# Neutron Star Heating from Dark Matter

## The relativistic frontier

Flip Tanedo

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Nirmal Raj

Haibo Yu

Based on
**arXiv:1911.13293**
**arXiv: 2004.09539**

UC RIVERSIDE

**30 June 2023**
CETUP*

THE INSTITUTE FOR UNDERGROUND SCIENCE AT SURF

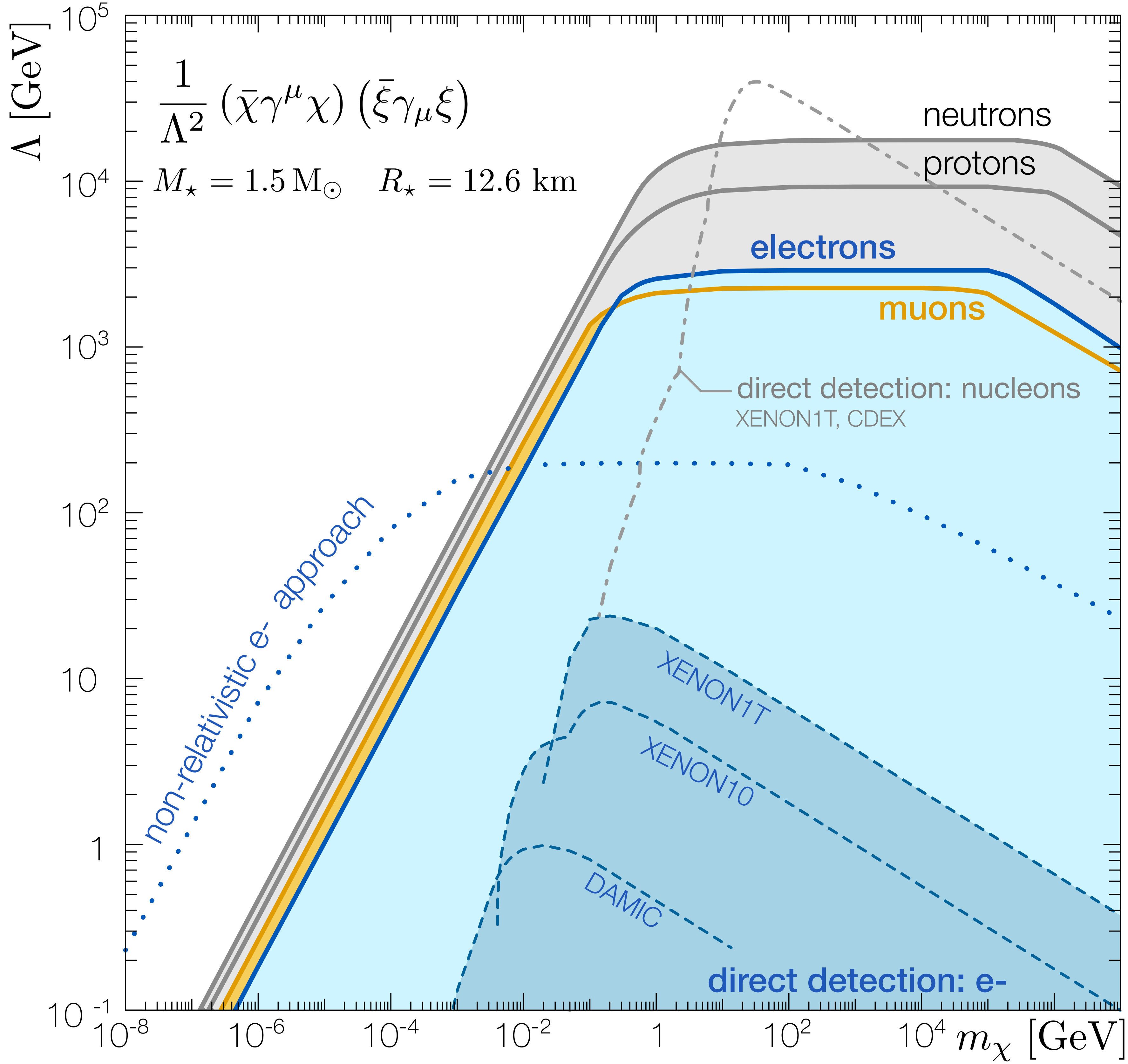
# Result

**Surprising reach of relativistic electrons**

can beat lepton direct detection

curiously similar reach to neutrons

dotted line?




Joglekar, Raj, FT, Yu, 1911.13293

# Direct Detection

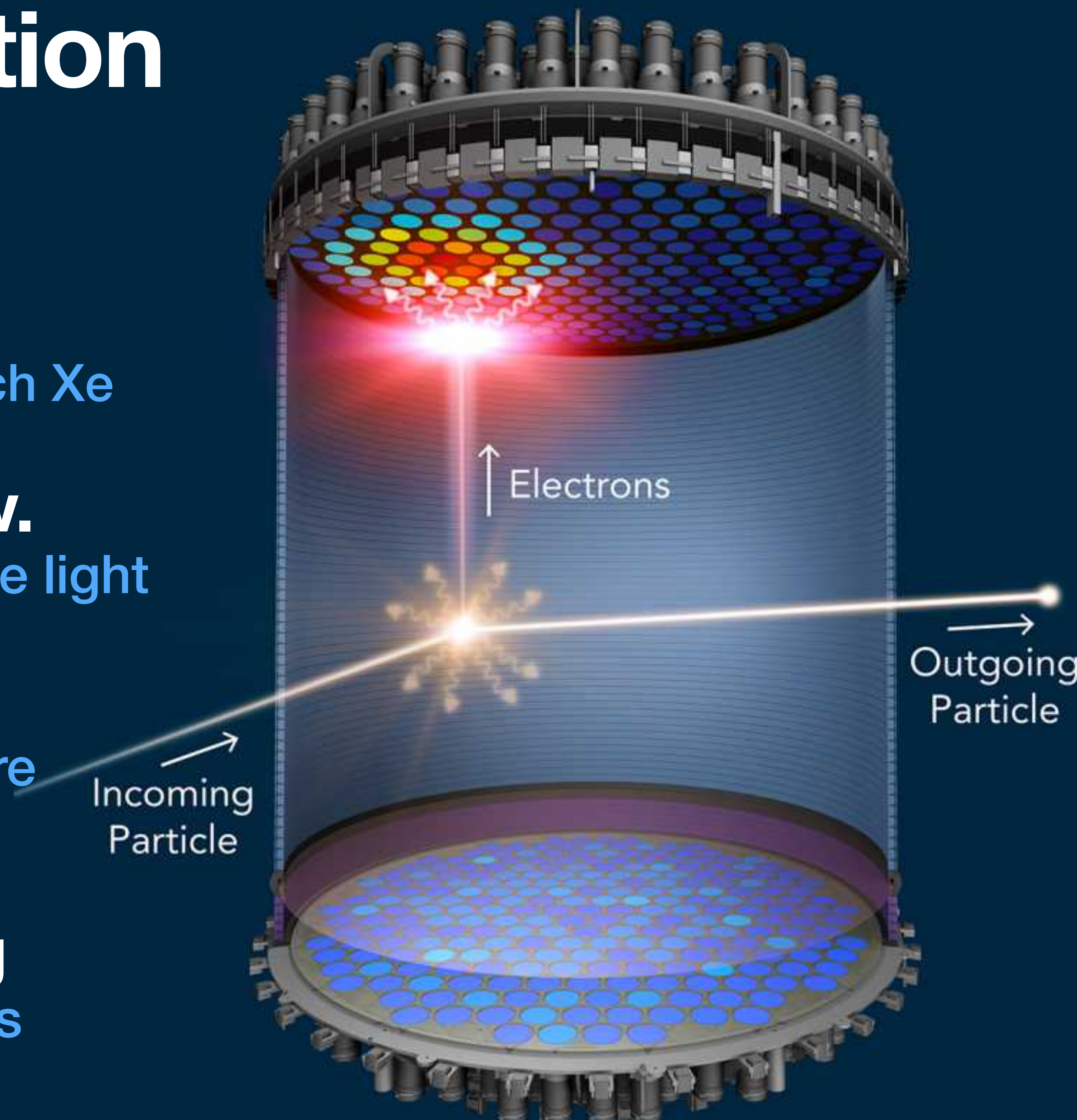
## Challenges

**Bigger = harder**
… and there's only so much Xe

**Dark matter is slow.**
… and worse, may also be light

**Neutrino fog.**
… in this talk, not a feature
(See Nityasa's talk)

**Overburden/ceiling**
… e.g. Jason & Chris talks




via LUX-LZ (kipac.stanford.edu/research/topics/direct-dark-matter-detection)

# This is a neutron star

$$M_\star = 1.5 \text{ M}_\odot$$

$$R_\star = 12.6 \text{ km}$$

*The Little Prince*, A. St.-Exupery via *The Atlantic*


$$M_\star = 1.5\ \mathrm{M}_\odot$$

$$R_\star = 12.6\ \mathrm{km}$$

# Particle physicist's view

$$M_\star = 1.5\ \mathrm{M}_\odot$$

$$R_\star = 12.6\ \mathrm{km}$$

Pretty big, pretty dense.
Full of **n**. Also **e⁻** .
(also lots of other stuff) ... see Zaki's talk


