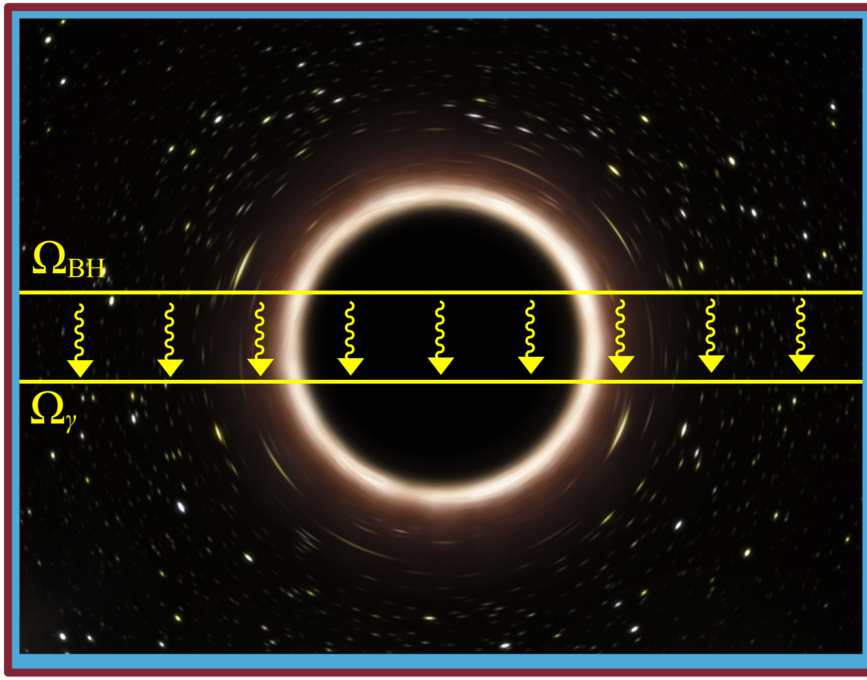
# Cosmic Stasis from Primordial-Black-Hole Evaporation and Its Phenomenological Implications



**Brooks Thomas**

LAFAYETTE COLLEGE

Work supported in part by NSF

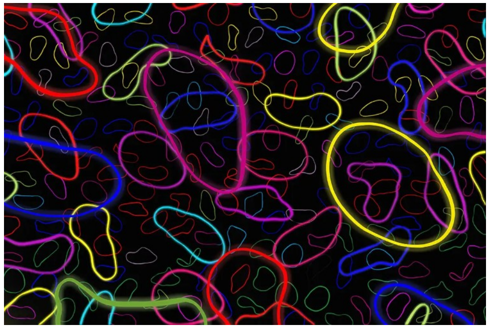
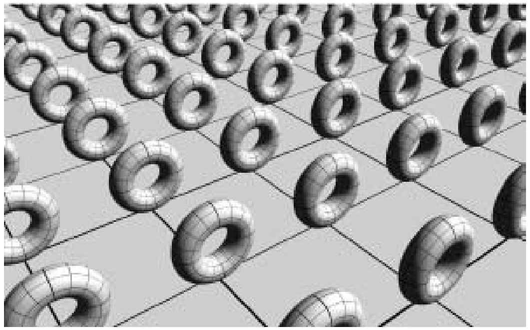
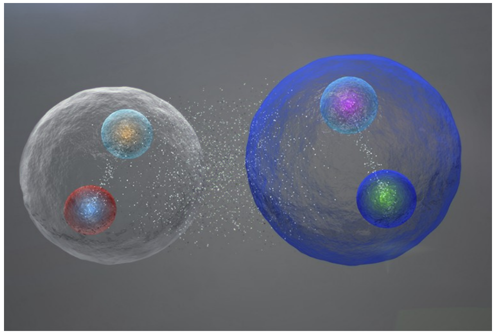
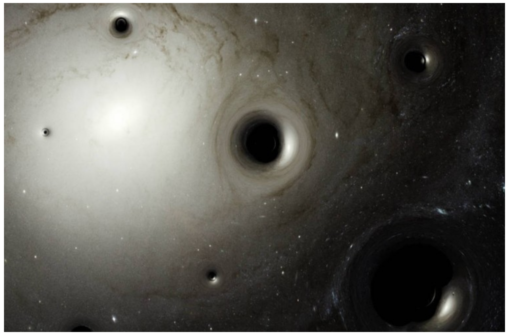
## Based on work done in collaboration with:

**Keith R. Dienes, Fei Huang, Lucien Heurtier, Doojin Kim, and Tim M. P. Tait [arXiv:2108.02204, 2212.01369]**

CETUP* 2023, Lead, South Dakota, June 23th, 2023

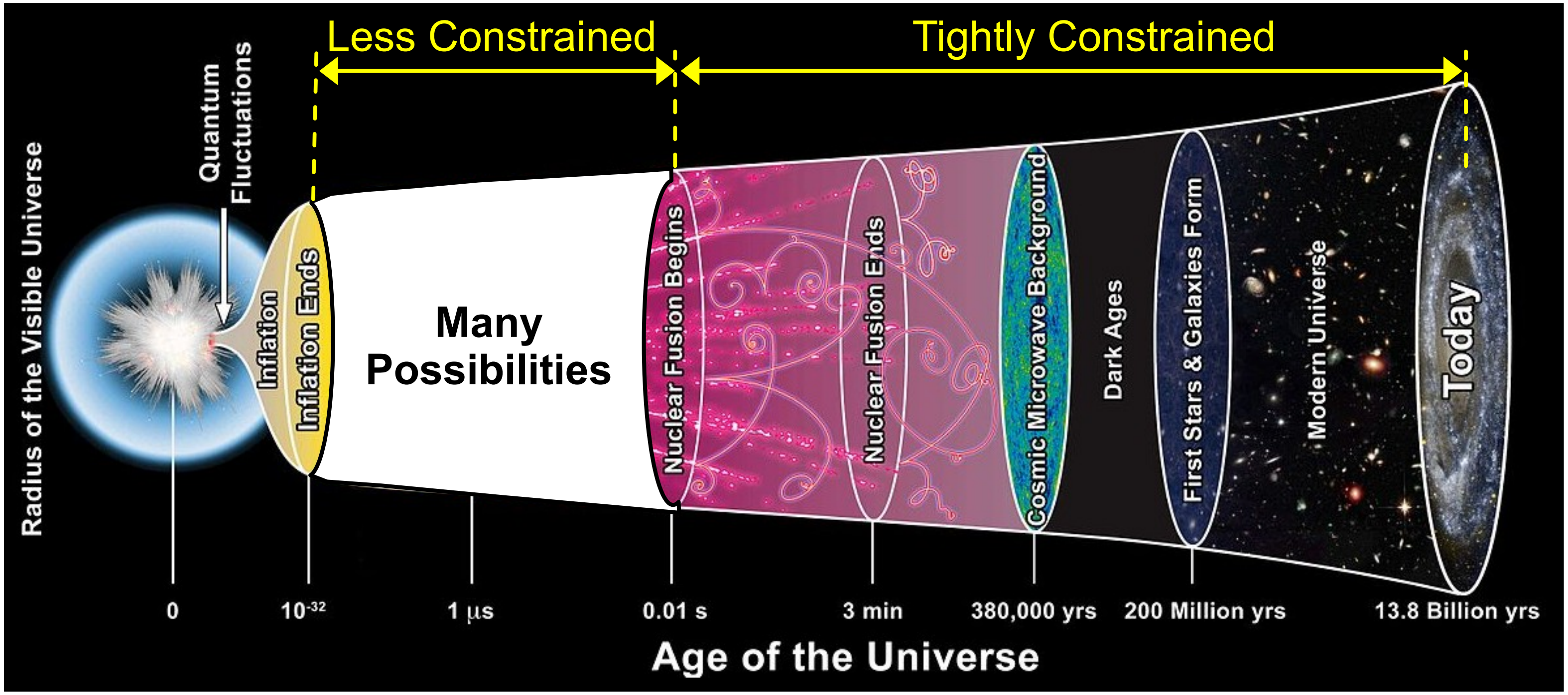
# Towers of Unstable States

- A wide variety of scenarios for new-physics predict **towers of massive, unstable states** with a broad spectrum of masses, cosmological abundances, and lifetimes.
- Such towers are a generic feature of, for example,...
  - String theory (string moduli, axions, etc.)
  - Theories with extra spacetime dimensions (KK towers)
  - Scenarios with confining dark/hidden-sector gauge groups (bound-state resonances)
  - Scenarios which lead to the production of primordial black holes with an extended mass spectrum (the black holes themselves)
- In some cases, such states can give rise to astrophysical signals, signals at colliders, etc.; in others, they are too heavy/short-lived.

