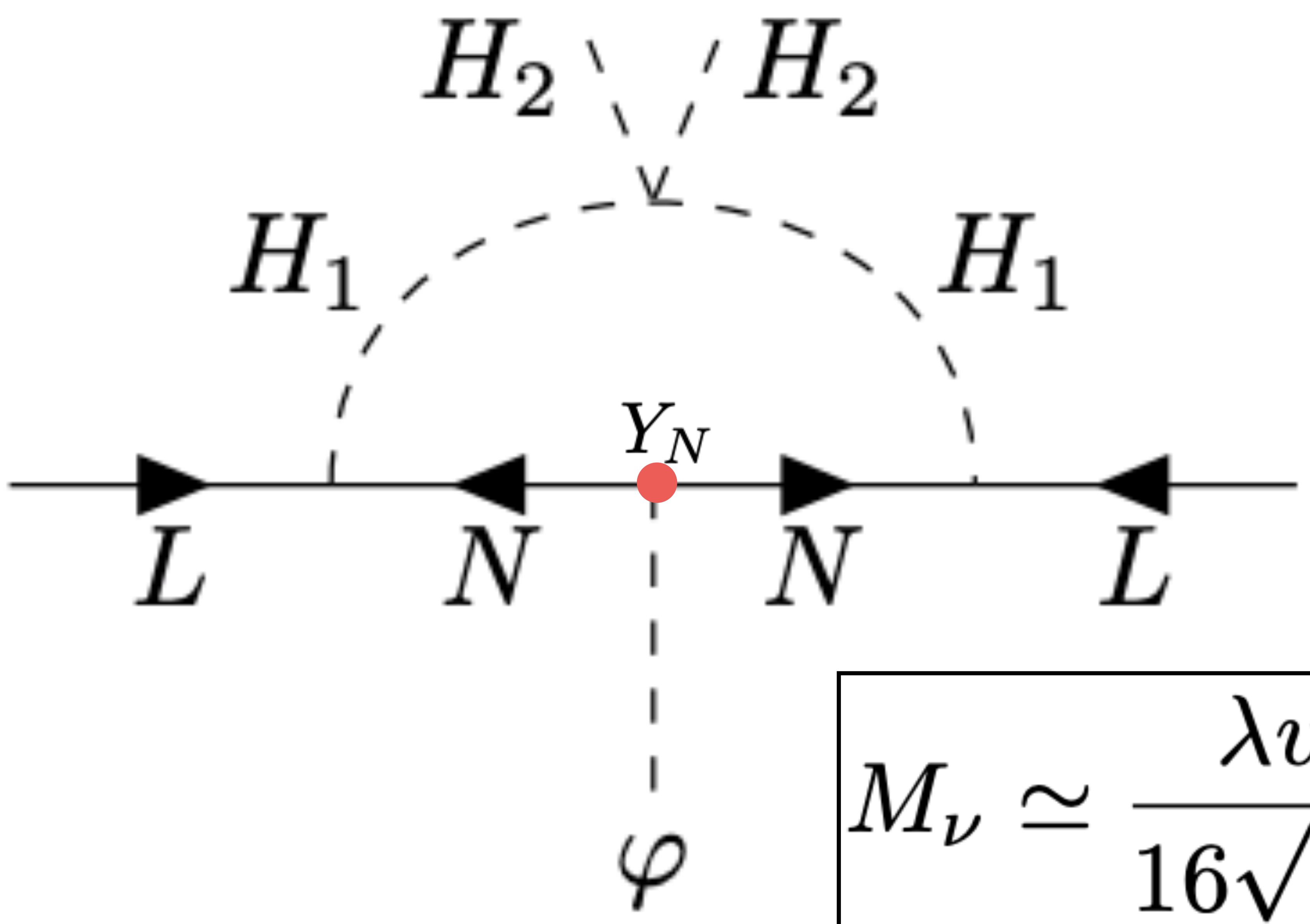

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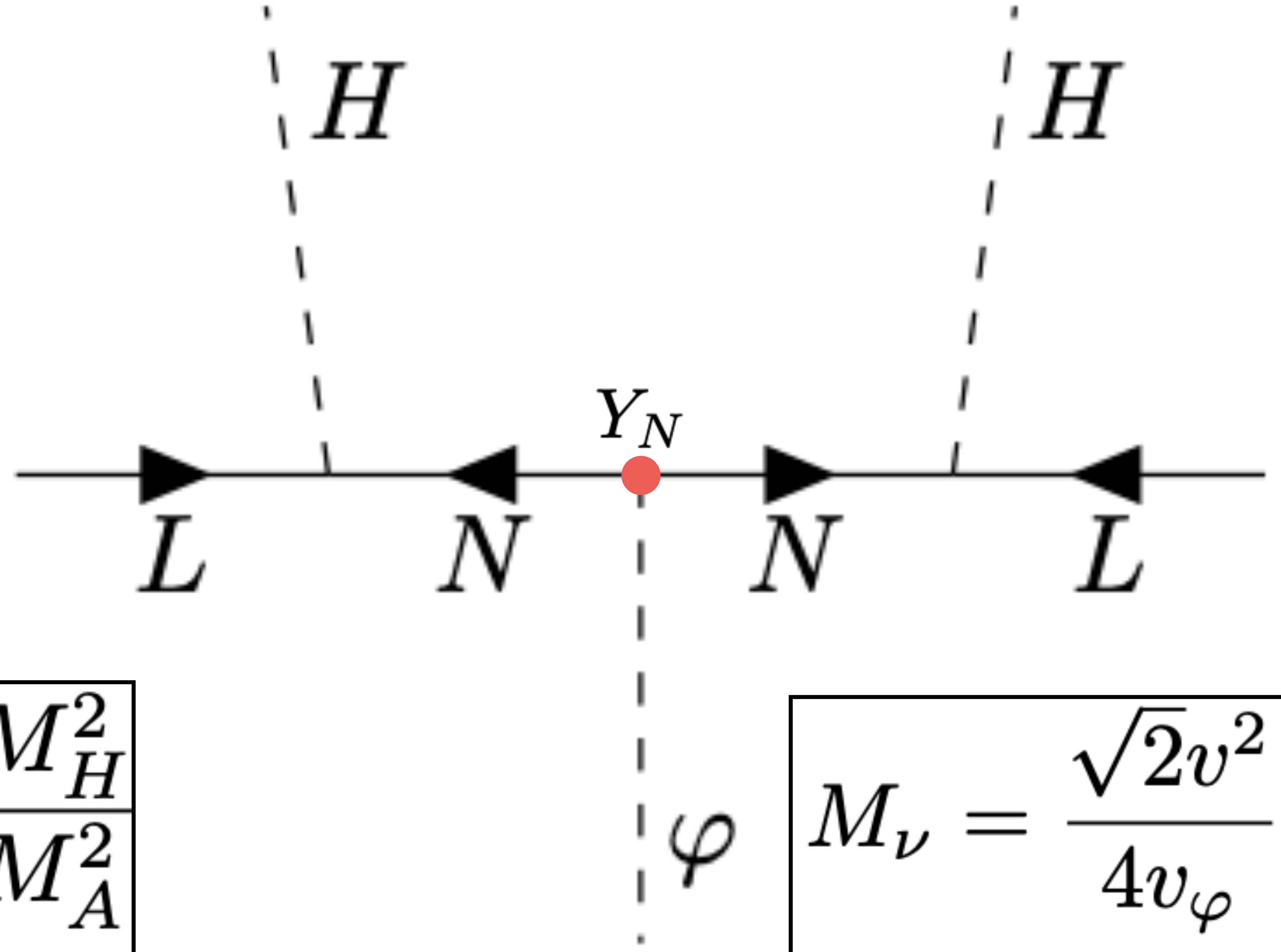