**n**

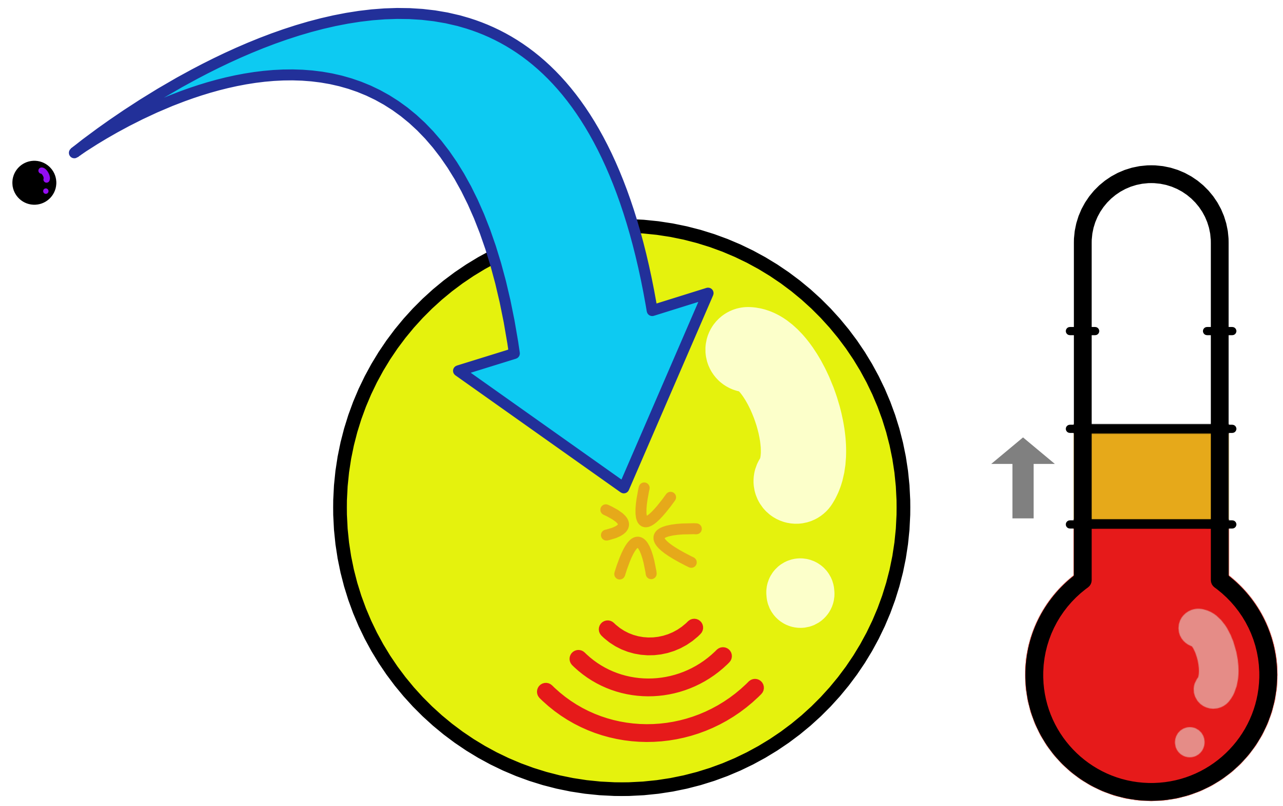

**e⁻**

Excellent *direct detection* target. What detector?

Julie Peasley, particlezoo.net

# Dark Matter Kinetic Heating

## Neutron stars as a laboratory



Dark matter captures on neutron star, deposits its kinetic energy, raises star temperature.

# Dark Kinetic Heating of Neutron Stars and An Infrared Window On WIMPs, SIMPs, and Pure Higgsinos

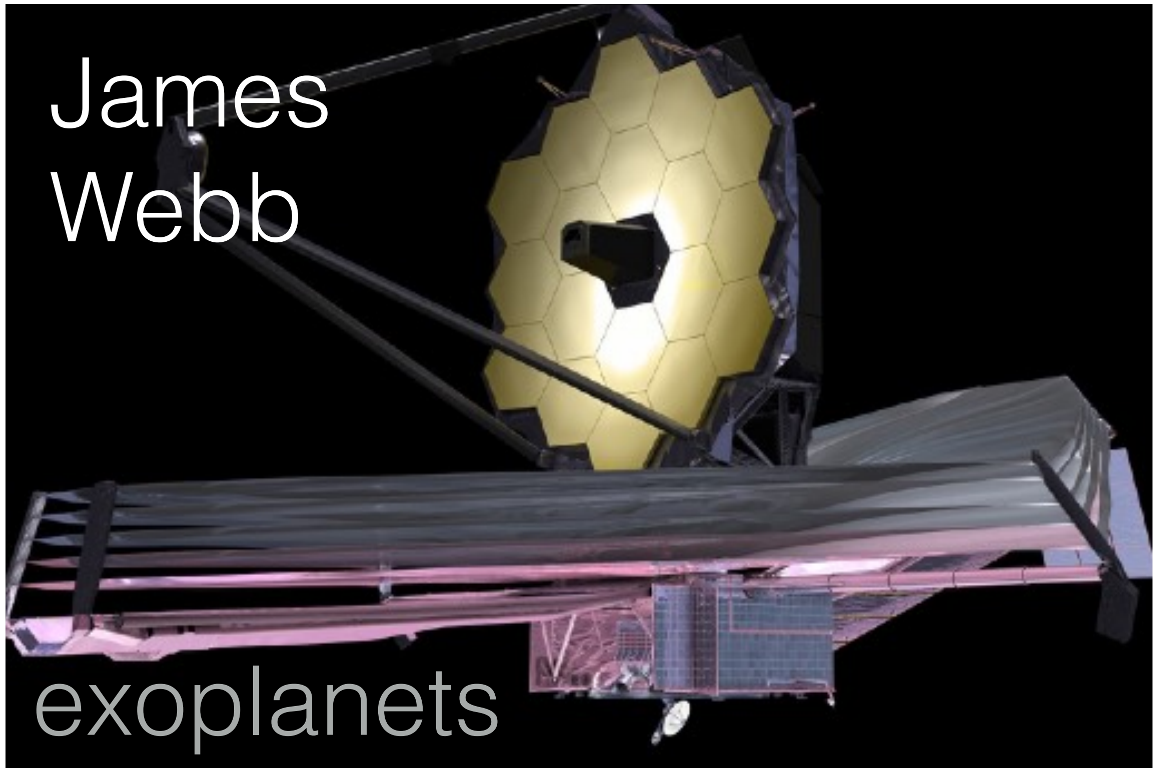
Masha Baryakhtar,[1] Joseph Bramante,[1] Shirley Weishi Li,[2] Tim Linden,[2] and Nirmal Raj[3]

[1] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario, N2L 2Y5, Canada*
[2] *CCAPP and Department of Physics, The Ohio State University, Columbus, OH, 43210, USA*
[3] *Department of Physics, University of Notre Dame, Notre Dame, IN, 46556, USA*

We identify a largely model-independent signature of dark matter interactions with nucleons and electrons. Dark matter in the local galactic halo, gravitationally accelerated to over half the speed of light, scatters against and deposits kinetic energy into neutron stars, heating them to infrared blackbody temperatures. The resulting radiation could potentially be detected by the James Webb Space Telescope, the Thirty Meter Telescope, or the European Extremely Large Telescope. This mechanism also produces optical emission from neutron stars in the galactic bulge, and X-ray emission near the galactic center, because dark matter is denser in these regions. For GeV - PeV mass dark matter, dark kinetic heating would initially unmask any spin-independent or spin-dependent dark matter-nucleon cross-sections exceeding $2 \times 10^{-45}$ cm$^2$, with improved sensitivity after more telescope exposure. For lighter-than-GeV dark matter, cross-section sensitivity scales inversely with dark matter mass because of Pauli blocking; for heavier-than-PeV dark matter, it scales linearly with mass as a result of needing multiple scatters for capture. Future observations of dark sector-warmed neutron stars could determine whether dark matter annihilates in or only kinetically heats neutron stars. Because inelastic inter-state transitions of up to a few GeV would occur in relativistic scattering against nucleons, elusive inelastic dark matter like pure Higgsinos can also be discovered.

1704.01577

# Detector


Detect radio pulses to identify nearby, *old* neutron stars.

Expect: ~100 w/in 50 pc

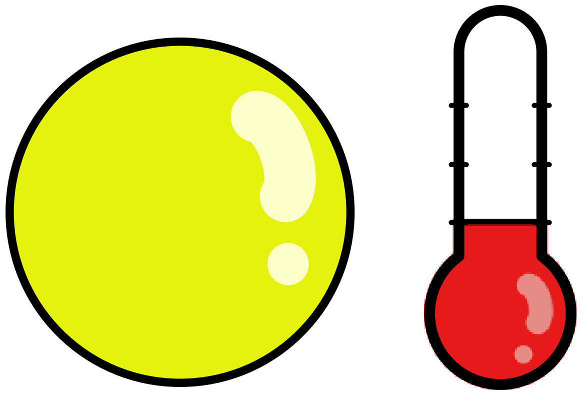
Measure temperature with infrared telescopes.

For 2σ:

$$10^5 \text{ sec} \left( \frac{d}{10\text{pc}} \right)^4$$

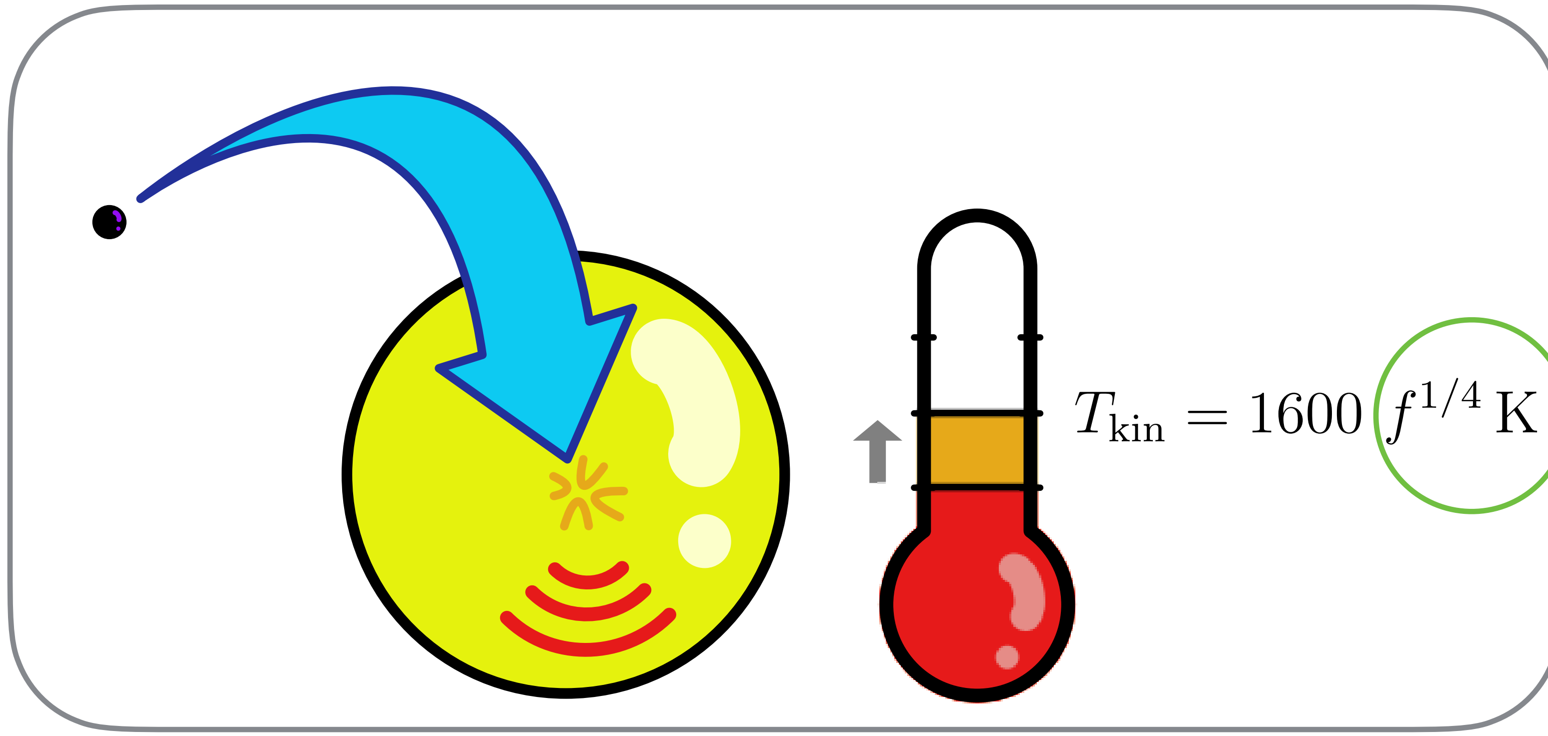
… and you may only need one.