# Cosmological Consequences

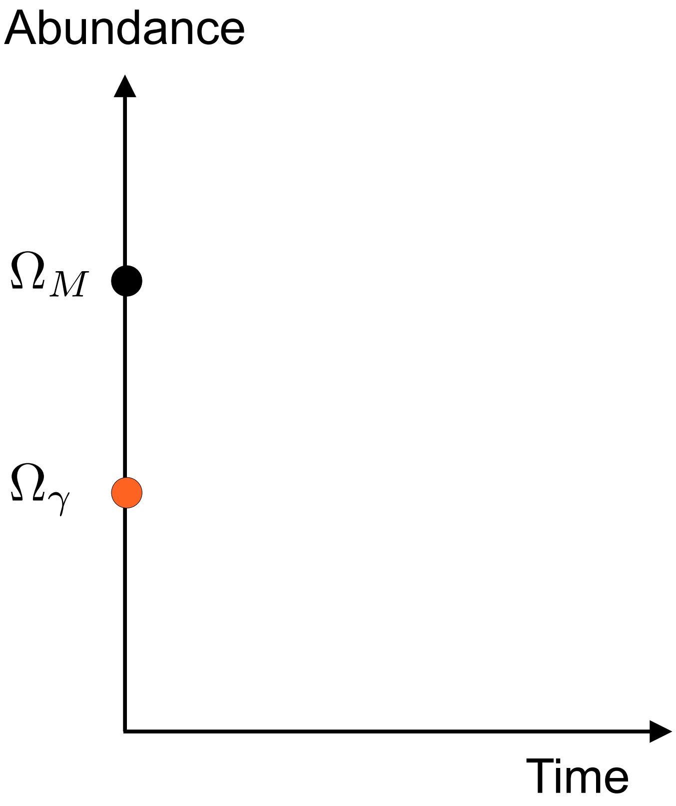
- The presence of such towers can have a significant impact on early-universe cosmology – even if the tower states are too heavy/short-lived to be accessible.



- Indeed, as discussed in the previous talk, such towers can give rise to **stable, mixed-component eras**: eras in which the abundances of multiple cosmological energy components (in this case, matter and radiation) remain effectively constant over an extended period.
- Moreover, these eras are **global attractors**: if the basic conditions under which they arise are satisfied, the universe will evolve toward them.

# Underpinnings of Stasis

- To see how a stasis era can arise, let us consider a universe effectively consisting of matter and radiation alone, with all other $\Omega_i$ negligible.



**Boltzmann Equations**

$$\frac{d\rho_M}{dt} = -3H\rho_M$$

$$\frac{d\rho_\gamma}{dt} = -4H\rho_\gamma$$

# Underpinnings of Stasis

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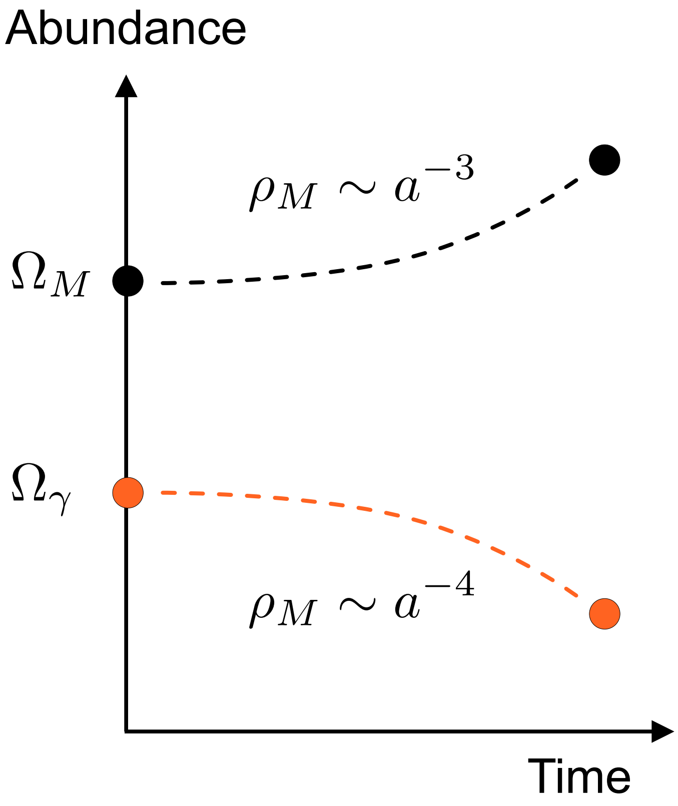

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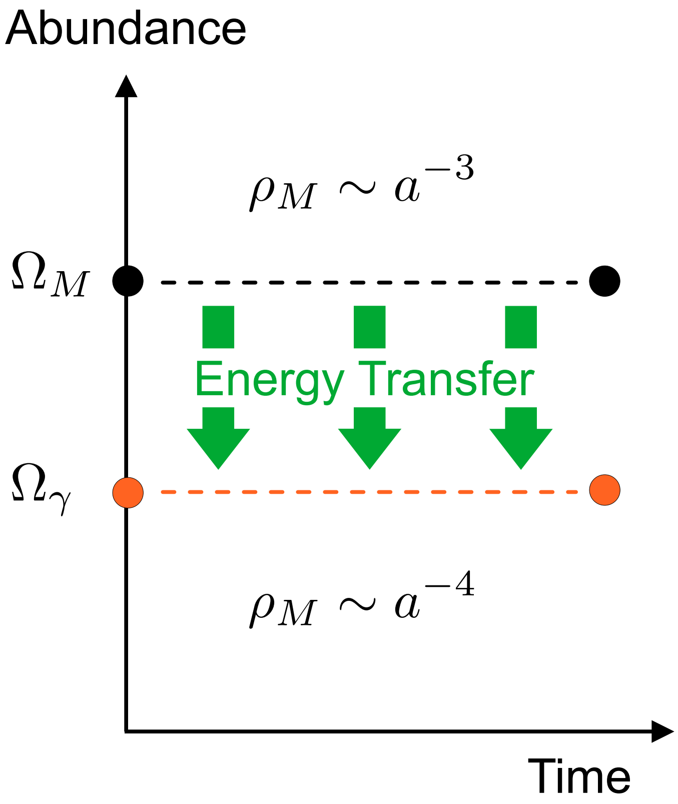
- Since $\rho_M$ and $\rho_\gamma$ scale differently under cosmic expansion, $\Omega_M$ typically increases, while $\Omega_\gamma$ decreases.
- In order to compensate for this effect, what's needed is a ***continuous transfer of energy density*** from matter to radiation.



**Boltzmann Equations**

$$\frac{d\rho_M}{dt} = -3H\rho_M - S(t)$$

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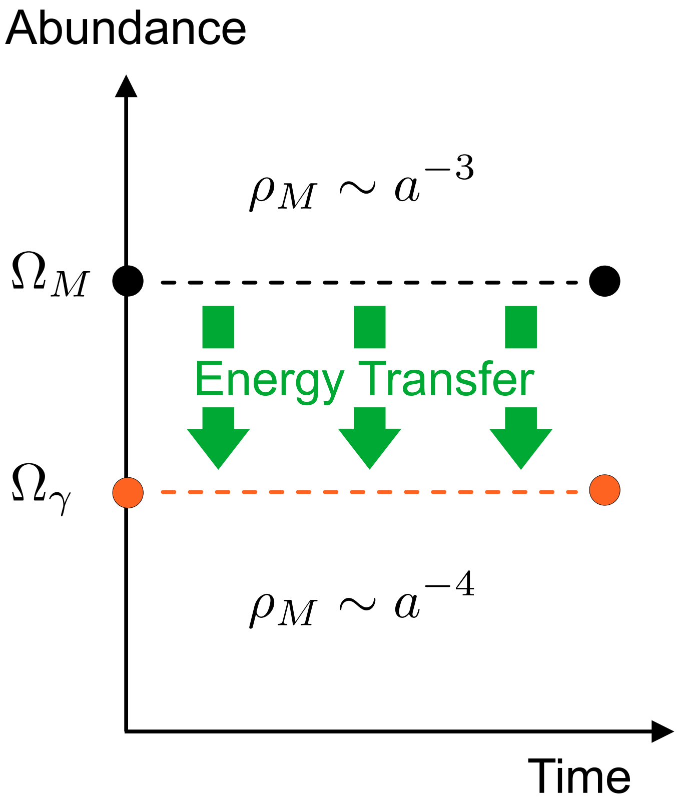

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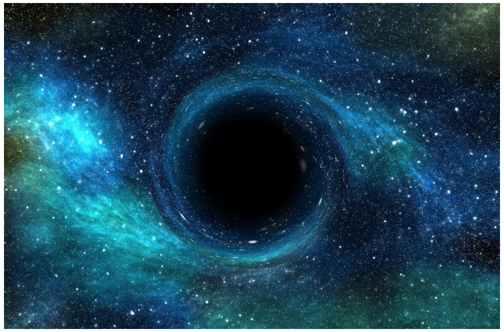
***Particle decays*** provide a natural mechanism for obtaining these source/sink terms.

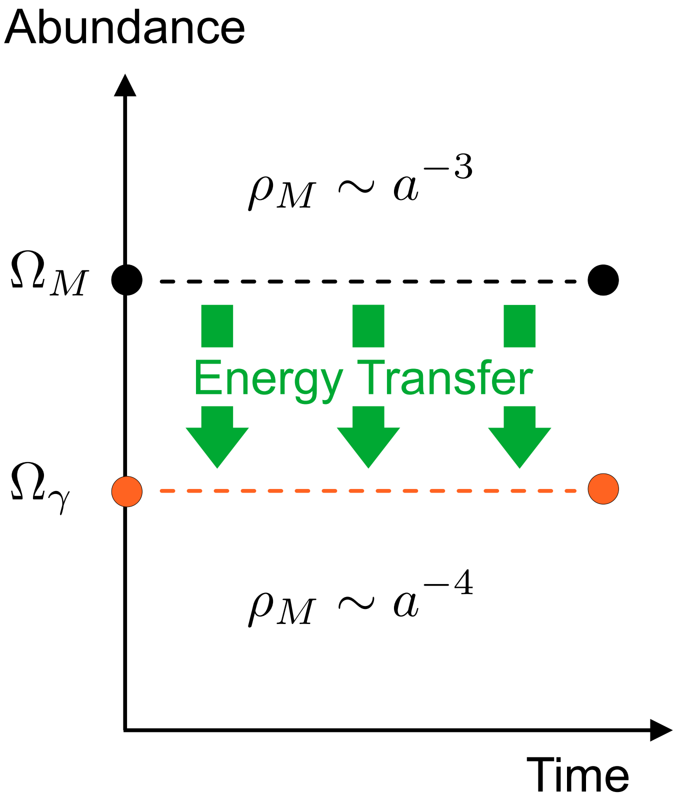
# Stasis from Primordial-Black-Hole Evaporation

- A population of ***primordial black holes*** (PBH), whose evaporation via Hawking radiation transfers energy density from matter to radiation, can likewise give rise to a period of stasis.