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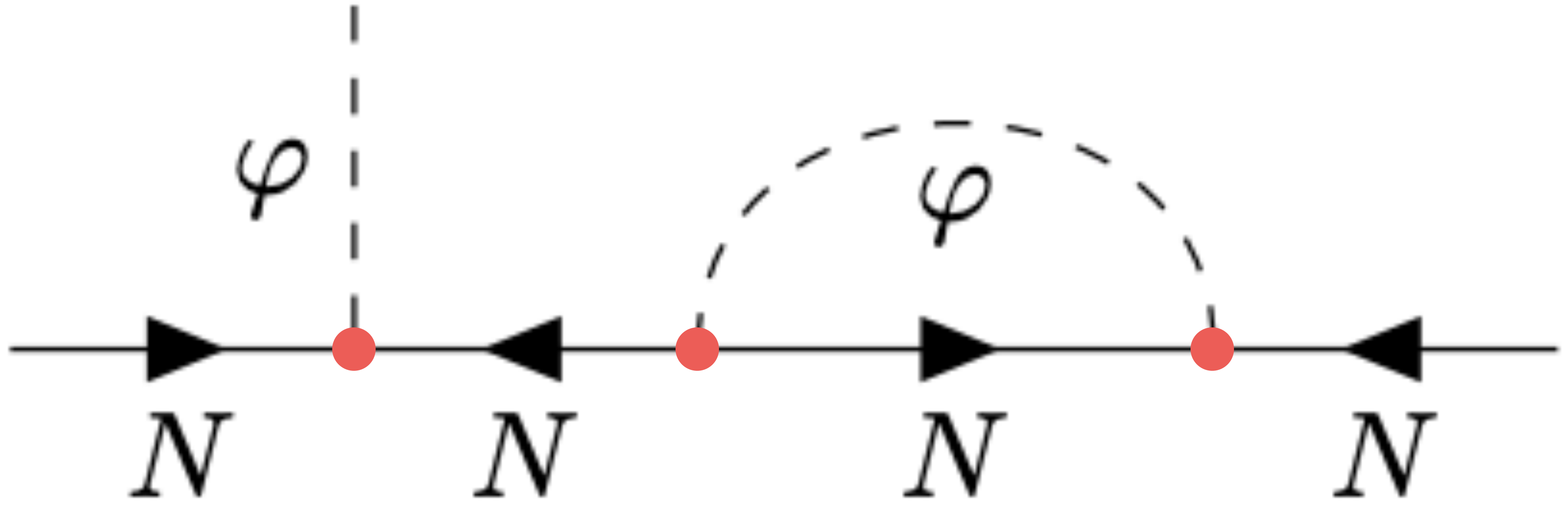
Diagram contributing to the running



$$16\pi^2 \beta(Y_N) \equiv 16\pi^2 \frac{dY_N}{d\ln|Q|} = 4Y_N \left[Y_N^2 + \frac{1}{2}\mathrm{Tr}(Y_N^2)\right]$$

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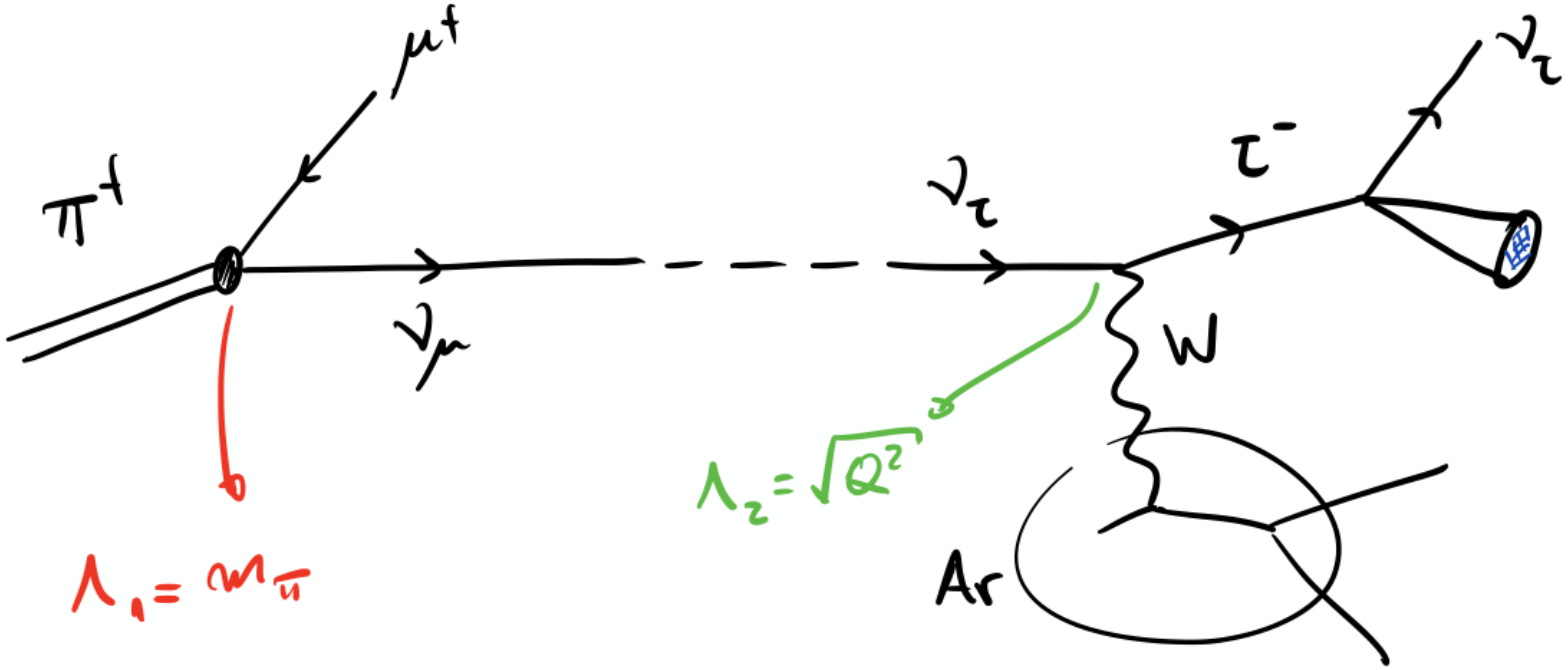
Energy dependent neutrino mixing



Standard case

$$A(\nu_\mu \to \nu_\tau) = \langle \nu_\tau | \exp(-iHL) | \nu_\mu \rangle$$
$$= \sum_i U_{\tau i} U^*_{\mu i} \exp\left(-\frac{i m_i^2 L}{2E}\right)$$

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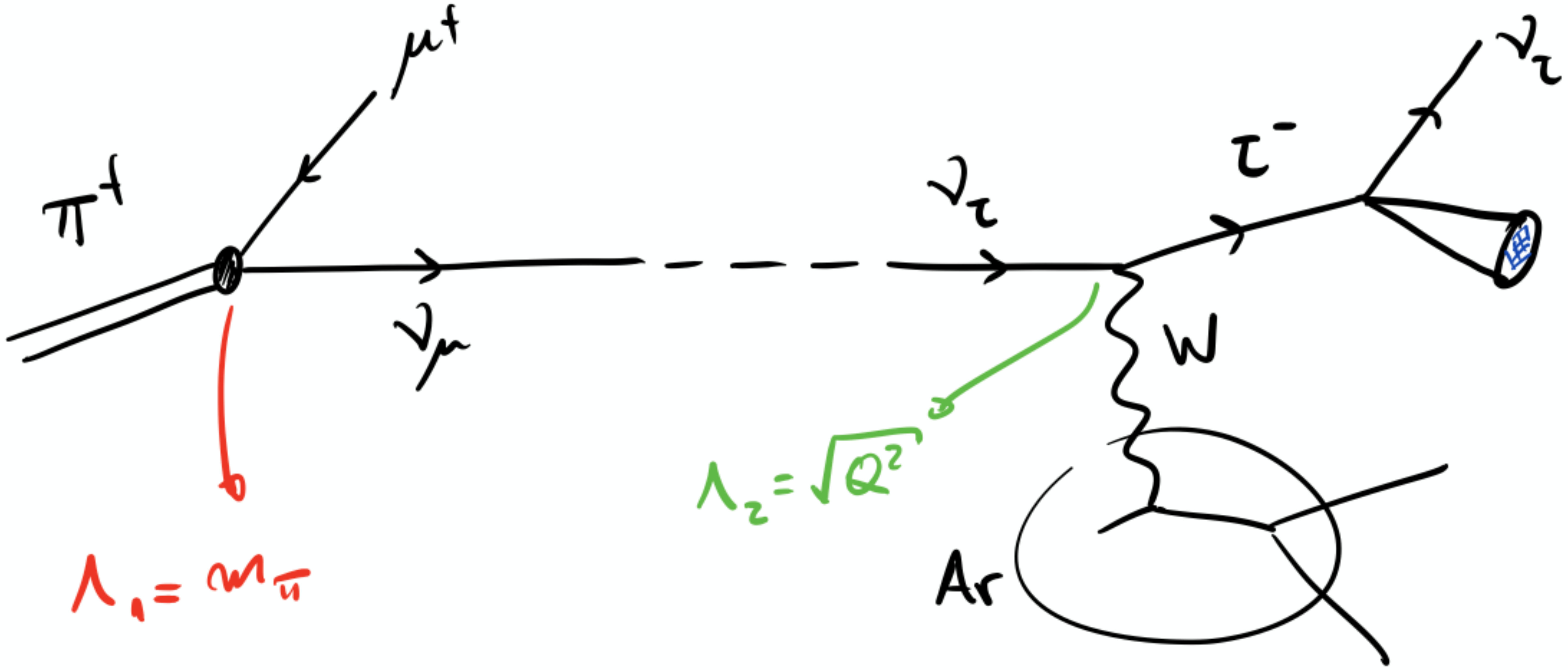