Image credits: FAST, JWST websites

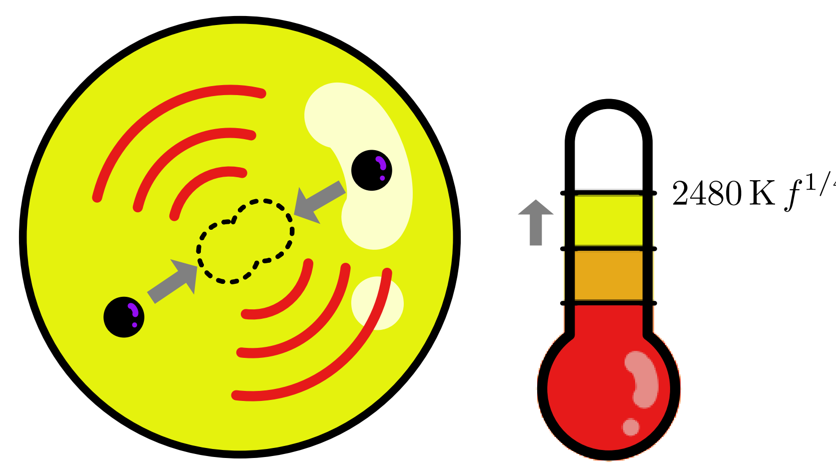


1000 K (20 Myr) (old and cold)

100 K (Gyr)

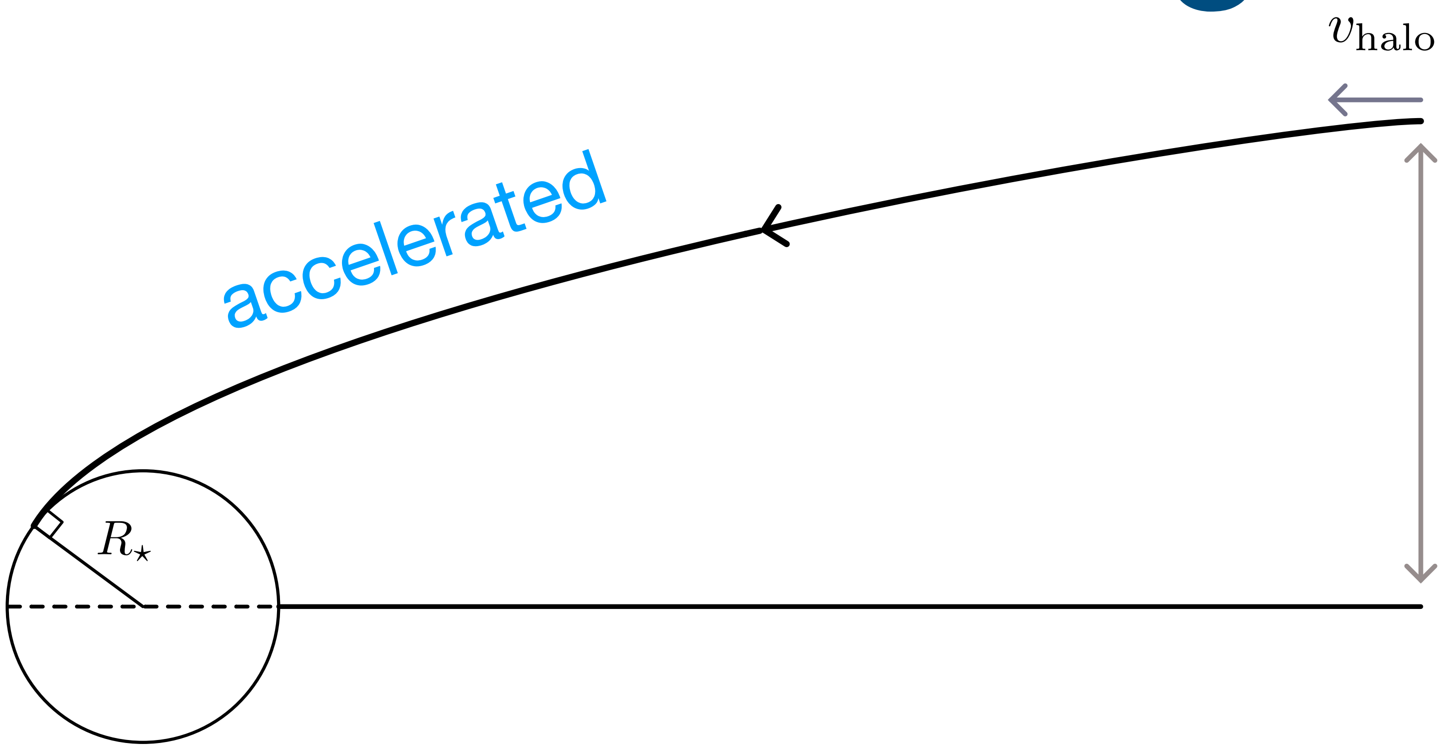



*f*: capture fraction relative to geometric cross section; same as the *f* in Jason's talk




extra credit
not in this talk

# How much heating?



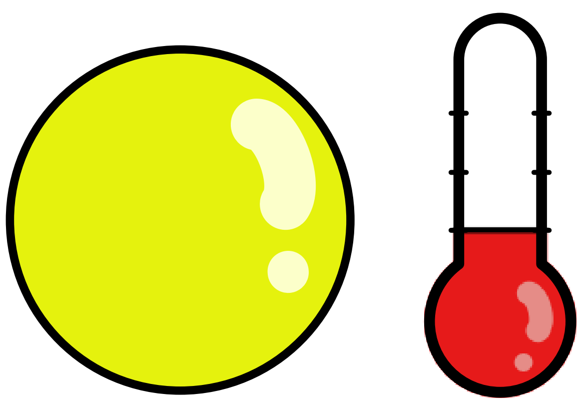

$$b_{\rm max} = R_\star \frac{v_{\rm esc}}{v_{\rm halo}} \gamma_{\rm esc}$$

$$\dot{M}_\chi = \pi b_{\rm max}^2 v_{\rm esc} \rho_\chi \approx 3.1 \times 10^{25}\ \frac{\rm GeV}{\rm s} \approx 55\ \frac{\rm g}{\rm s}\ .$$

$$\dot{K} = (\gamma_{\rm esc} - 1)\, \dot{M} f \approx 6.5 \times 10^{24}\ {\rm GeV\ s^{-1}}\ .$$

$$T_{\rm kin} = 1600\, f^{1/4}\, {\rm K}$$

vs.



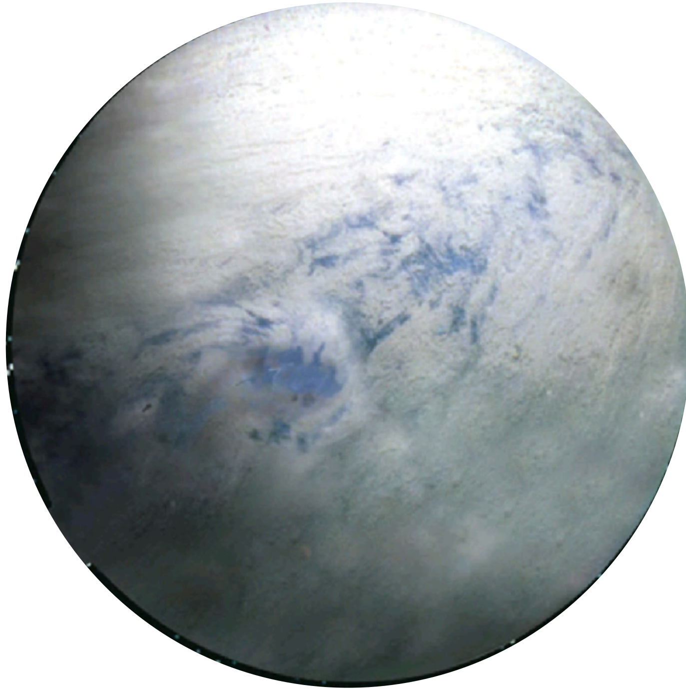
No heating:

100 K (Gyr)

$b_{max}$: see your favorite GR text

no DM heating **Hoth**

DM kinetic heating **Mustafar**



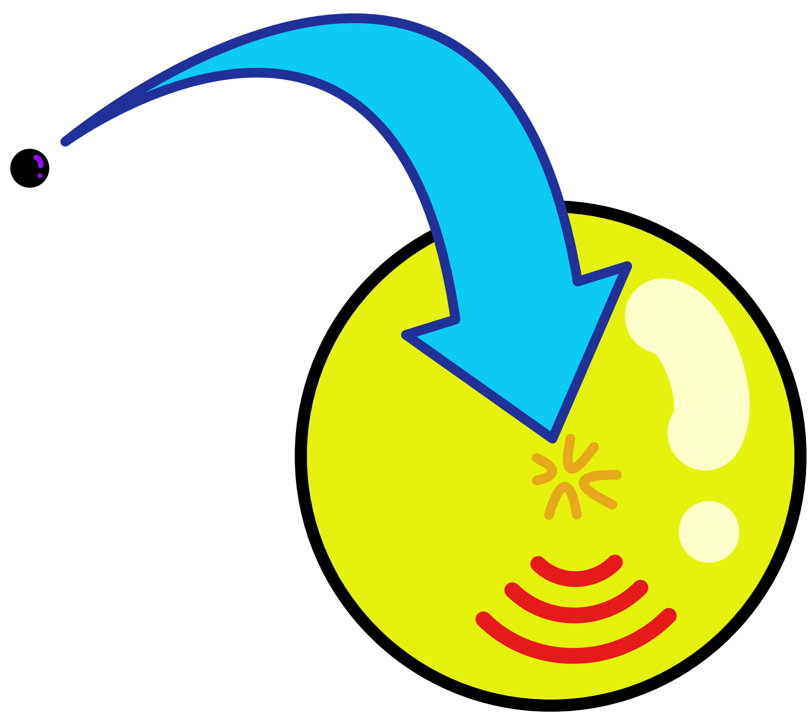
T ~ 100 K

T ~ 1600 K

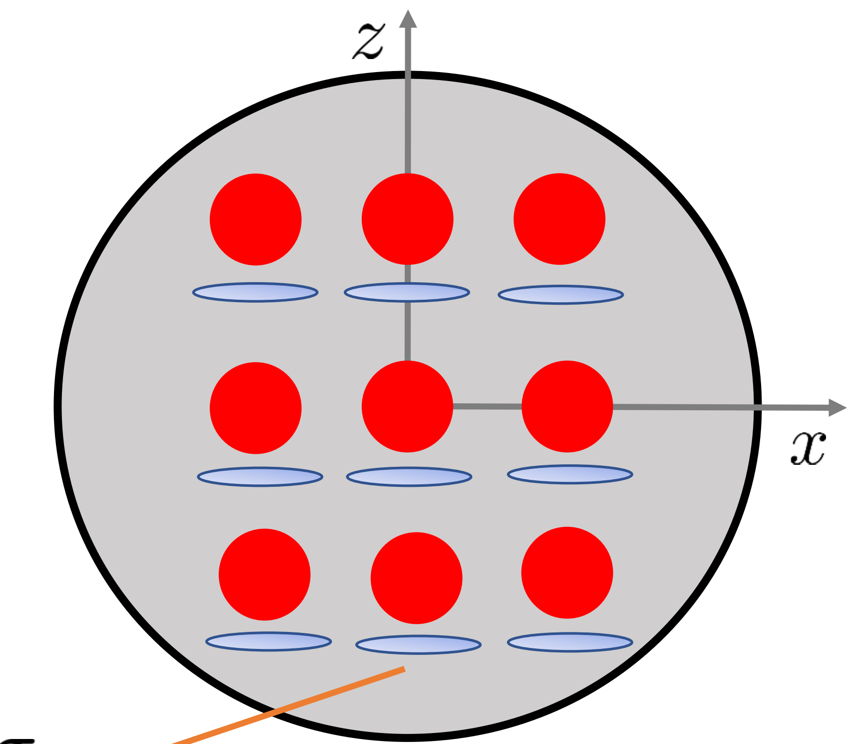
Images: Wookiepedia, *The Little Prince*

# Capture Cross Section

## Measure relative to geometric cross section



$$T_{\rm kin} = 1600\, f^{1/4}\ \mathrm{K}$$



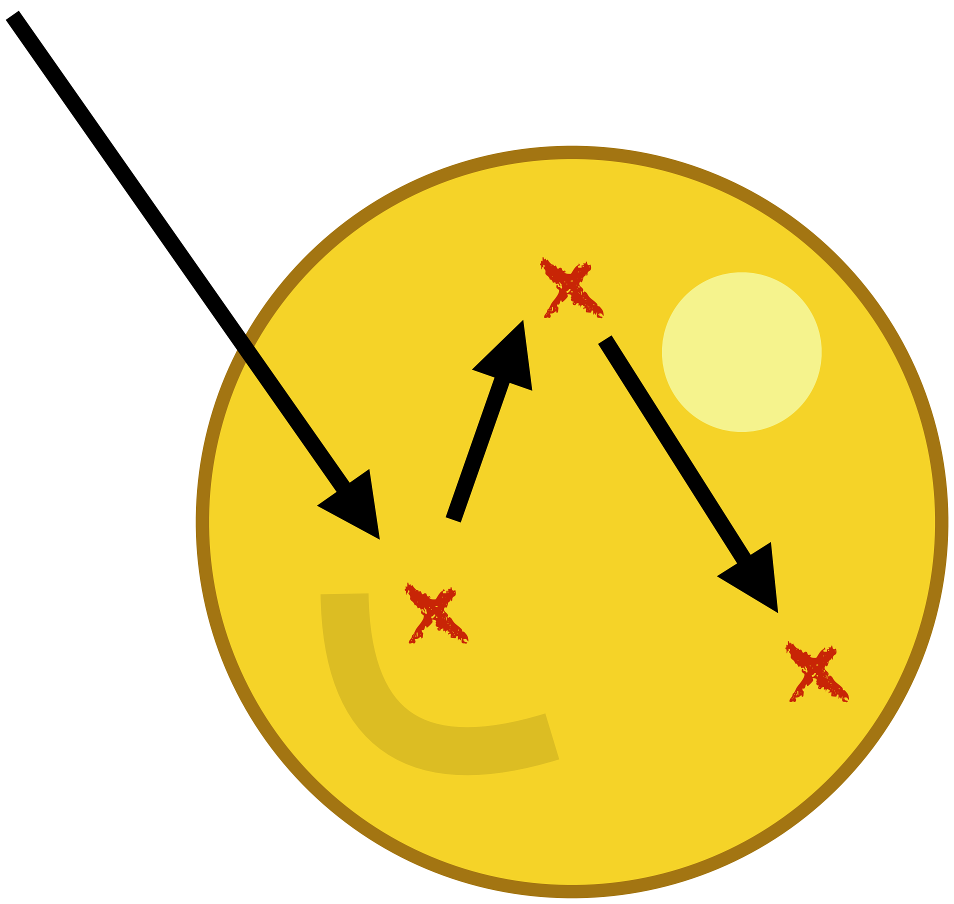

$$f = \frac{\sigma_{\chi n}}{\sigma_{\rm geo}}$$


Breaks down for extreme masses

Image (right): Aniket Joglekar

# Break Down #1

## Not enough ΔE



Heavy dark matter does not transfer enough energy.
(c.f. why we like Xenon)

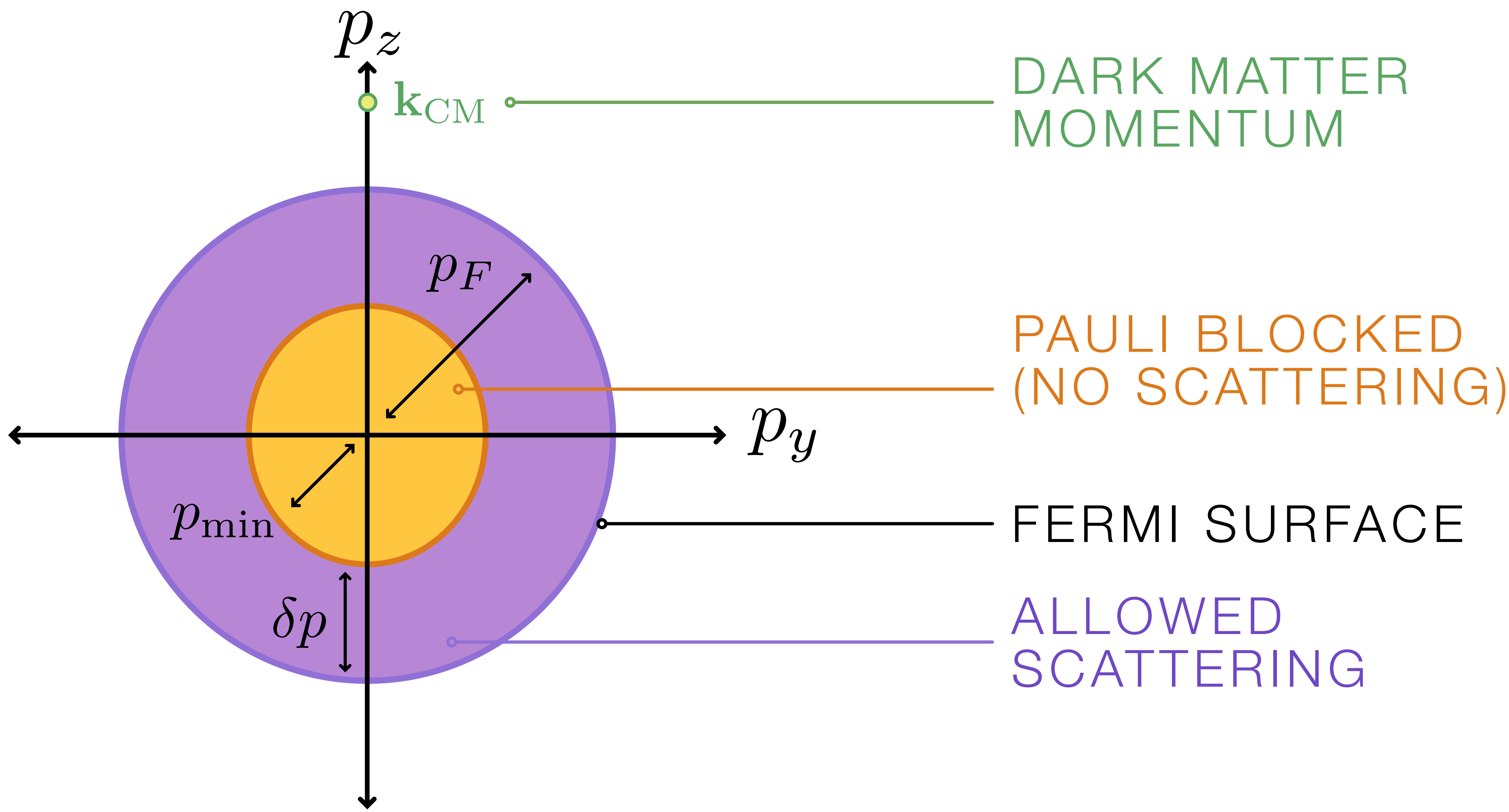
…multiple scatters required to capture in the star.
c.f. Jason & Chris talks

Non-relativistic limit; n.b. saturates in large $m_\chi$ limit

$$\Delta E = \frac{m_\mathrm{T} m_\chi^2}{m_\chi^2 + m_\mathrm{T}^2 + 2\gamma_\mathrm{esc} m_\chi m_\mathrm{T}} \frac{v_\mathrm{esc}^2}{1 - v_\mathrm{esc}^2} (1 - \cos\psi)$$