**[Dienes, Huang, Heurtier, Kim, Tait, BT ‘22]**



- In this case, **Hawking radiation** provides the mechanism via which energy density is transferred from matter to radiation.



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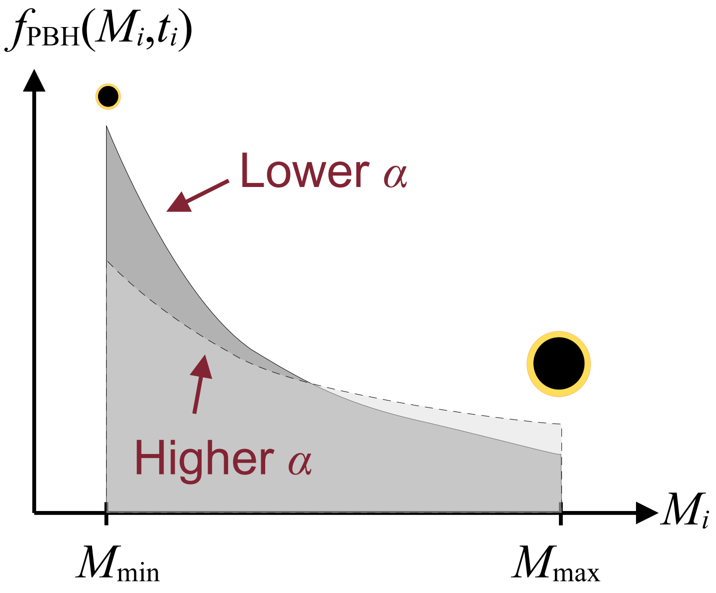
# Initial PBH Mass Spectrum



- Let's consider a population of primordial black holes (PBHs) with the mass spectrum

$$\boxed{f_{\rm BH}(M_i,t_i) = \begin{cases} CM_i^{\alpha-1} & \text{for } M_{\rm min} \leq M_i \leq M_{\rm max} \\ 0 & \text{otherwise} \end{cases}}$$

- Such an **extended mass spectrum** arises naturally in scenarios in which the PBHs form after inflation via the collapse of perturbations with a scale-invariant power spectrum.

**[Carr '75; Green, Liddle '97; Kim, Lee, MacGibbon '99; Bringmann, Keifer, Polarski '02; Carr et. al. '17]**

- The value of $\alpha$ is determined by the equation-of-state parameter $w_{\rm c}$ for the universe during the epoch wherein the PBHs form.

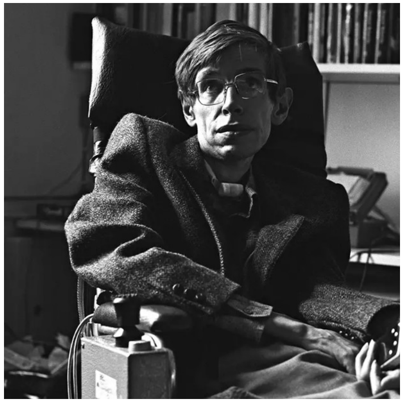
$$\boxed{\alpha = -\frac{3w_c+1}{w_c+1}} \xrightarrow{-1/3 \leq w_c \leq 1} \boxed{-2 \leq \alpha \leq 0}$$

- Observational considerations likewise place constraints on the values of $M_{\rm min}$ and $M_{\rm max}$:

**[Carr, Kohri, Sendouda, Yokoyama '09; Keith, Hooper, Blinov, McDermott '20; Carr, Kohri, Sendouda, Yokoyama '21; Akrami et al. (Planck) '20]**

$$\boxed{\underbrace{0.1\text{g} \lesssim}_{\text{Planck upper bound on } H_\star} M_{\rm min} < M_{\rm max} \underbrace{\lesssim 10^9\text{g}}_{\text{Heaviest PBH evaporate completely before BBN}}}$$

# Evaporation



- **Hawking radiation** provides a mechanism via which energy density can be transferred from the PBHs (which behave like massive matter) to radiation. [Hawking, '74; Hawking '75]

$$\boxed{T_{\rm BH} = \frac{1}{8\pi G M} \sim 1.06~{\rm GeV}\left(\frac{10^{13}~{\rm g}}{M}\right)}$$

- The rate of change of the mass $M$ of a single PBH due to this effect is

[MacGibbon, Webber, '90; MAcGibbon '91]

$$\boxed{\frac{dM}{dt} \equiv -\varepsilon(M)\frac{M_P^4}{M^2}}$$

Graybody factor: for this range of $M$, $\varepsilon(M) \approx \varepsilon$ is approximately constant.

- The time at which a PBH evaporates completely (i.e., at which $M = 0$) as a result of this effect is

$$\boxed{\tau(M_i) \equiv \frac{{M_i}^3}{3\varepsilon M_P^4}}$$

- As a result, the PBH mass spectrum subsequently evolves according to a Boltzmann equation of the form

$$\boxed{\frac{d\rho_{\rm BH}}{dt} + 3H\rho_{\rm BH} = \int_0^\infty dM\, f_{\rm BH}(M,t)\frac{dM}{dt}}$$

# Boltzmann Evolution

- The evolution of the Hubble parameter $H(t)$ is given by the Friedmann acceleration equation, which in thes case takes the form

$w_{\text{BH}} = 0$

$$\frac{dH}{dt} = -H^2 - \frac{4\pi G}{3}\left[\rho_{\text{BH}}(1 + 3\cancel{w_{\text{BH}}}) + \rho_\gamma(1 + w_\gamma)\right]$$

- Expressed in terms of the cosmological abundance $\Omega_{\text{BH}} \equiv \rho_{\text{BH}}/\rho_{\text{crit}}$, the system of equations governing the expansion of the universe is

$$\frac{dH}{dt} = -\frac{1}{2}H^2(4 - \Omega_{\text{BH}})$$
$$\frac{d\Omega_{\text{BH}}}{dt} = -\Gamma_{\text{BH}}(t)\,\Omega_{\text{BH}} + H\left(\Omega_{\text{BH}} - \Omega_{\text{BH}}^2\right)$$

...where we have defined

$$\Gamma_{\text{BH}}(t) \equiv -\frac{\int_0^\infty f_{\text{BH}}(M,t)\frac{dM}{dt}\,dM}{\int_0^\infty f_{\text{BH}}(M,t)M\,dM}$$

- Alternatively, one can change variables and express this system of equations in terms of $\Omega_{\text{BH}}$ and its time-averaged value $\langle\Omega_{\text{BH}}\rangle$ since the time $t_i$ at which the PBH spectrum was initially established:

$$\langle\Omega_{\text{BH}}\rangle \equiv \frac{1}{t - t_i}\int_{t_i}^{t} dt'\,\Omega_{\text{BH}}(t')$$

# PBH-Induced Stasis is a Global Attractor

- One can show that not only do these equations admit a **stasis solution**, but that this stasis solution is a **global attractor**.

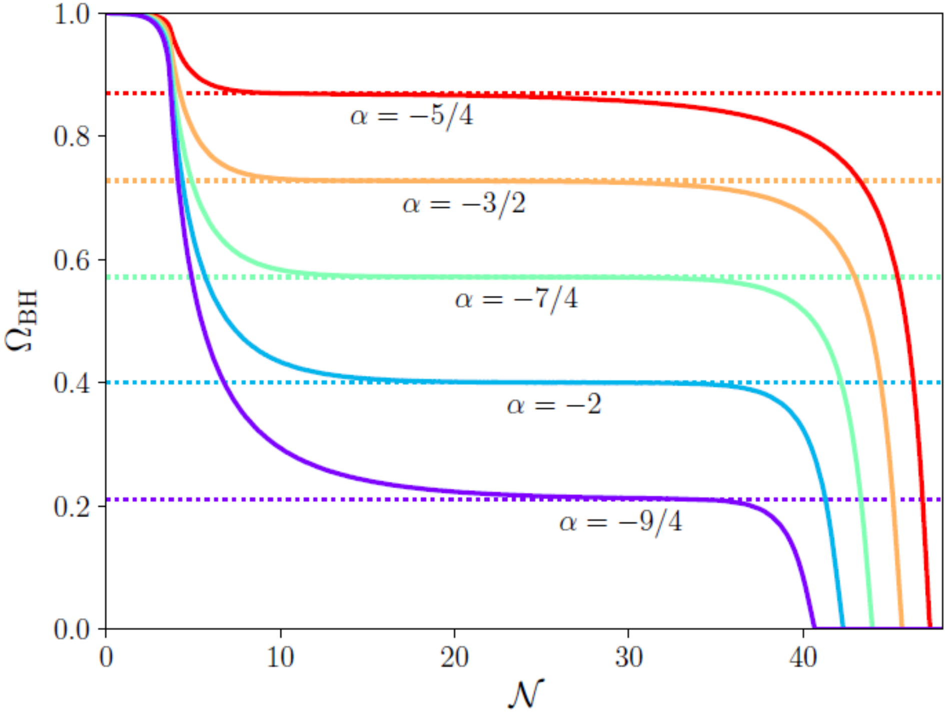
[Barrow, Copeland, Liddle '91; Dienes, Huang, Heurtier, Kim, Tait, BT '22]