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$$\nu_\alpha(Q^2) = U_{\alpha i}(Q^2)\nu_i$$

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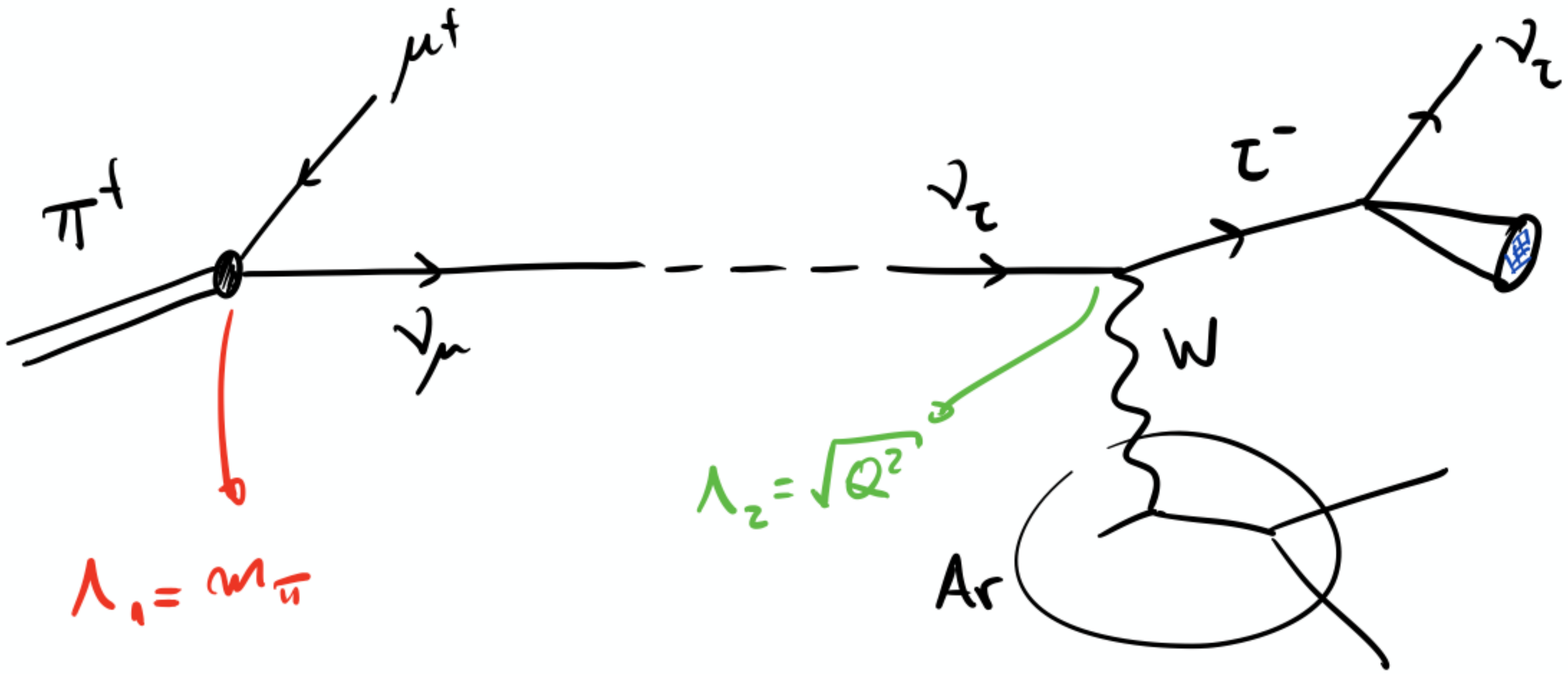

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Energy dependent neutrino mixing

What are the effects we would be looking for?

I will use two flavor oscillations to show simplified formulae

$$P_{e\mu} = P_{\mu e} = \sin^2(\theta_p - \theta_d) + \sin 2\theta_p \sin 2\theta_d \sin^2\left(\frac{\Delta m^2 L}{4E} + \frac{\beta}{2}\right)$$

$$U(Q^2) = \begin{pmatrix} \cos\theta(Q^2) & \sin\theta(Q^2) \\ -\sin\theta(Q^2) & \cos\theta(Q^2) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\tilde{\beta}(Q^2)} \end{pmatrix}$$

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production detection

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production detection

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production (p) detection (d)

1) Zero distance transitions, though it is 2nd order
2) Finite L: $P_{\alpha\beta} \neq P_{\bar{\alpha}\bar{\beta}}$, as the phase flips sign (*apparent* CPT violation)
3) Distortions of oscillation probability, since $\theta_{p,d}$ depend on energy

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production detection

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3) Distortions of oscillation probability, since $\theta_{p,d}$ depend on energy
4) New sources of CP violation

$$P_{\mu e} - P_{\bar{\mu}\bar{e}} \simeq -8J\Delta_{21}\sin^2\left(\frac{\Delta_{31}}{2}\right)\left[1 + \left(2\frac{\epsilon_{12}}{\sin 2\theta_{12}} + \epsilon_\alpha \frac{c_\delta}{s_\delta}\right)\frac{\cot(\Delta_{31}/2)}{\Delta_{21}}\right]$$

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Energy dependent neutrino mixing

What are the effects we would be looking for?

Short baseline constraints:

Experiment	E (GeV)	$\sqrt{Q_d^2}$ (GeV)	channel	constraint
ICARUS [64]	17	3.94	$\nu_\mu \to \nu_e$	3.4×10^{-3}
CHARM-II [65]	24	4.70	$\nu_\mu \to \nu_e$	2.8×10^{-3}
NOMAD [61–63]	47.5	6.64	$\nu_\mu \to \nu_e$	7.4×10^{-3}
			$\nu_\mu \to \nu_\tau$	1.63×10^{-4}
NuTeV [66, 67]	250	15.30	$\nu_\mu \to \nu_e$	5.5×10^{-4}
			$\nu_e \to \nu_\tau$	0.1
			$\nu_\mu \to \nu_\tau$	9×10^{-3}

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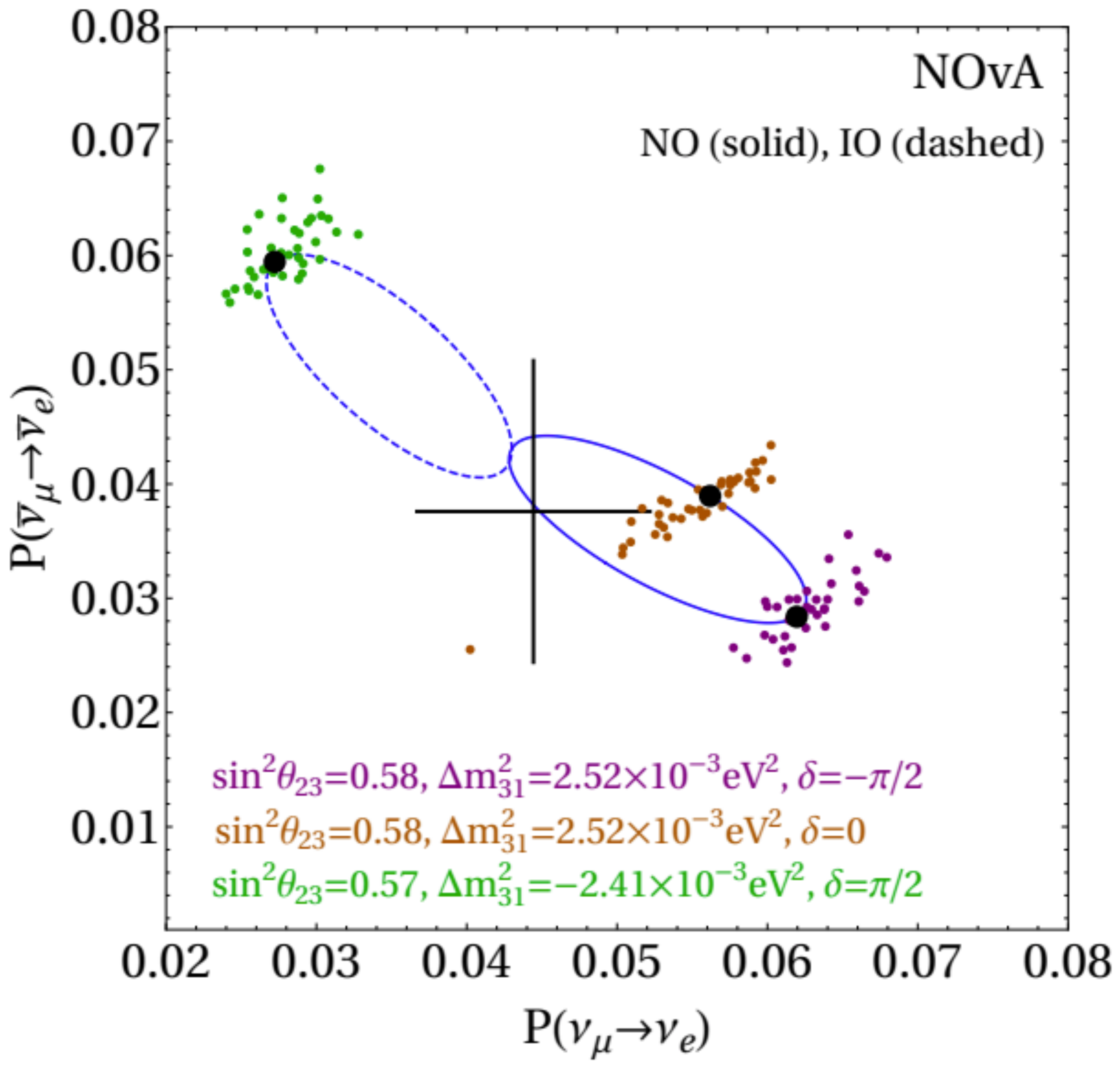