# Break Down #2

## Pauli Blocking




NON-RELATIVISTIC

MOMENTUM SPACE

Light DM can't overcome Pauli blocking

# Break Down of the geometric σ

$m_\chi$

**Beyond threshold σ, no increase in capture**

TOO HEAVY

$$\sigma_{\text{thres.}}^{\text{multi}} \approx \frac{m_\chi}{10^6\ \text{GeV}} \sigma_{\text{thres.}}$$

not enough energy transfer

$10^6$ GeV

GOLDILOCKS

$$\sigma_{\text{thres.}} = \frac{\text{geometric cross section}}{\text{number of targets}} = \pi R_\star^2 \frac{m_{\text{T}}}{M_\star}$$

GeV

TOO LIGHT

$$\sigma_{\text{thres.}}^{\text{Pauli}} = \frac{1}{3}\frac{p_F}{\delta p}\sigma_{\text{thres.}} \approx \frac{\text{GeV}}{m_\chi}\sigma_{\text{thres.}}$$

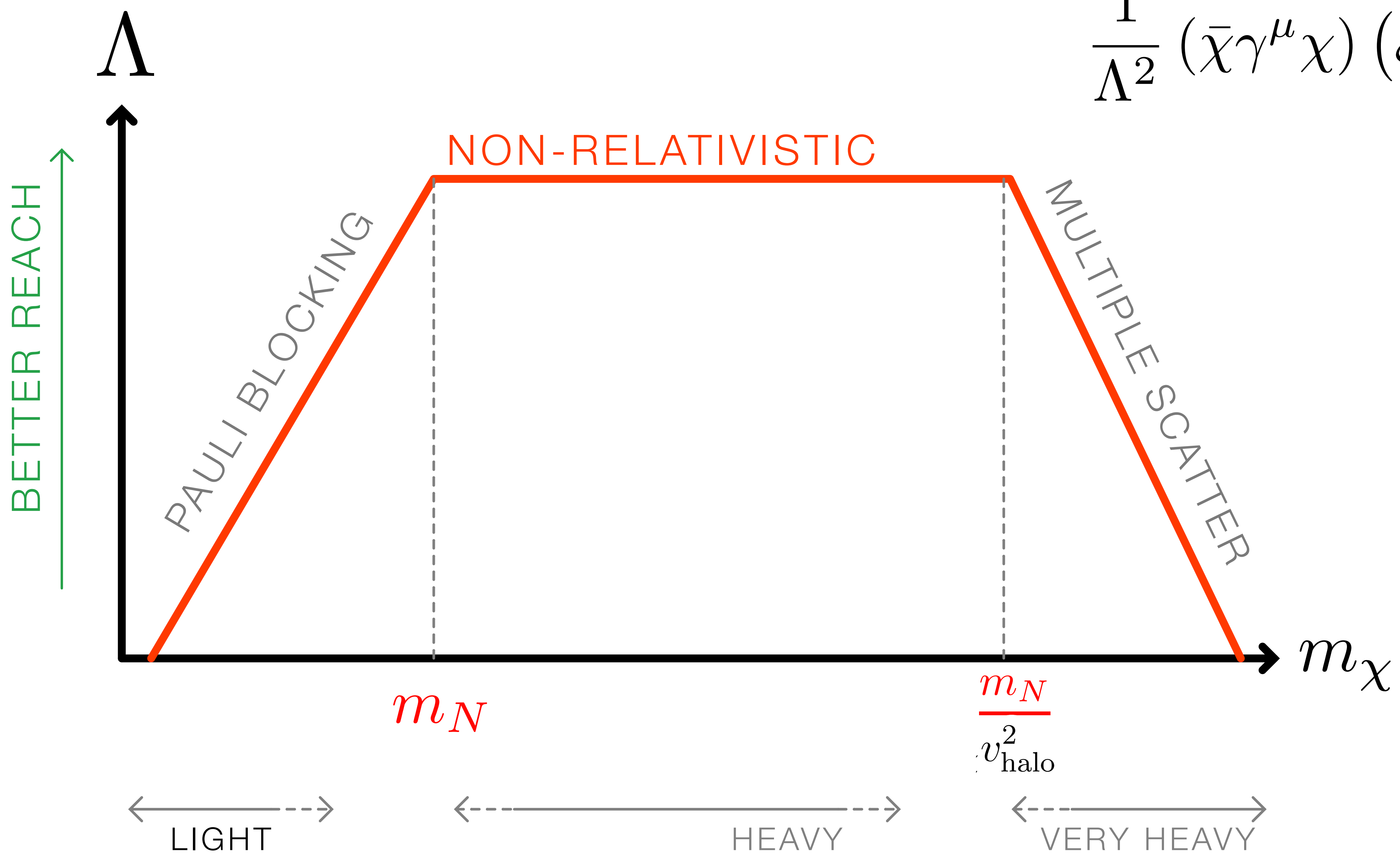
not enough phase space

$$\delta p \approx (\gamma_\chi - 1) m_\chi v_\chi$$

# The reach of kinetic heating

## Standard Picture

$$\frac{1}{\Lambda^2}\left(\bar{\chi}\gamma^\mu\chi\right)\left(\bar{\xi}\gamma_\mu\xi\right)$$



# What else can we do?

(leptophilic)


Julie Peasley, particlezoo.net

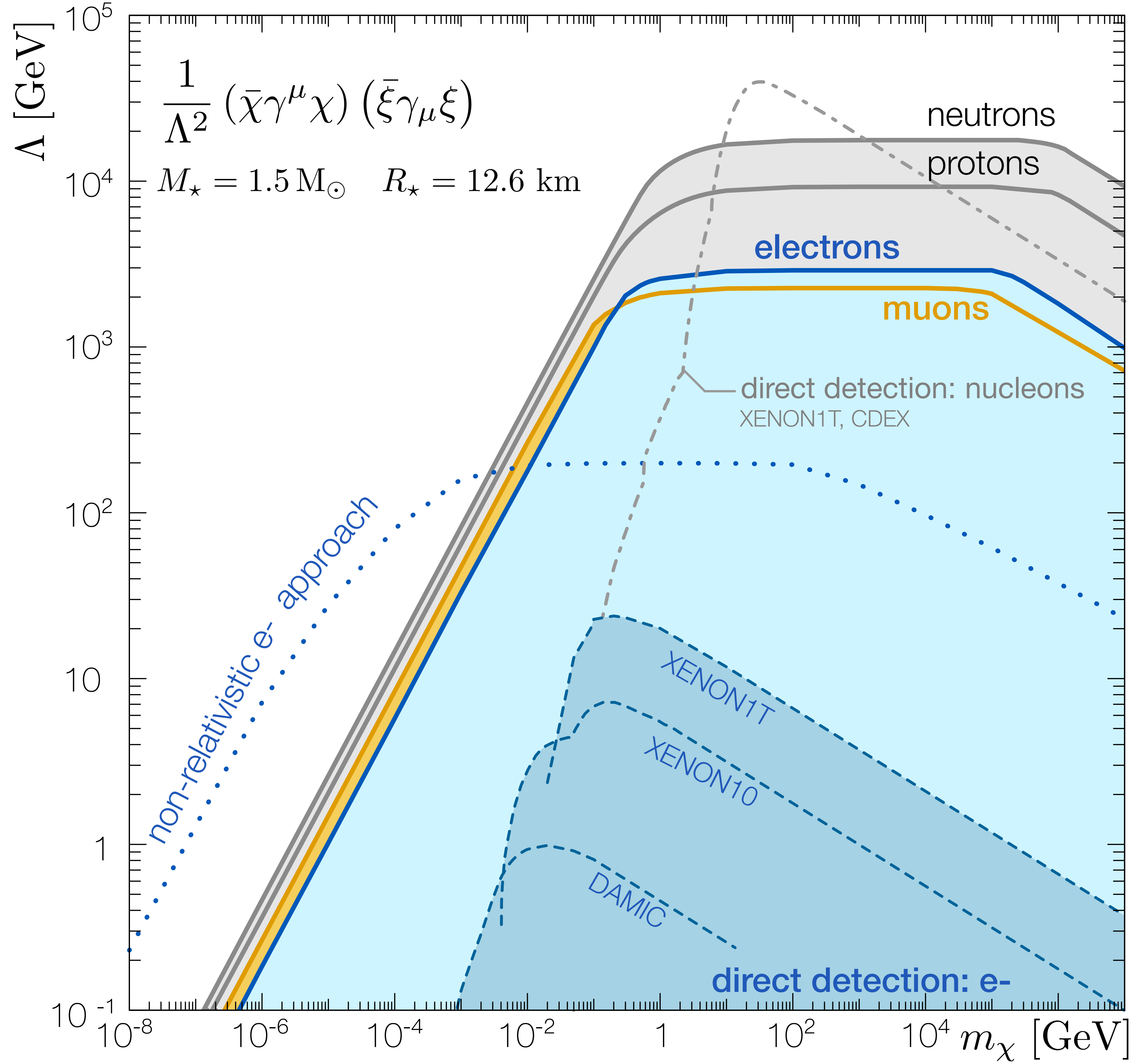
# Result

**Surprising reach of relativistic electrons**

can beat lepton direct detection

curiously similar reach to neutrons

dotted line?




Joglekar, Raj, FT, Yu, 1911.13293

# Non-relativistic estimate

treat electrons as little neutrons

## Capture of Leptophilic Dark Matter in Neutron Stars

Nicole F. Bell,[a] Giorgio Busoni[b] and Sandra Robles[a]

**Abstract.** Dark matter particles will be captured in neutron stars if they undergo scattering interactions with nucleons or leptons. These collisions transfer the dark matter kinetic energy to the star, resulting in appreciable heating that is potentially observable by forthcoming infrared telescopes. While previous work considered scattering only on nucleons, neutron stars contain small abundances of other particle species, including electrons and muons. We perform a detailed analysis of the neutron star kinetic heating constraints on leptophilic dark matter. We also estimate the size of loop induced couplings to quarks, arising from the exchange of photons and Z bosons. Despite having relatively small lepton abundances, we find that an observation of an old, cold, neutron star would provide very strong limits on dark matter interactions with leptons, with the greatest reach arising from scattering off muons. The projected sensitivity is orders of magnitude more powerful than current dark matter-electron scattering bounds from terrestrial direct detection experiments.