- The effective equation-of-state parameter $\overline{w}$ for the universe as a whole during the stasis epoch and the PBH abundance $\Omega_{\rm BH}$ are determined by the value of $\alpha$:

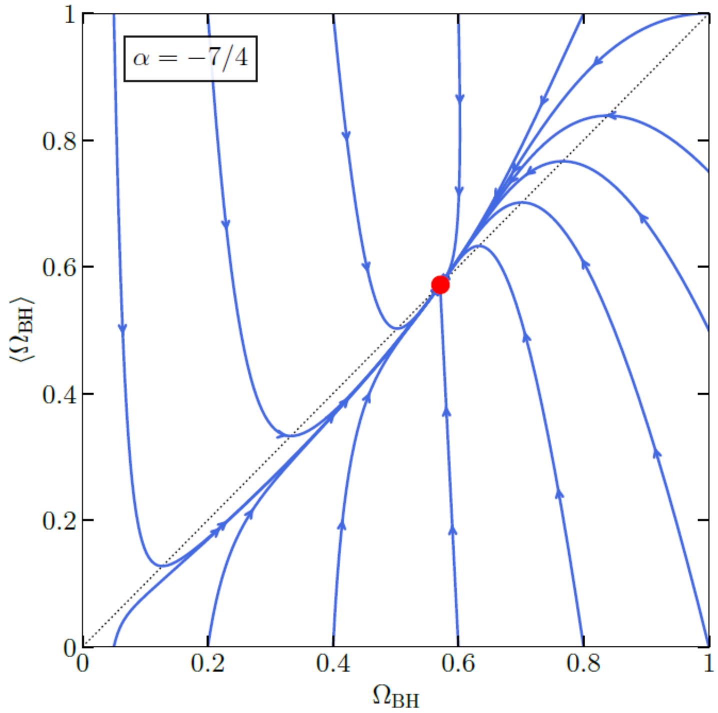
$$\boxed{\overline{w} = -\frac{\alpha+1}{\alpha+7}}$$

$$\boxed{\overline{\Omega}_{\rm BH} = \frac{4\alpha+10}{\alpha+7}}$$

**Stasis from PBH Evaporation**



**Attractor Behavior**



# Duration of the Stasis Epoch

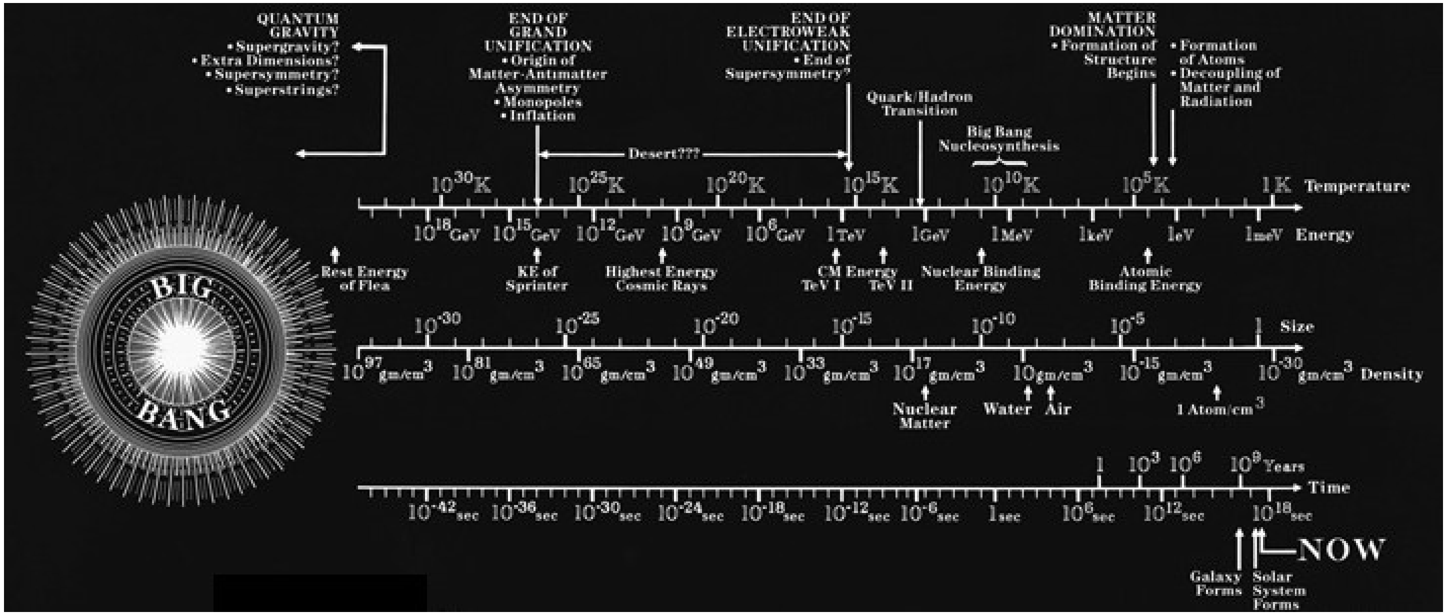
- The duration of this PBH-induced stasis epoch, expressed in terms of the number of $e$-folds of cosmic expansion that it spans, is given by

$$\boxed{\mathcal{N}_s \approx \log\left[\frac{a(\tau(M_{\rm max}))}{a(\tau(M_{\rm min}))}\right] \approx \frac{\alpha+7}{3}\log\left(\frac{M_{\rm max}}{M_{\rm min}}\right)}$$

- For $M_{\rm min} = 0.1$ g at its minimum and $M_{\rm max} = 10^9$ g at its maximum, this yields a stasis epoch of duration

$$\boxed{\mathcal{N}_s \lesssim 23\left(\frac{\alpha+7}{3}\right)}$$

- This is a significant duration indeed – potentially spanning a range of temperatures $\mathcal{O}(\text{MeV}) \lesssim T \lesssim \mathcal{O}(10^{11}\text{ GeV})$!



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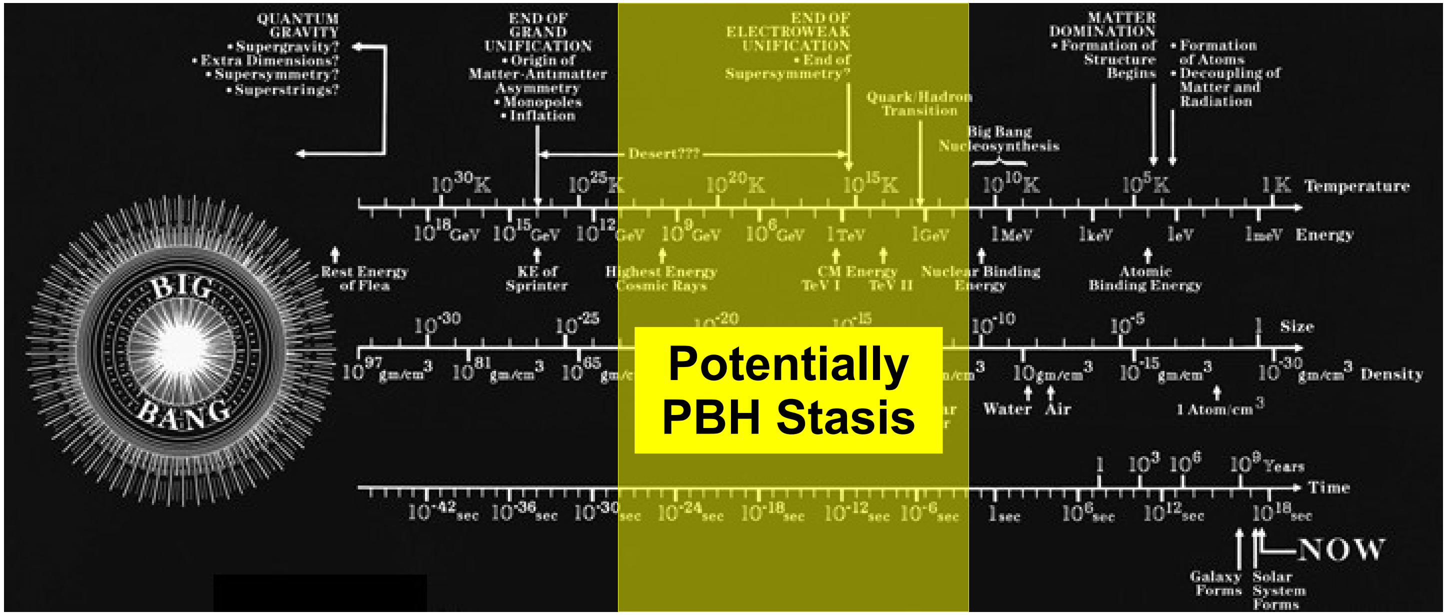