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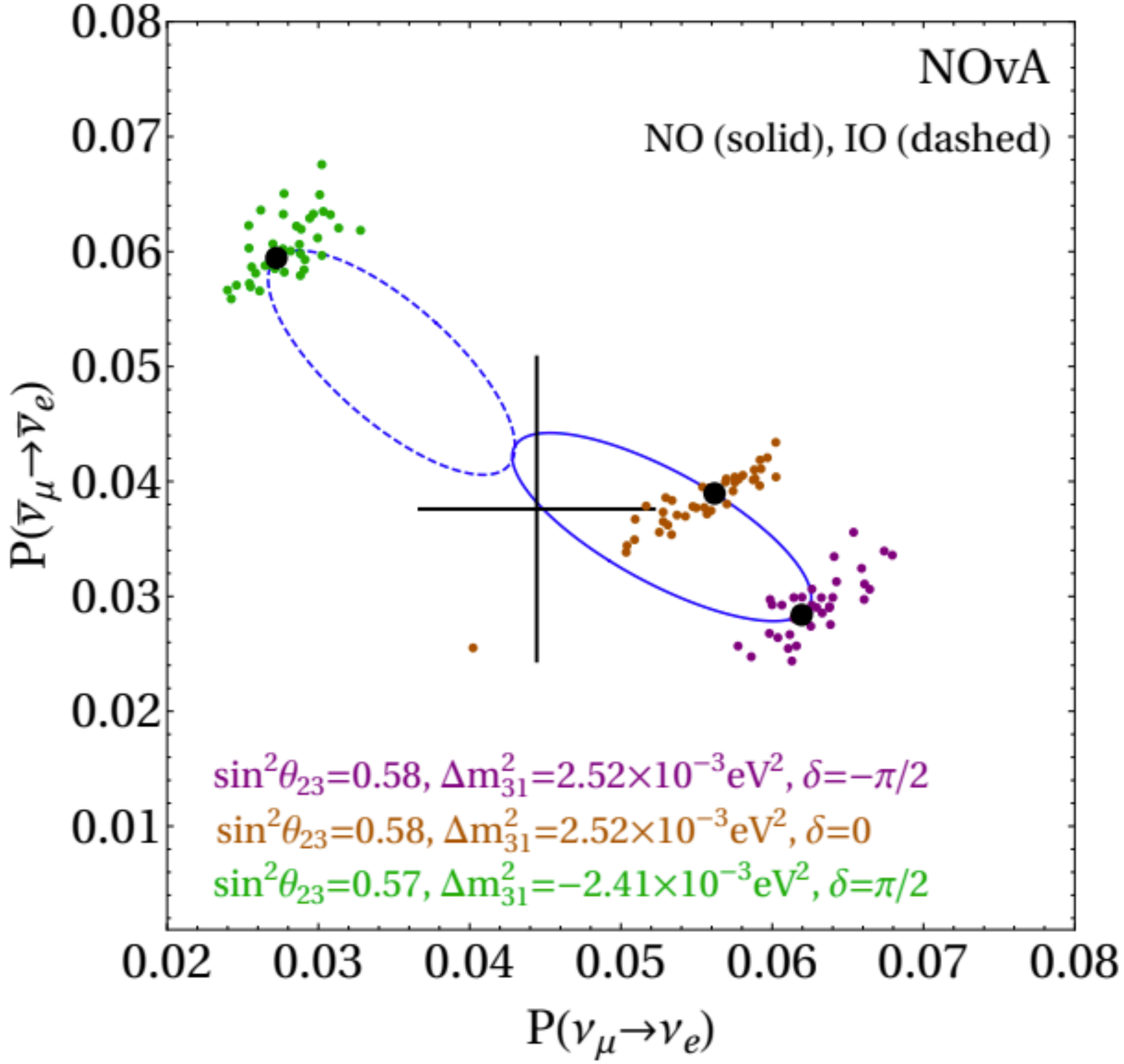