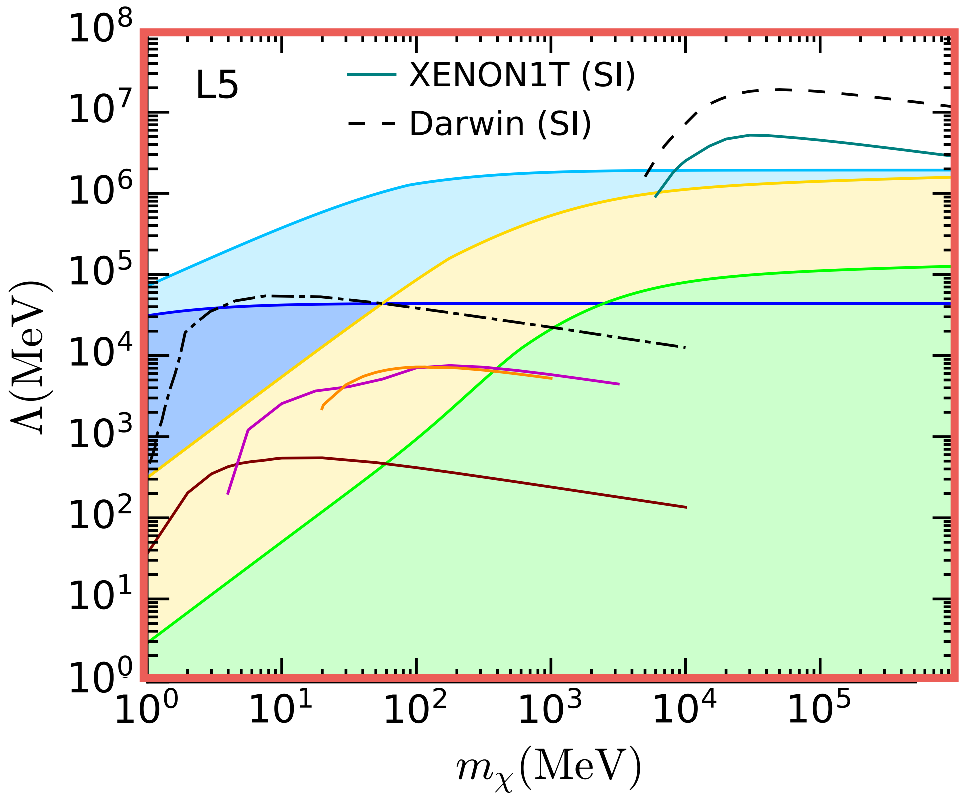
KILLER APP

leptophilic dark matter

result: muons very promising, electrons are okay

1904.09803

# NR vs full relativistic



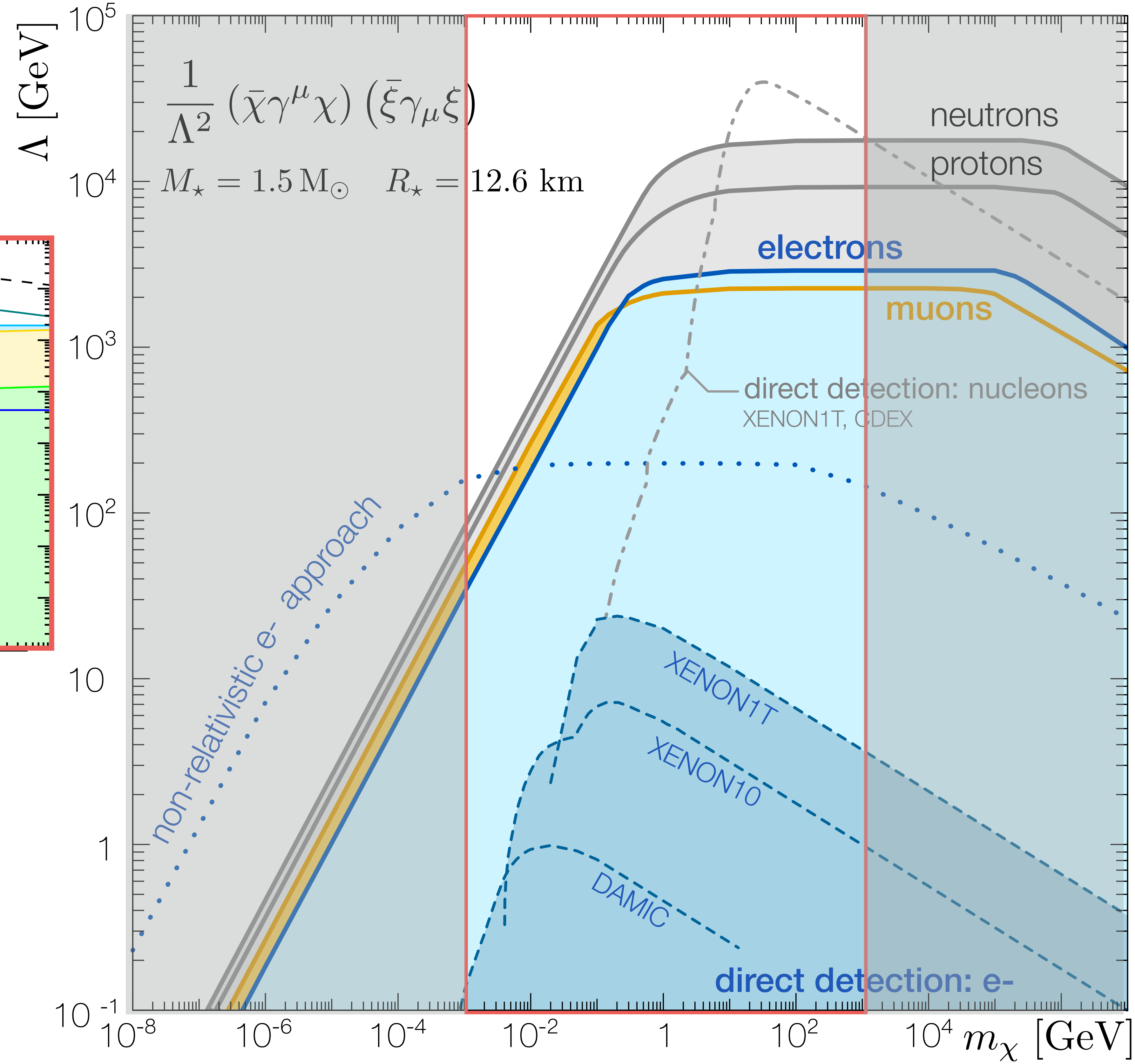

NS (BSk24-1)