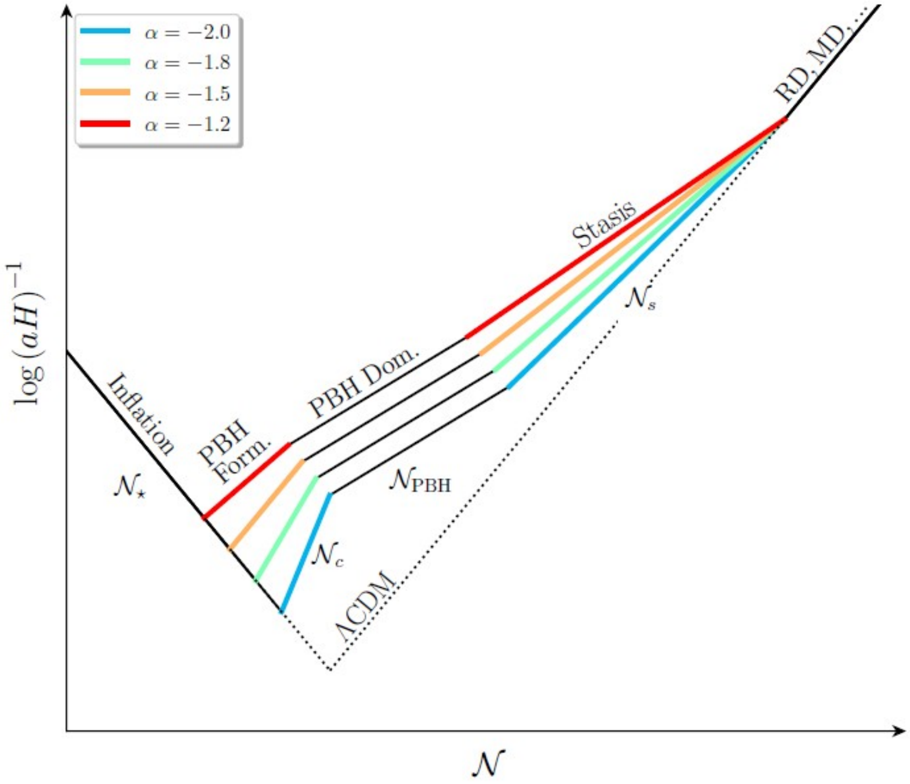
**Potentially PBH Stasis**

Thus, events such as the electroweak phase transition could have occurred during such a stasis epoch!

# Cosmic Expansion History

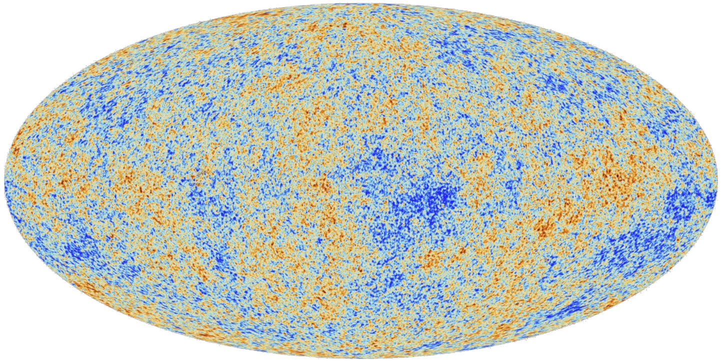
- In cosmologies involving an epoch of PBH-induced stasis, the cosmological timeline includes a series of several different epochs after cosmic inflation ends. Sequentially, these are:
  - The epoch during which the **PBHs are generated**, wherein the equation-of-state parameter $w_c$ determines $\alpha$.
  - An epoch during which the PBHs come to dominate the energy density of the universe. This epoch is **matter-dominated** $(w = 0)$.
  - The **stasis epoch**, which begins once the lightest PBHs begin to evaporate, and wherein $w = \overline{w}$.

**Comoving Hubble Horizon**



  - The usual **RD epoch** with $w = 1/3$, which begins after the heaviest PBHs evaporate and stasis ends. Once this epoch begins, the expansion history coincides with that of the standard cosmology.

# Inflationary Observables



- In the simplest inflationary scenarios, primordial perturbations of the inflaton field give rise to the pattern of inhomogeneities observed in the cosmic microwave background (CMB).
- However, **modifications of the cosmological timeline** beween the end of inflation and last scattering can alter predictions for CMB observables.
- The primary such observables are the **tensor-to-scalar ratio** $r$ and **spectral index** $n_s$ that characterize the primordial perturbation spectrum.
- For example, in single-field, slow-roll models of inflation, these obserables are directly related to the slow-roll parameters $\varepsilon$ and $\eta$:

$$\boxed{\begin{aligned} n_s &= 1 - 6\epsilon + 2\eta \\ r &= 16\epsilon \end{aligned}}$$

where

$$\boxed{\epsilon \equiv \frac{M_P^2}{16\pi}\left[\frac{V'(\phi_\star)}{V(\phi_\star)}\right]^2 \qquad \eta \equiv \frac{M_P^2}{8\pi}\left|\frac{V''(\phi_\star)}{V(\phi_\star)}\right|}$$

- The quantity $\phi_\star$ denotes the value of the inflaton field at the time at which a perturbation with wavenumber equal to the pivot scale $k_\star$ exits the horizon. Following Planck, we take $k_\star = 0.002\ \mathrm{Mpc}^{-1}$. **[Akrami et al. (Planck) '20]**

# Inflationary Observables

- In order to determine $\phi_\star$ we note that in the slow-roll approximation, the Hubble parameter $H_\star$ and scale factor $a_\star$ at the time at which this same mode exist the horizon are related to $\phi_\star$ by

$$\boxed{H_\star^2 \approx \frac{8\pi V(\phi_\star)}{3M_P^2}}$$

and

$$\boxed{\log\left(\frac{a_\text{end}}{a_\star}\right) = \frac{8\pi}{M_P^2}\int_{\phi_\text{end}}^{\phi_\star} \frac{V(\phi)}{V'(\phi)}d\phi}$$

- Combining these relations yields the integro-differential equation

$$\boxed{\frac{8\pi}{M_P^2}\int_{\phi_\text{end}}^{\phi_\star} \frac{V(\phi)}{V'(\phi)}d\phi = \frac{1}{2}\log\left(\frac{8\pi a_\text{now}^2 V(\phi_\star)}{3M_P^2 k_\star^2}\right) - \log\left(\frac{a_\text{now}}{a_\text{end}}\right)}$$

...which can be solved for a given form of $V(\phi)$.

- In order to illustrate how $r$ and $n_s$ are modified in cosmologies involving an epoch of PBH-induced stasis, it is useful to work in the context of a concrete model for the inflaton potential… or two. We'll choose

① **Polynomial potentials**:

$$\boxed{V(\phi) \sim |\phi|^p}$$

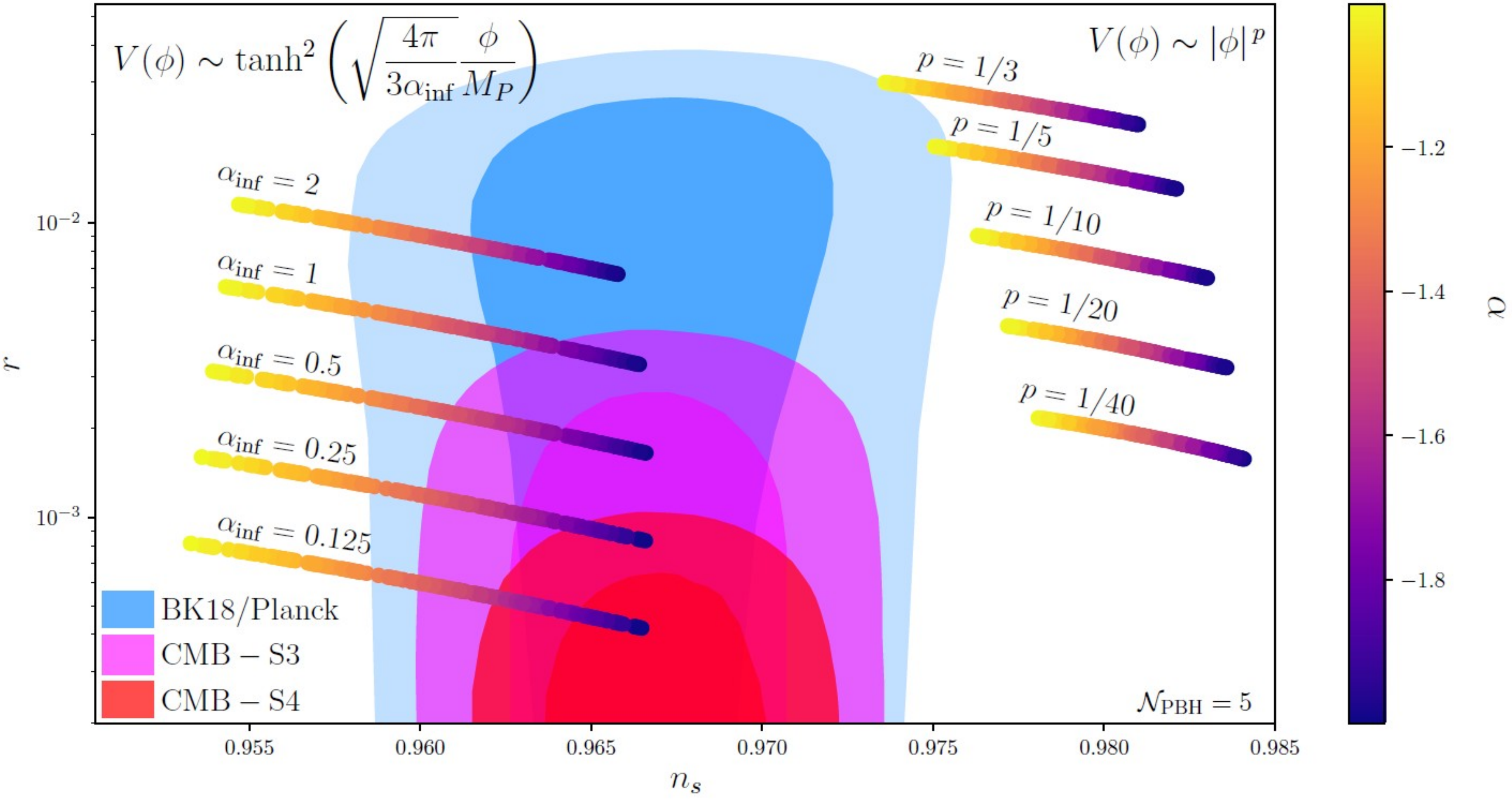
② **T-Model $\alpha$-attractors**:

[Kallosh, Linde '13]

$$\boxed{V(\phi) \sim \tanh^{2n}\left(\sqrt{\frac{4\pi}{3\alpha_\text{inf}}}\frac{\phi}{M_P}\right)}$$

# Inflationary Observables: Results

- In general, the modifications of the cosmological timeline associated with PBH-induced stasis serve to increase $r$ and decrease $n_s$.