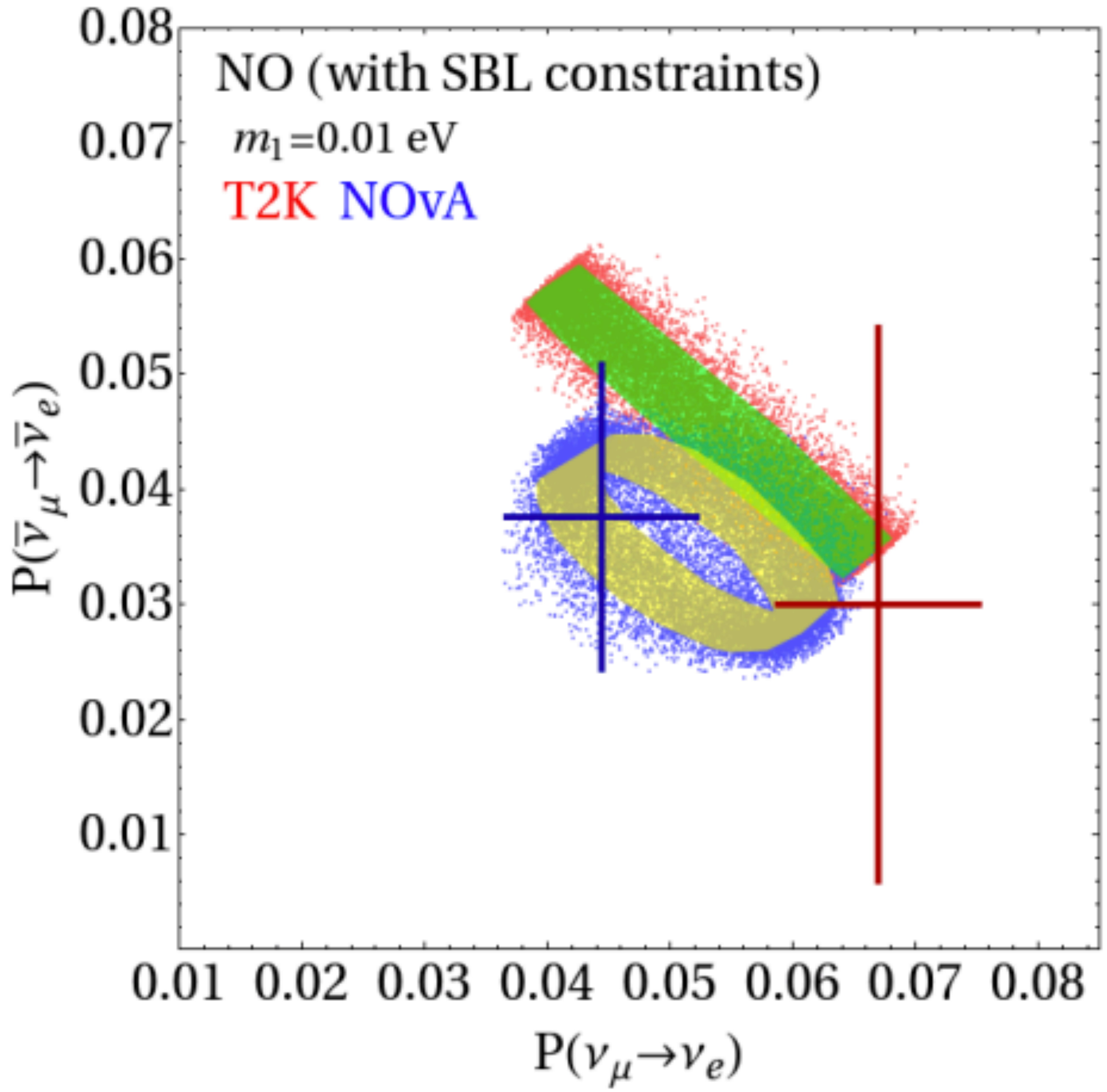

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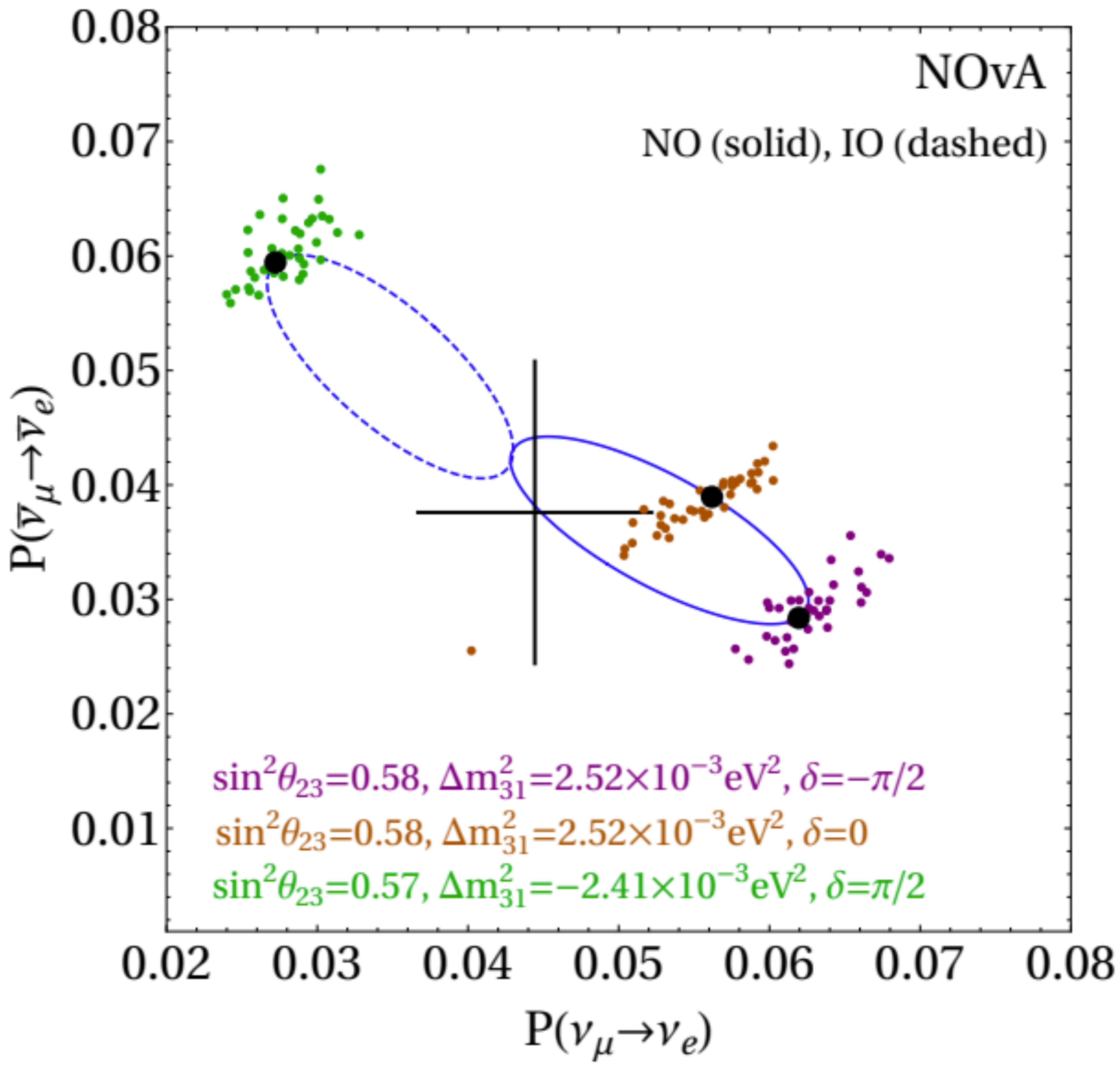

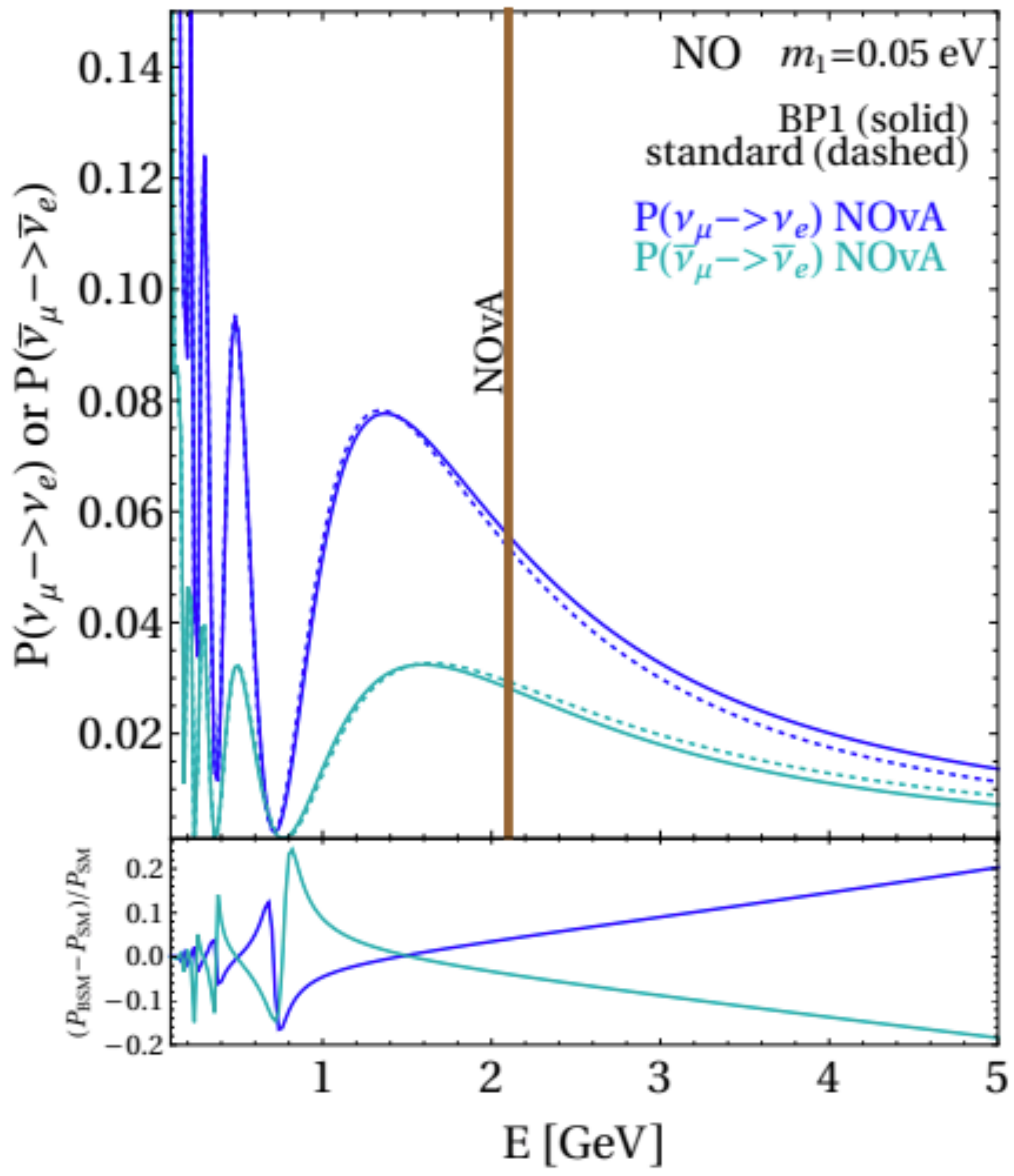

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Energy dependent neutrino mixing

Cosmogenic neutrinos: flavor composition

$$P_{\nu_\alpha\to\nu_\beta} = P_{\nu_\beta\to\nu_\alpha} = \delta_{\alpha\beta} - 2\sum_{k>j}\mathrm{Re}\left[U^*_{\alpha k}U_{\beta k}U_{\alpha j}U^*_{\beta j}\right] = \sum_{j=1}^{n}\left|U_{\alpha j}\right|^2\left|U_{\beta j}\right|^2$$