$e^-$ $T_{kin}^{\infty,th} = 1700\ K$

$\mu^-$ $T_{kin}^{\infty,th} = 1700\ K$

$n$ $T_{kin}^{\infty,th} = 1700\ K$

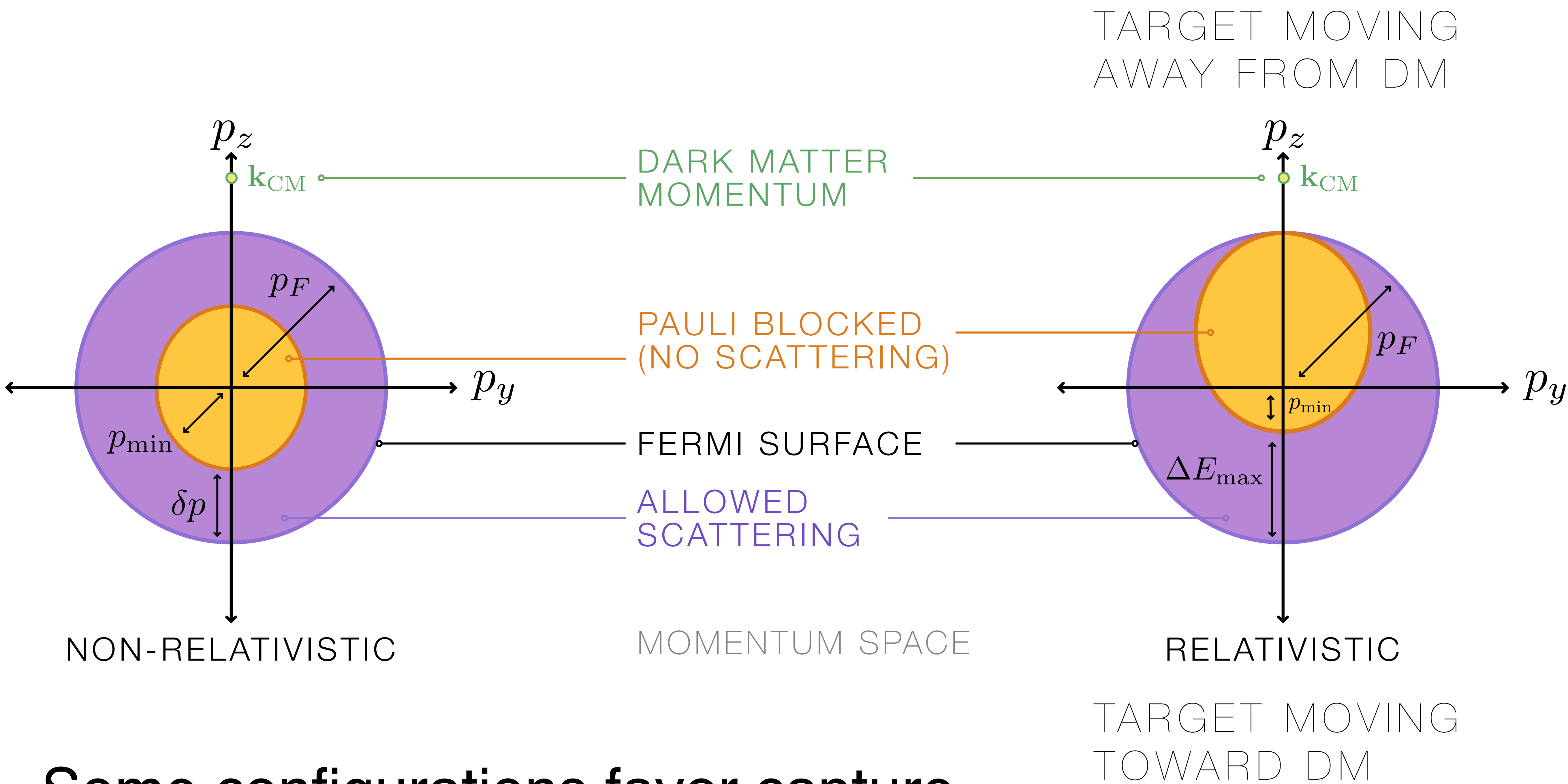
$p$ $T_{kin}^{\infty,th} = 1700\ K$




Bell et al. 1904.09803, Joglekar et al. 1911.13293

# What was missing #1

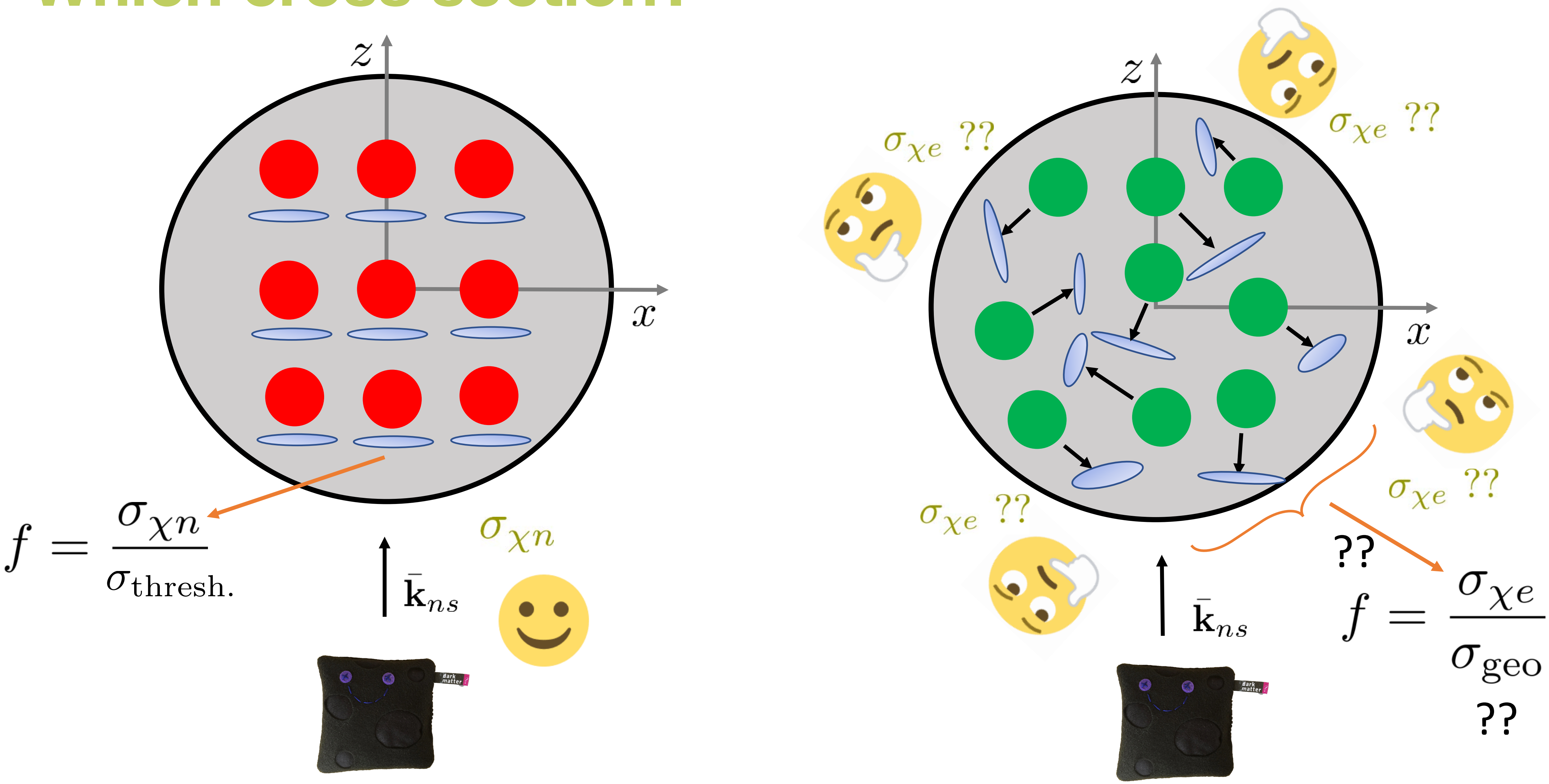
## Pauli Blocking for Relativistic Targets




Some configurations favor capture.

# What was missing #2

## Which cross section?

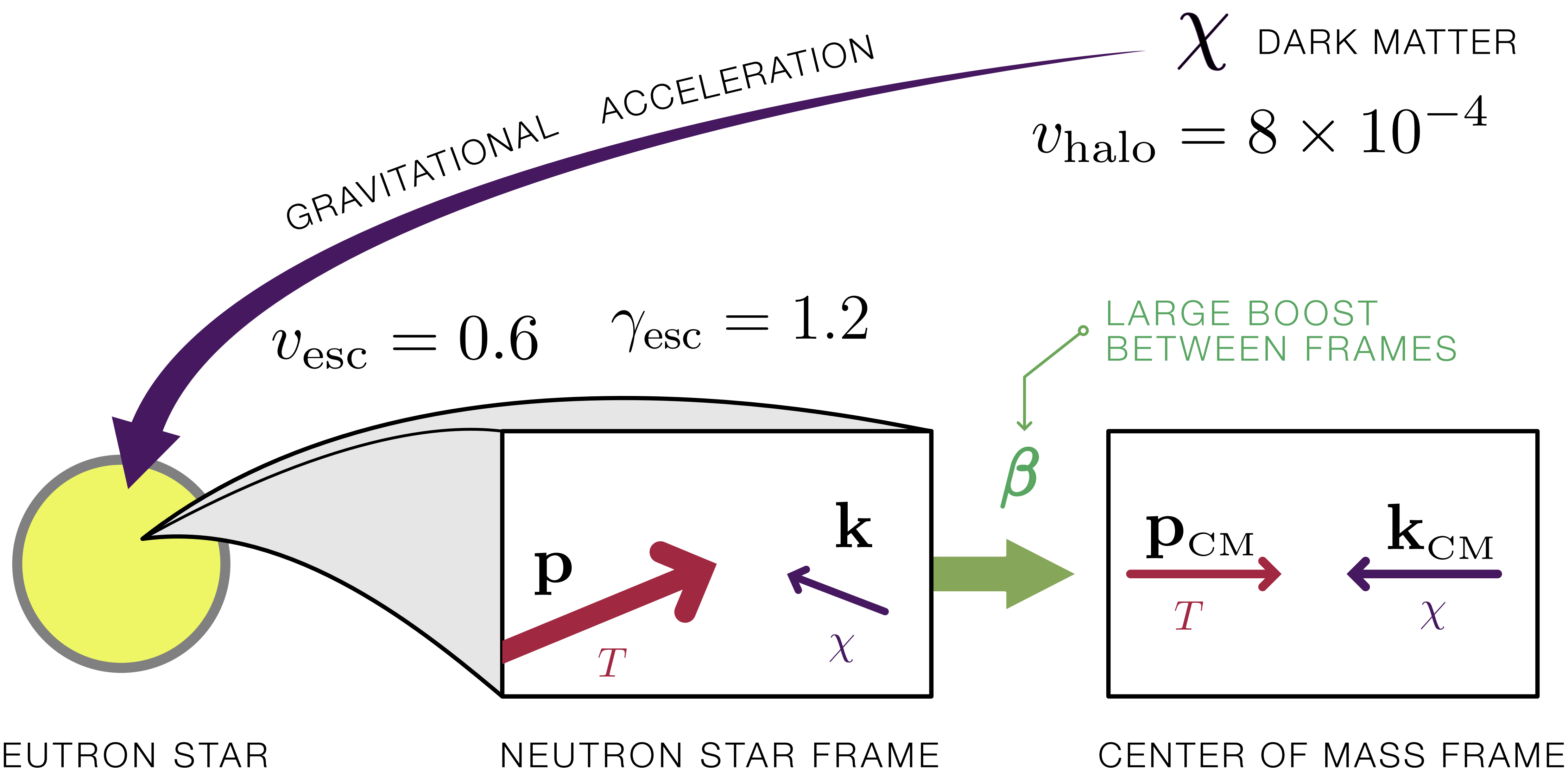



Cross section depends on kinematics (frame)

Images: Aniket Joglekar

# A matter of frame

## Not a fixed target experiment



# Formalism

## Lorentz Invariant

# OF SCATTERS:

$$d\nu = d\sigma\, v_{\rm rel}\, dn_{\rm T}\, dn_\chi\, \Delta V\, \Delta t$$

$$\Delta t \approx 3.2\ R_\star$$

CAPTURE EFFICIENCY

$$df = \left.\frac{d\nu}{dN_\chi}\right|_{\rm capture}$$

CAPTURE CONDITIONS
ΔE & Pauli Block

# OF INCIDENT DM:

$$dN_\chi = dn_\chi \Delta V$$

df, dv, dN are each separately Lorentz invariant

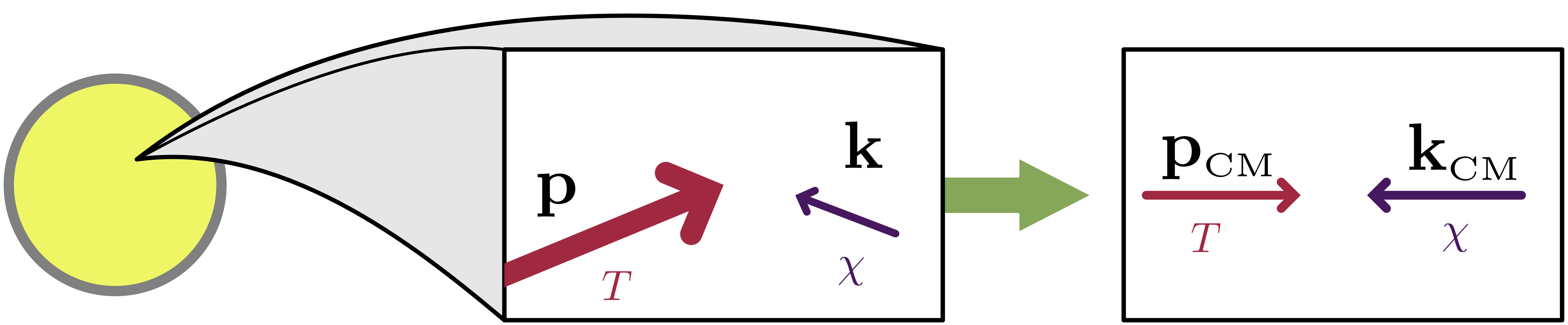
# Frames & Lorentz Invariance

LORENTZ INVARIANT

$$df = d\sigma\, v_{\rm rel}\; dn_{\rm T}\, \Delta t \,\big|_{\rm capture}$$

CENTER OF MASS FRAME?