- As a result, depending on the inflationary model in question, tensions between the predictions for these observables and CMB data may be **either eased or exacerbated**.

# Gravitational-Wave Background

- The cosmological modifications associated with a PBH-induced stasis epoch affect the **gravitational-wave (GW) background** in several ways.
- Perhaps most importantly, the modified expansion history alters the contribution to the GW background generated by other sources.
- For concreteness, we'll consider the simple case of a **stochastic GW background** which is homogeneous, isotropic, Gaussian, and unpolarized.
- The differential GW energy density per logarithmic comoving wavenumber $k$ for this case is:

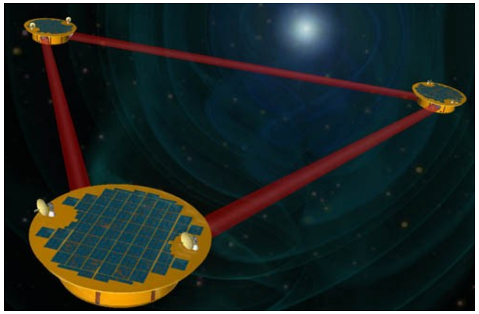
$$\frac{d\rho_{\rm GW}(a)}{d\log k} = \frac{k^2 h_k^2(a)}{16\pi G a^2}$$

[Caprini, Figueroa '18]

- The differential amplitude $h_k(a)$ depends on when the pertubation mode re-enters the horizon:

$$h_k(a) = \frac{a_k}{a} h_k(a_k)$$

Advanced LIGO



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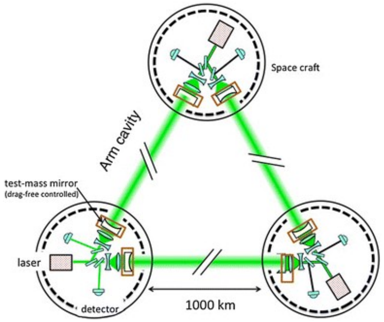


DECIGO

# Gravitational-Wave Background

- During an epoch wherein $w$ is constant, the wavenumber $k$ which enters the horizon at scale factor $a_k$ scales with $a_k$ according to the relation

$$\boxed{k = a_k H_k \propto a_k^{-(1+3w)/2}}$$

- This implies that: $\boxed{\dfrac{d\rho_{\rm GW}(a)}{d\log k} \propto a^{-4} h_k^2(a_k) k^{\xi(w)}}$ where $\boxed{\xi(w) \equiv \dfrac{2(3w-1)}{(3w+1)}}$

- In the standard cosmology, wherein the universe remains radiation-dominated ($w = 1/3$) from the end of reheating until matter-radiation equality, $\xi(w) = 0$ throughout the entire duration.

- Thus, the resulting present-day GW spectrum – or, more precisely, the differential present-day GW abundance per unit physical frequency $f$ – is flat (i.e., $f$-independent) and given by **[Caprini, Figueroa '18]**

$$\boxed{\frac{d\Omega_{\rm GW}^{\rm sc}}{d\log f} = \Omega_\gamma(a_{\rm now}) \left(\frac{g_{\star S}(T_{\rm eq})}{g_{\star S}(T_k)}\right)^{4/3} \frac{g_\star(T_k)}{24\pi^2} \frac{H_\star^2}{M_P^2}}$$

# Gravitational-Wave Background

- By contrast, cosmology involving a PBH-induced stasis epoch with $w = \overline{w}$ – as well as a PBH-production epoch with $w = w_c$ and a PBH-dominated epoch with $w = 0$ – can **differ significantly** from this result.
- In particular, in such a modified cosmology, the corresponding present-day GW spectrum is given by

$$\frac{d\Omega_{\rm GW}}{d\log f} = \frac{d\Omega^{\rm sc}_{\rm GW}}{d\log f} \times \begin{cases} 1 & f \leq f_s \\ \left(\frac{f}{f_s}\right)^{\xi(\overline{w})} & f_s < f \leq f_{\rm PBH} \\ \left(\frac{f_{\rm PBH}}{f_s}\right)^{\xi(\overline{w})} \left(\frac{f}{f_{\rm PBH}}\right)^{-2} & f_{\rm PBH} < f \leq f_f \\ \left(\frac{f_{\rm PBH}}{f_s}\right)^{\xi(\overline{w})} \left(\frac{f}{f_{\rm PBH}}\right)^{-2} \left(\frac{f}{f_f}\right)^{\xi(w_c)} & f_f < f \leq f_{\rm end} \\ 0 & f_{\rm end} < f \ , \end{cases}$$

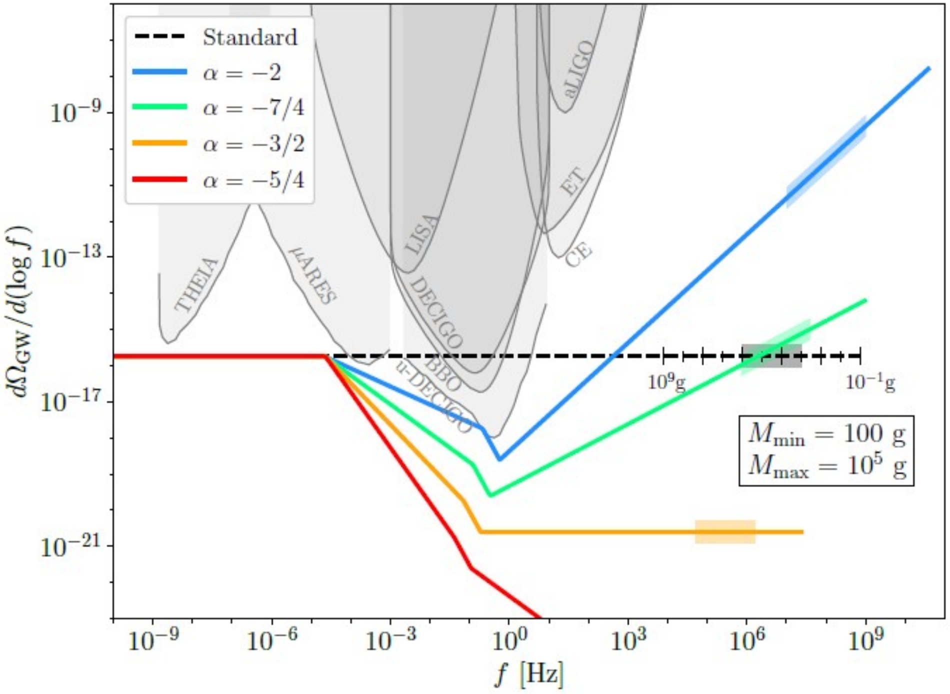
Spectrum obtained in the standard cosmology for the same $H_\star$

Piecewise function with different power-law exponents within different frequency intervals corresponding to different cosmological epochs.

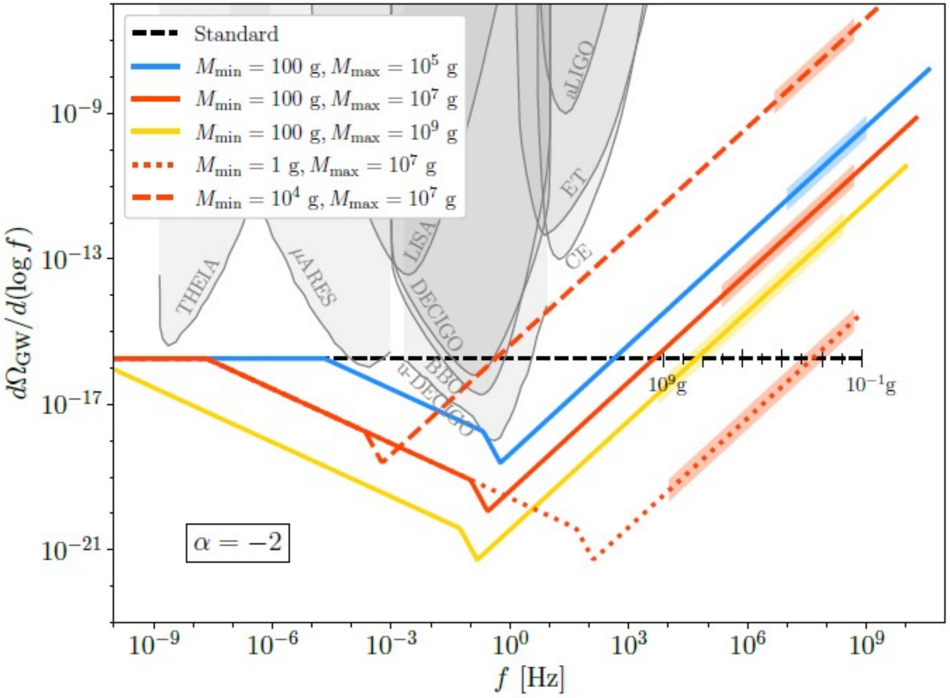
# Gravitational-Wave Background: Results

- Given the sensitivities of planned, proposed, and existing **gravitational-wave observatories**, these modifications can have significant implications for the detection of the stochastic GW background.