These neutrinos come from so far that they decohere

Even if we do not know the flavor composition at the source, the possible flavor composition at detection is constrained and is related to the mixing matrix

$$P_{\nu_\alpha\to\nu_\beta} = \sum_{j=1}^{3}\left|U_{\alpha j}(Q_p^2)\right|^2\left|U_{\beta j}(Q_d^2)\right|^2$$

$$X_\beta = \sum_\alpha P_{\nu_\alpha\to\nu_\beta}X_\alpha^{\mathrm{prod}}$$

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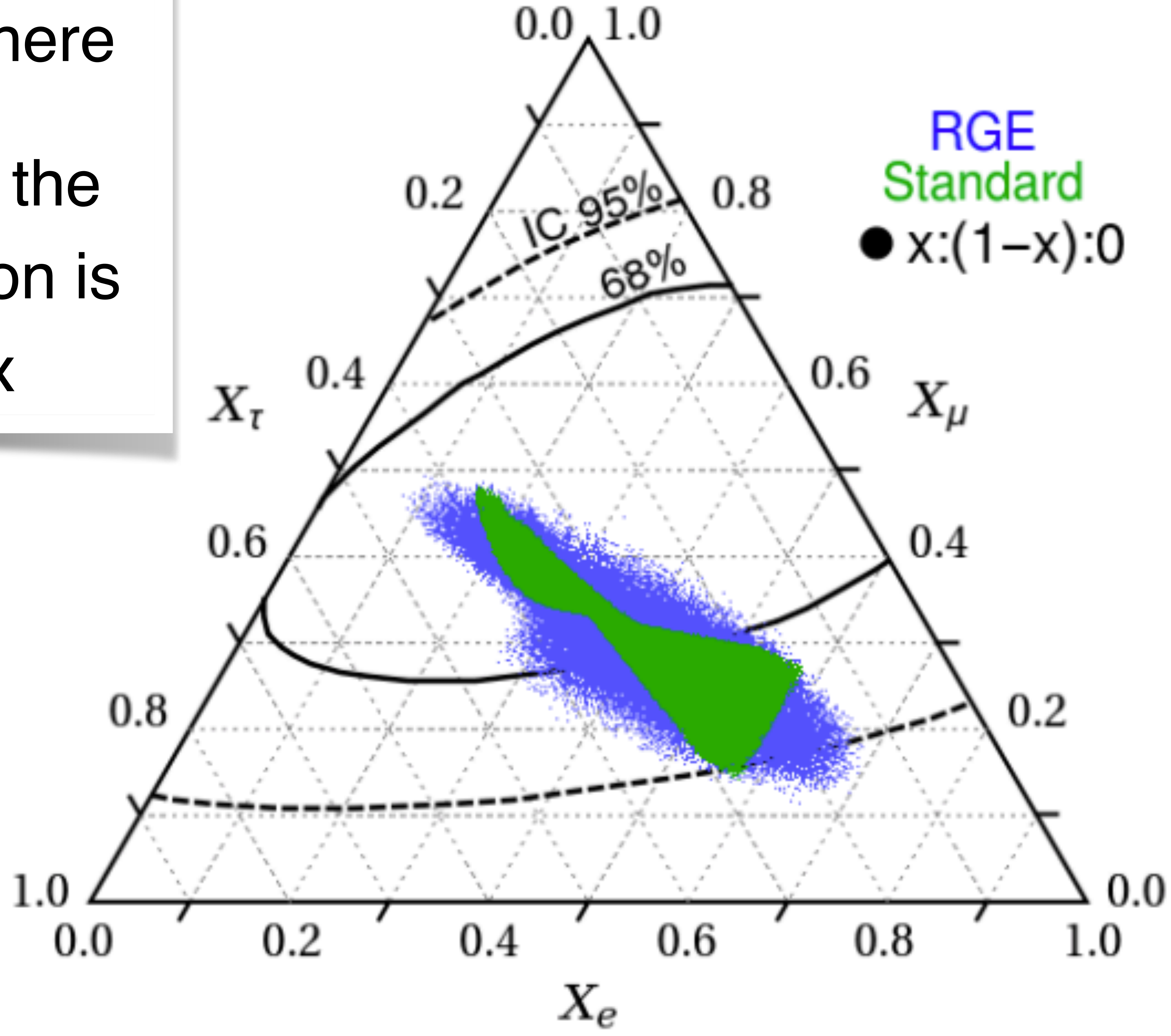

Can a low scale mass model
alleviate the sterile neutrino tension?

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Energy dependent neutrino mixing

An general framework for light steriles

$$\mathcal{L} \supset \frac{C_\alpha}{\Lambda} \overline{L}_\alpha H S N + M N' N + \text{h.c.}$$

$$M_\nu = \begin{pmatrix} \times & \times & \times & \mu_e & 0 \\ \times & \times & \times & \mu_\mu & 0 \\ \times & \times & \times & \mu_\tau & 0 \\ \mu_e & \mu_\mu & \mu_\tau & 0 & M \\ 0 & 0 & 0 & M & 0 \end{pmatrix}$$

leads to active-sterile mixing.

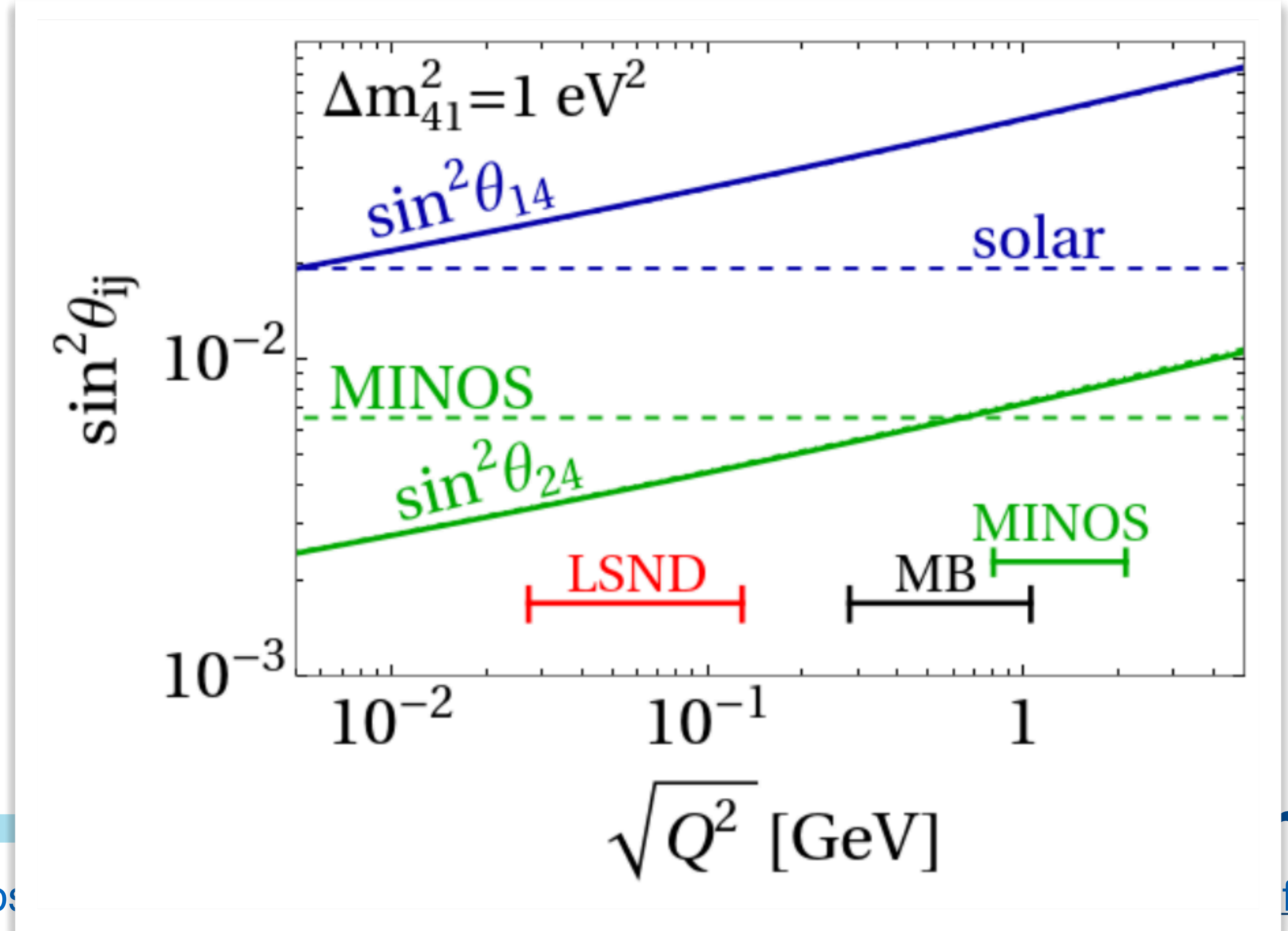
$$\tan\theta_{14} \simeq \frac{\mu_e}{M}, \qquad \tan\theta_{24} \simeq \frac{\mu_\mu}{M}$$