## Effect of Varying $\alpha$



## Effect of Varying $M_{\rm min}$ and $M_{\rm max}$



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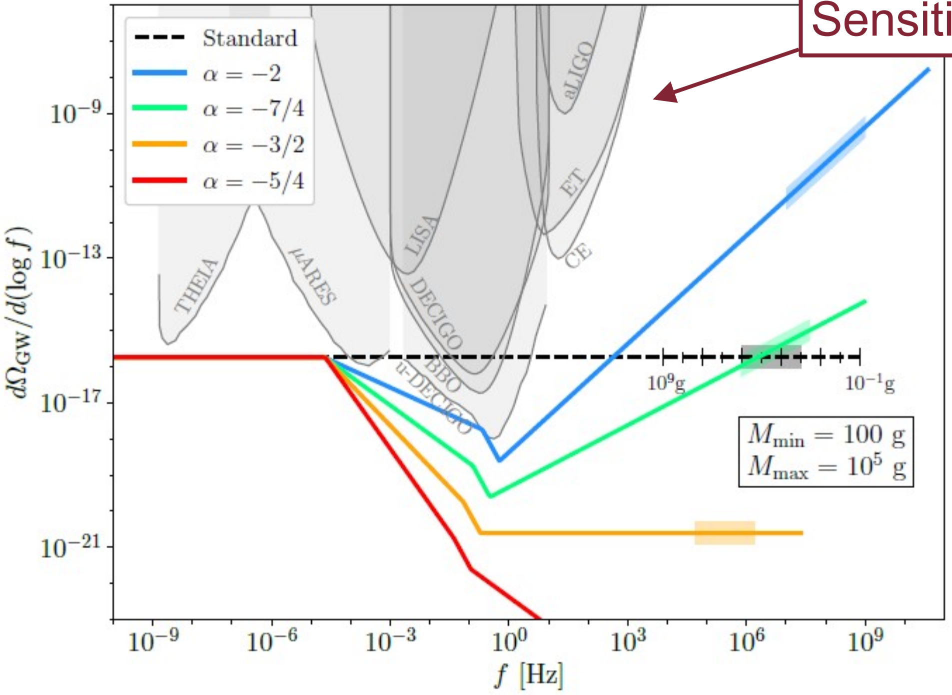
**Effect of Varying α**

Detector Sensitivities



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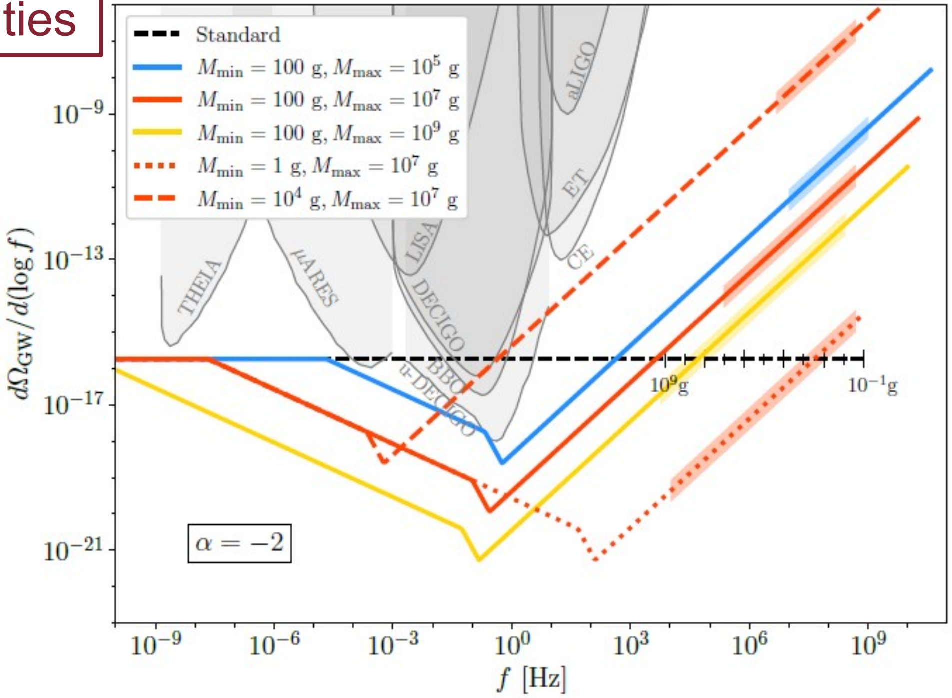
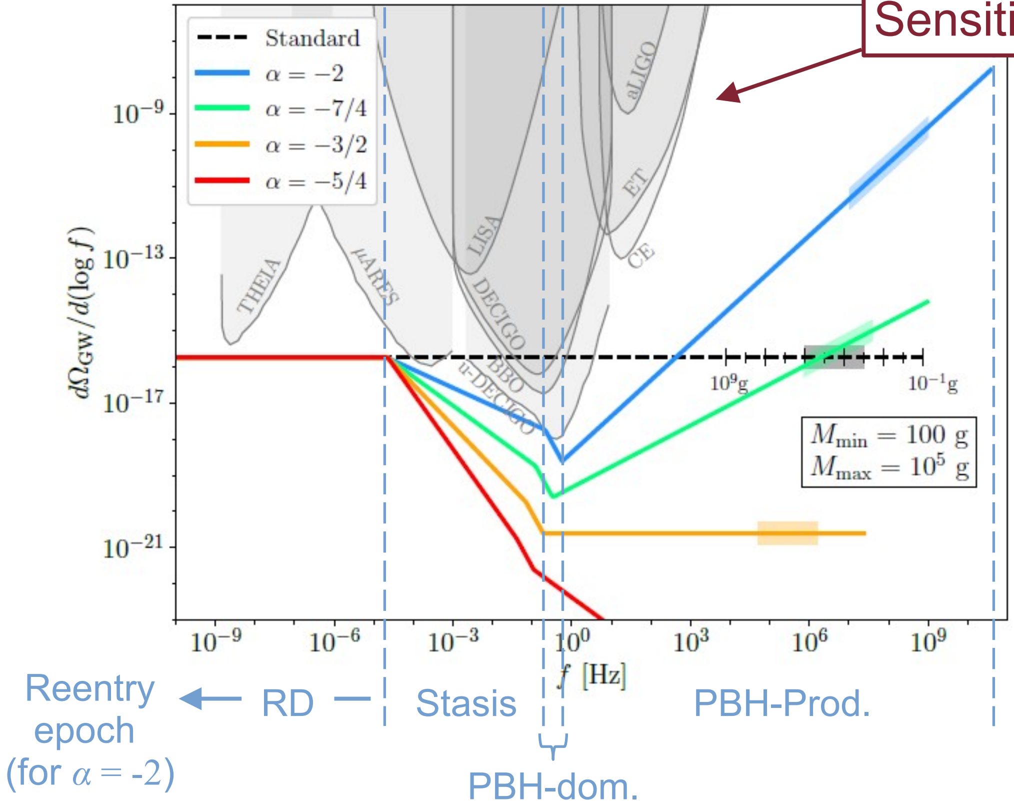


# Gravitational-Wave Background: Results

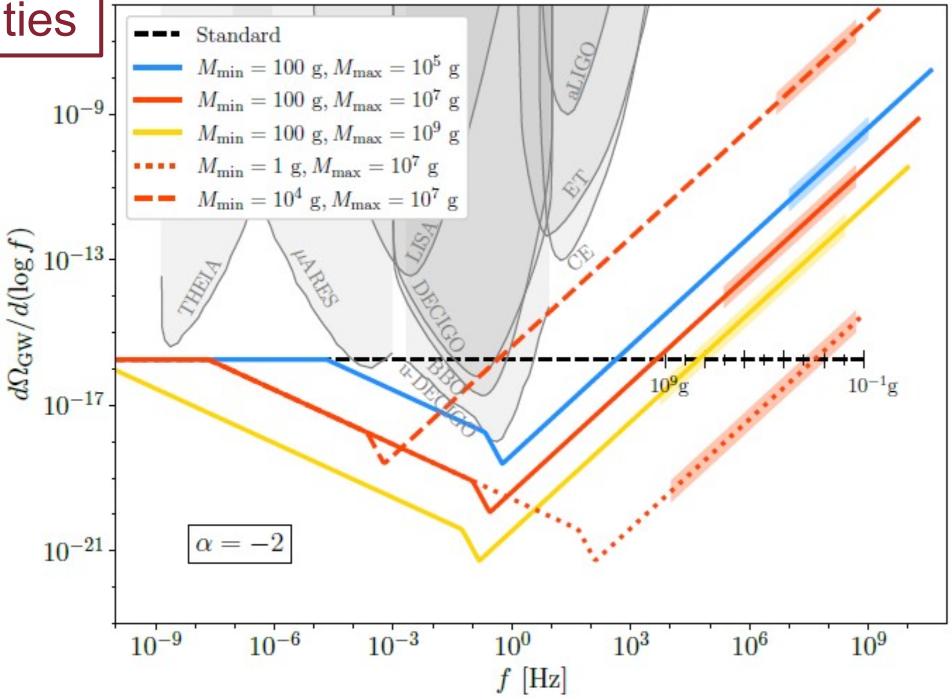
- Given the sensitivities of planned, proposed, and existing **gravitational-wave observatories**, these modifications can have significant implications for the detection of the stochastic GW background.

**Effect of Varying α**

Detector Sensitivities



**Effect of Varying $M_{\rm min}$ and $M_{\rm max}$**



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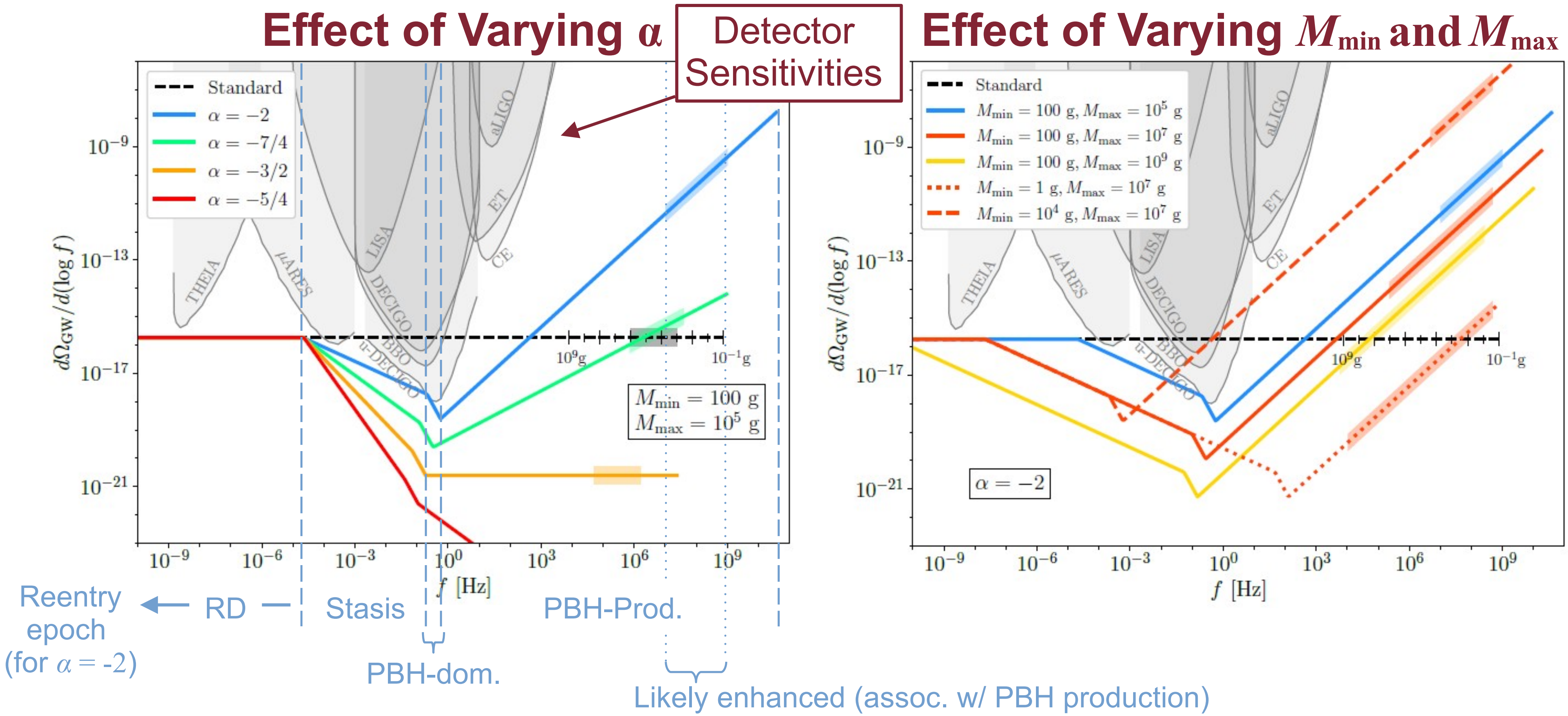

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Detector Sensitivities

**Effect of Varying $M_{\rm min}$ and $M_{\rm max}$**

Reentry epoch (for $\alpha = -2$)

RD

Stasis

PBH-Prod.

PBH-dom.

Likely enhanced (assoc. w/ PBH production)

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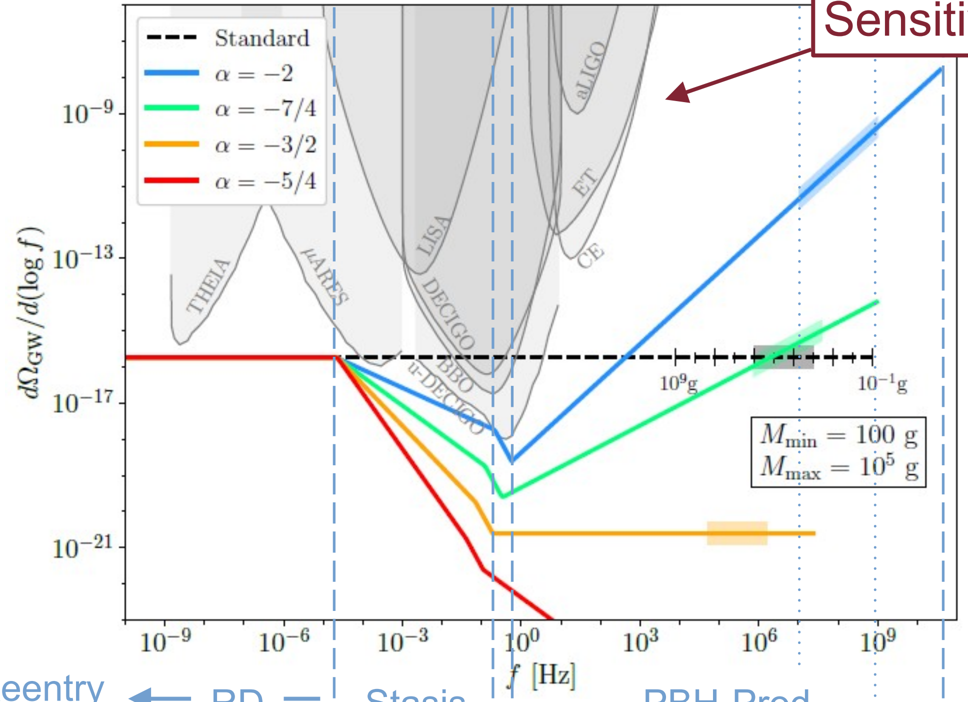

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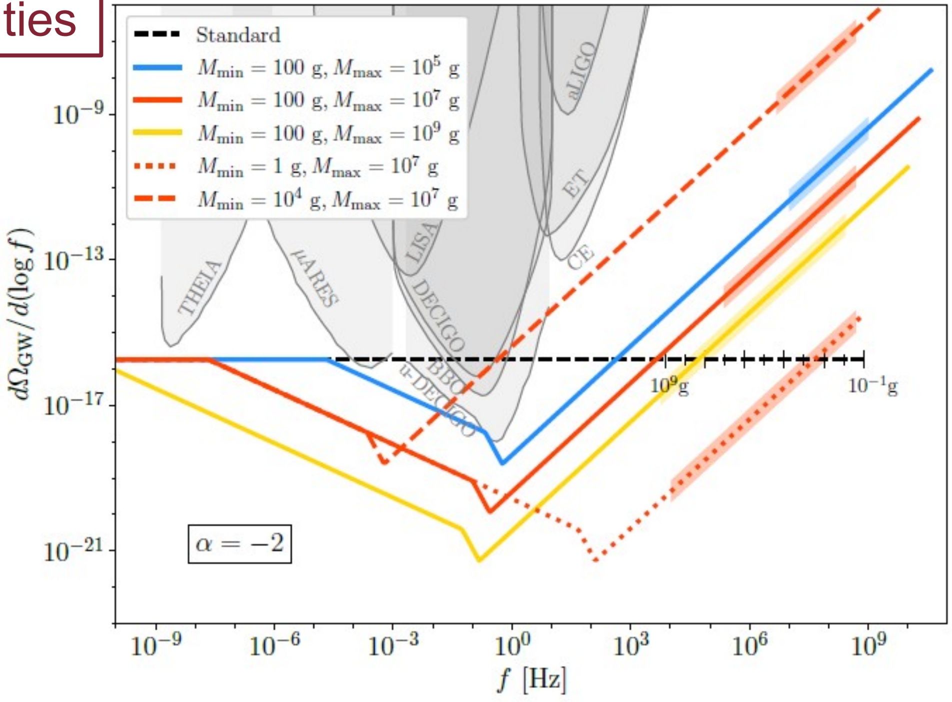

**The Upshot**: A GW signal can be amplified – or hidden – as a result of PBH-induced stasis. Correlations between slopes in different regions provide an observational handle on $\alpha$, $M_{\min}$, and $M_{\max}$.

# Summary

- ***Stable, mixed-component cosmological eras*** – *i.e.* **stasis eras** – are indeed a viable cosmological possibility – and one that can arise naturally in many extensions of the Standard Model.
- For example, we have seen that a population of **primordial black holes** with an extended mass spectrum can give rise to a stasis era.
- PBH-induced stasis is a ***global attractor***, and achieving it does not require any fine-tuning of initial conditions.
- A period of PBH-induced stasis can have a variety of cosmological implications. These include both effects on **inflationary observables** and characteristic modifications of the **gravitational-wave spectrum**.