A new interaction in sterile sector

$$\mathcal{L} \supset g' \overline{N} \not{Z}' N - g' \overline{N}' \not{Z}' N'$$

leads to running of sterile masses

$$M(\mu) = M(\mu_0) \left(1 - \frac{5 g'(\mu_0)^2}{24\pi^2} \ln\left(\frac{\mu}{\mu_0}\right)\right)^{9/4}$$



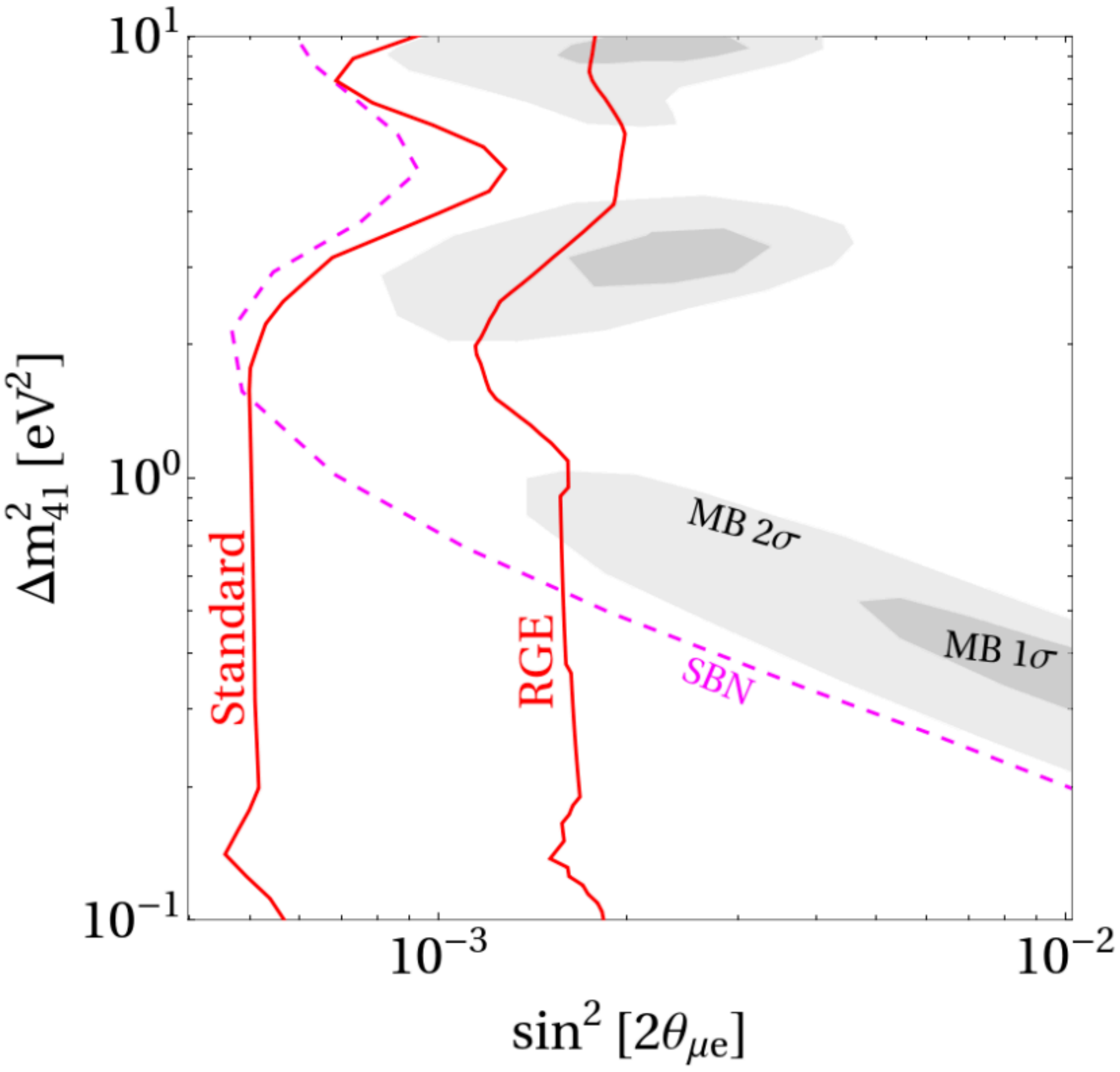
Energy dependent neutrino mixing

An general framework for light steriles can enhance θ_{14} at MiniBooNE scales compared to solar neutrino scales (which provide the dominant constraints).

SBN lives at essentially the same scales of MiniBooNE.

Impact on LSND is marginal.

IceCube is a low scale experiment in this context…



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Conclusions

If the neutrino mass model takes place at low scales,
it can induce quantum corrections that affect neutrino oscillations

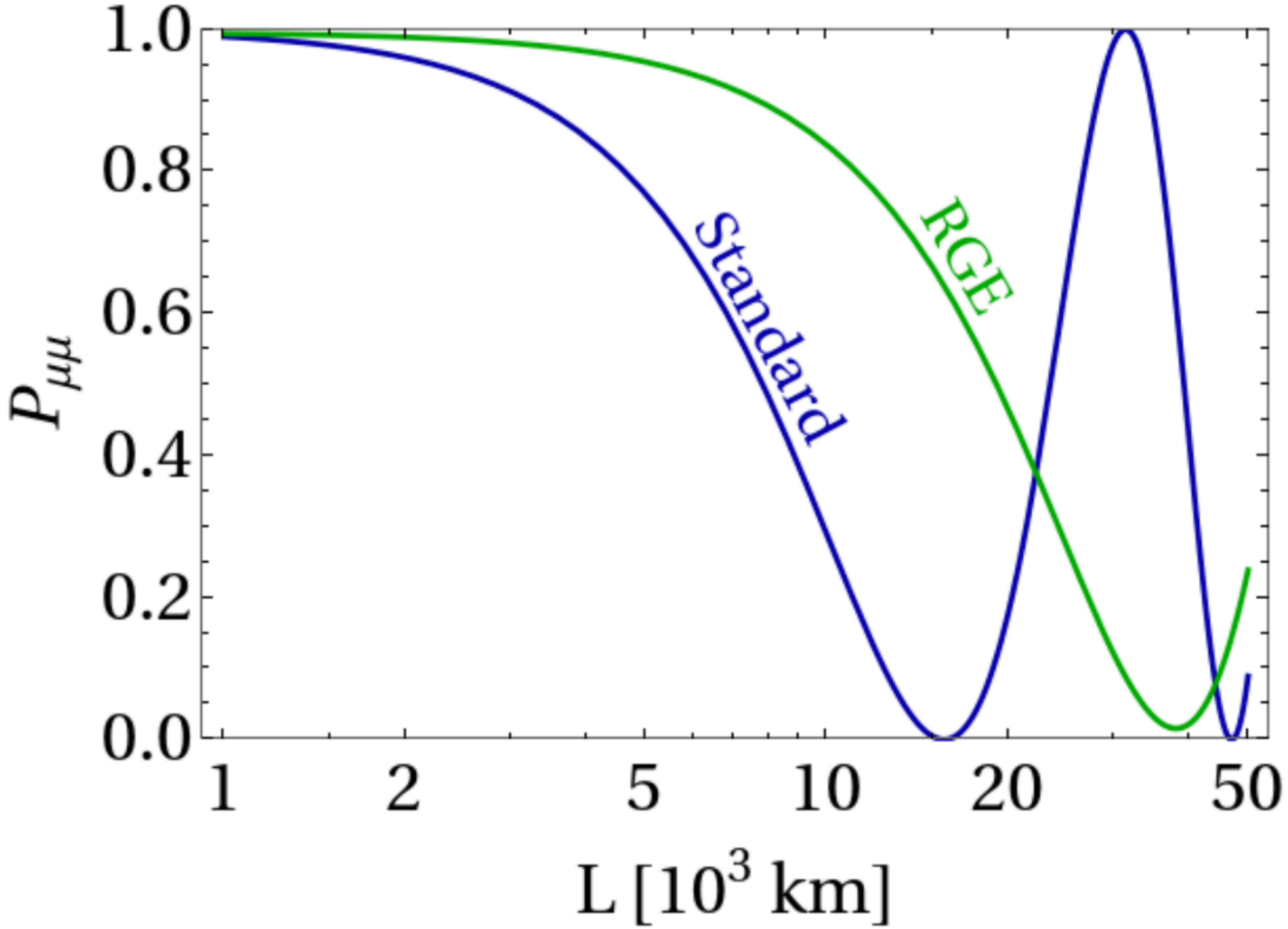
This boils down to producing and detecting neutrinos via different mixing matrices

Several effects are present: zero baseline transitions, apparent CPT violation, enhanced CP violation, overall distortions on the oscillation probabilities, changes on flavor composition of cosmogenic neutrinos

DUNE and IceCube-gen2 are in a very special position to probe this framework

Running can alleviate the tension in the sterile-nu interpretation of the MB anomaly

Backup



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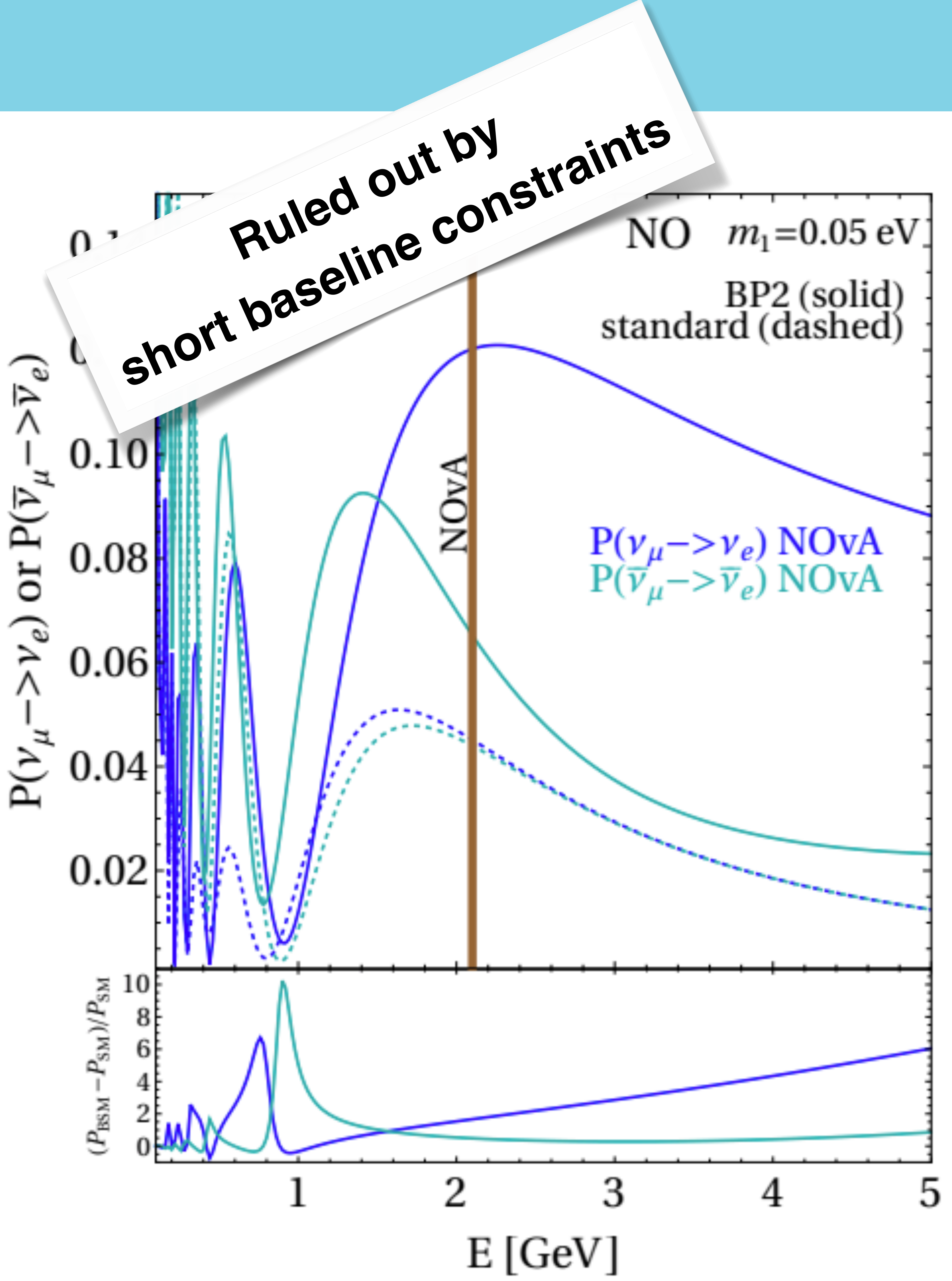
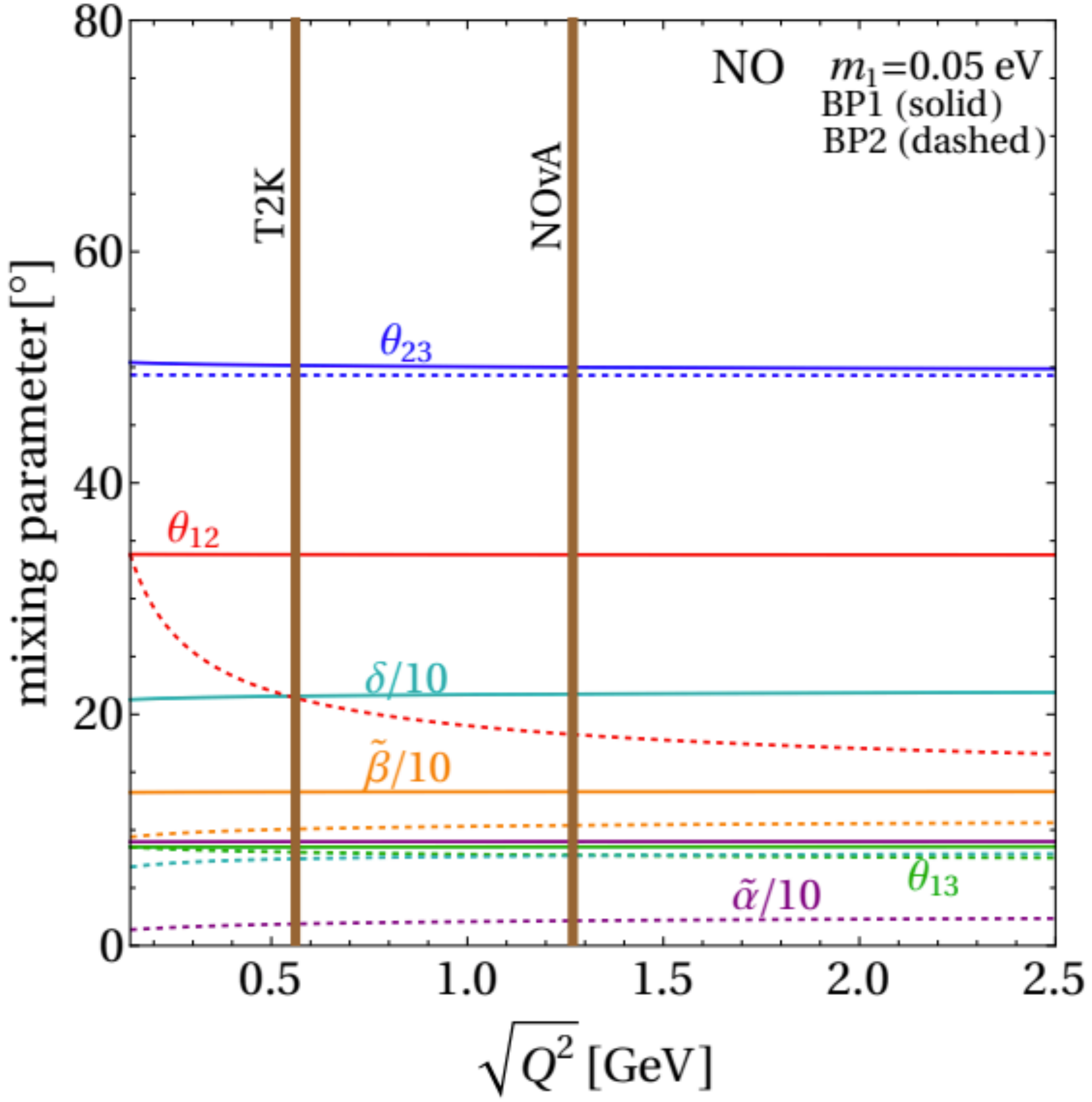
Backup





pmachado@fnal.gov