NEUTRON STAR FRAME?



# Reminder: Möller velocity

RELATIVISTIC
RELATIVE VELOCITY

$$d\sigma \, v_{\text{rel}} = d\sigma_{\text{CM}} \, v_{\text{Møl}}$$

FRAME | CM FRAME | FRAME

ANY FRAME

$$v_{\text{Møl}} = \frac{\sqrt{(p\cdot k)^2 - m_{\text{T}}^2 m_\chi^2}}{E_p E_k}$$

see, e.g. Cannoni 1605.00569 (but you probably saw this first in Gelmini & Gondolo "Cosmic abundances of stable particles," 1990)

# Lorentz Invariant Capture Efficiency

LORENTZ
INVARIANT

$$df = d\sigma\, v_{\text{rel}}\ dn_{\text{T}}\, \Delta t\, \big|_{\text{capture}}$$

CENTER OF MASS FRAME

NEUTRON STAR FRAME

$$df = d\sigma_{\text{CM}}\ v_{\text{Møl}}\, dn_{\text{T}}\, \Delta t\, \big|_{\text{capture}}$$

CENTER OF MASS FRAME

NEUTRON STAR FRAME

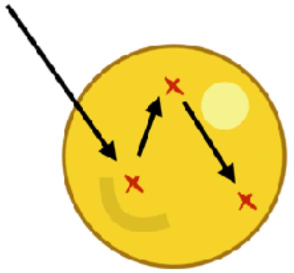
# Apply Capture Conditions

$$df = d\sigma_{\text{CM}}\, v_{\text{Møl}}\, dn_{\text{T}}\, \Delta t \,\Big|_{\text{capture}}$$

$$df = \sum_{N_{\text{hit}}} d\sigma_{\text{CM}}\, v_{\text{Møl}}\, dn_{\text{T}}\, \frac{\Delta t}{N_{\text{hit}}}$$

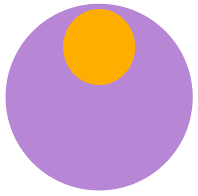
$$\times\ \Theta\left(\Delta E - \frac{E_{\text{halo}}}{N_{\text{hit}}}\right) \Theta\left(\frac{\Delta E_{\text{min}}}{N_{\text{hit}}+1} - \Delta E\right)$$

$$\times\ \Theta\left(\Delta E + E_p - E_{\text{F}}\right)$$



energy transfer leads to capture in N hits

Pauli blocking of final state



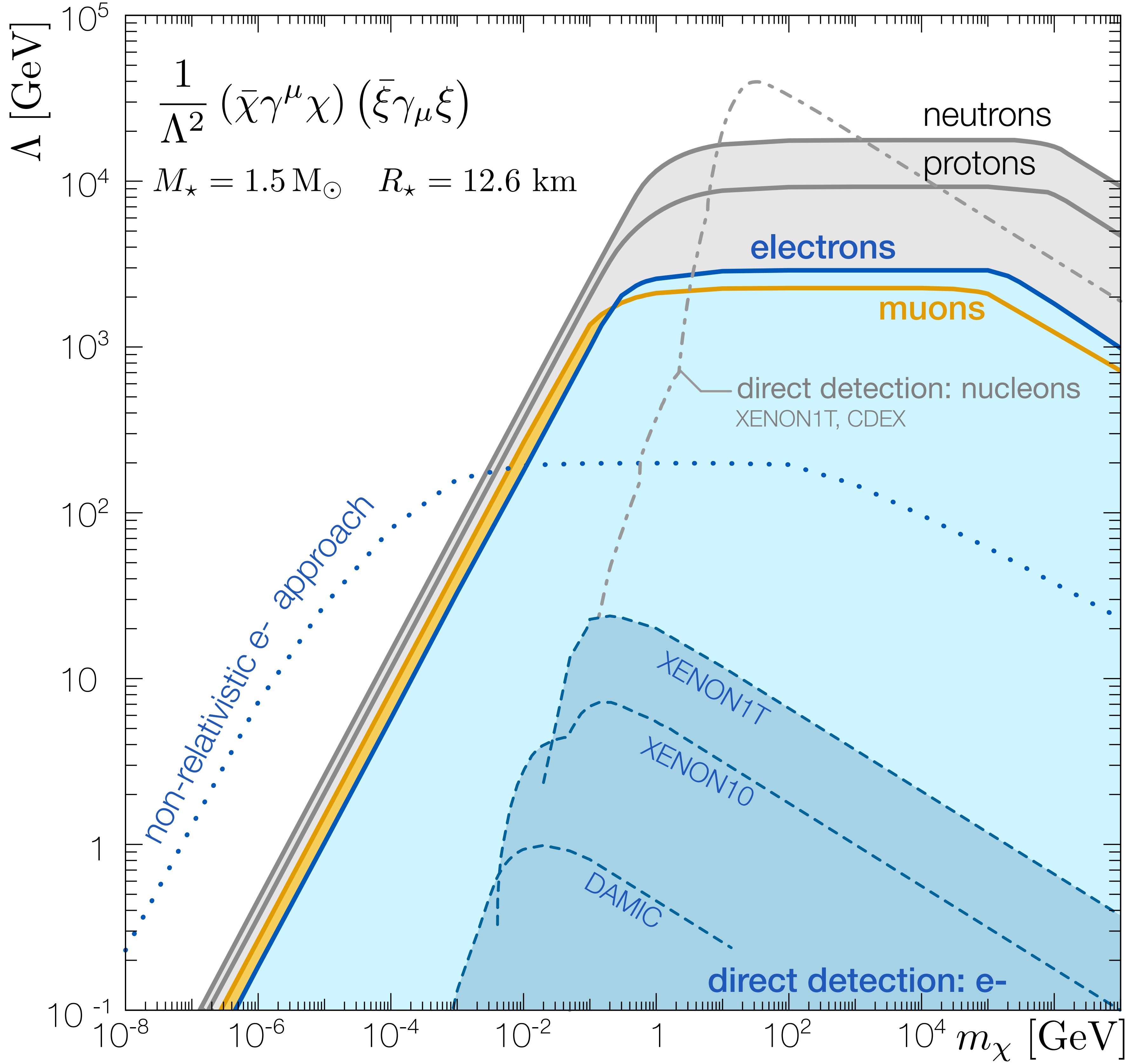
# Result

**Surprising reach of relativistic electrons**

can beat lepton direct detection

curiously similar reach to neutrons

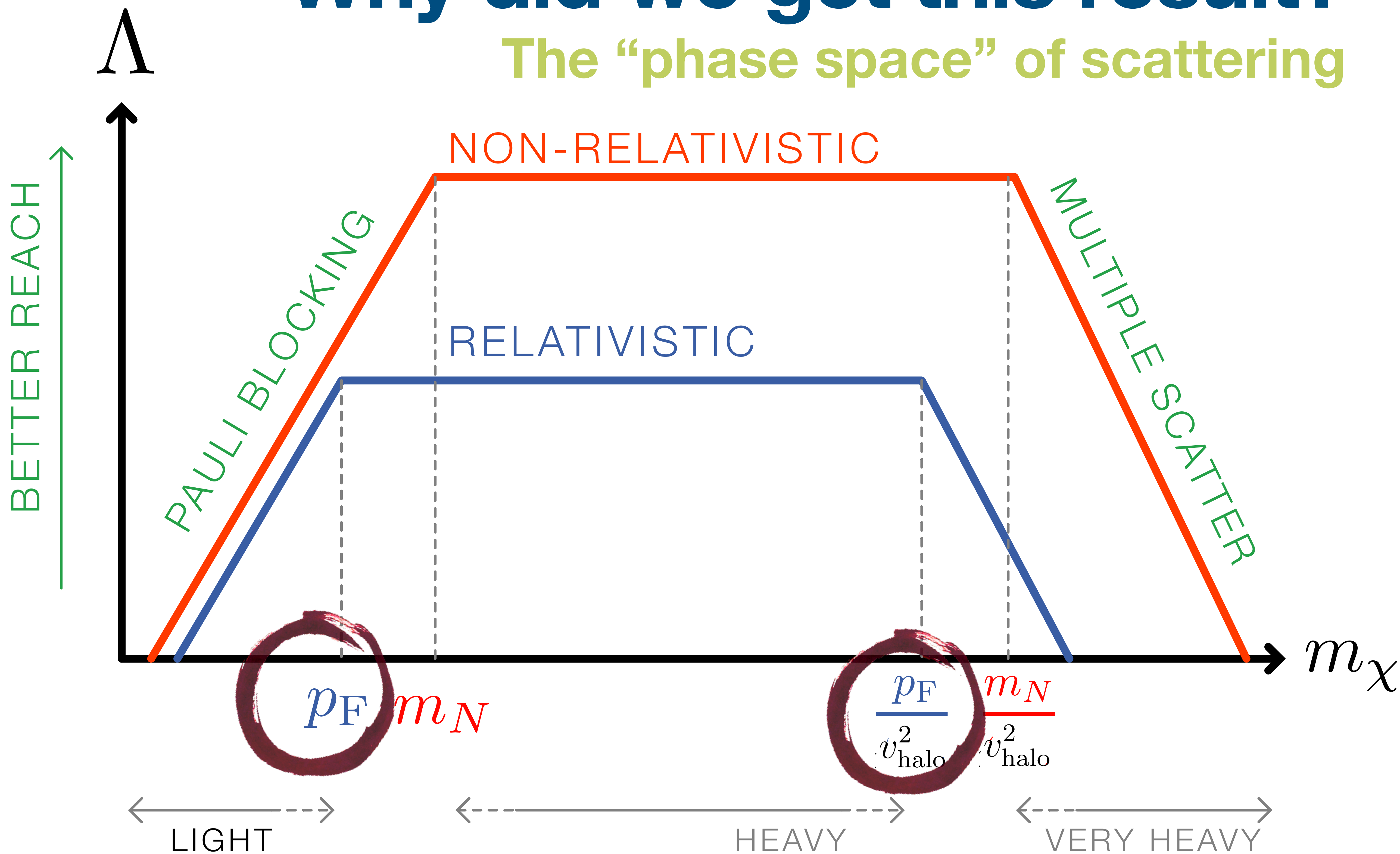
dotted line?




Joglekar, Raj, FT, Yu, 1911.13293

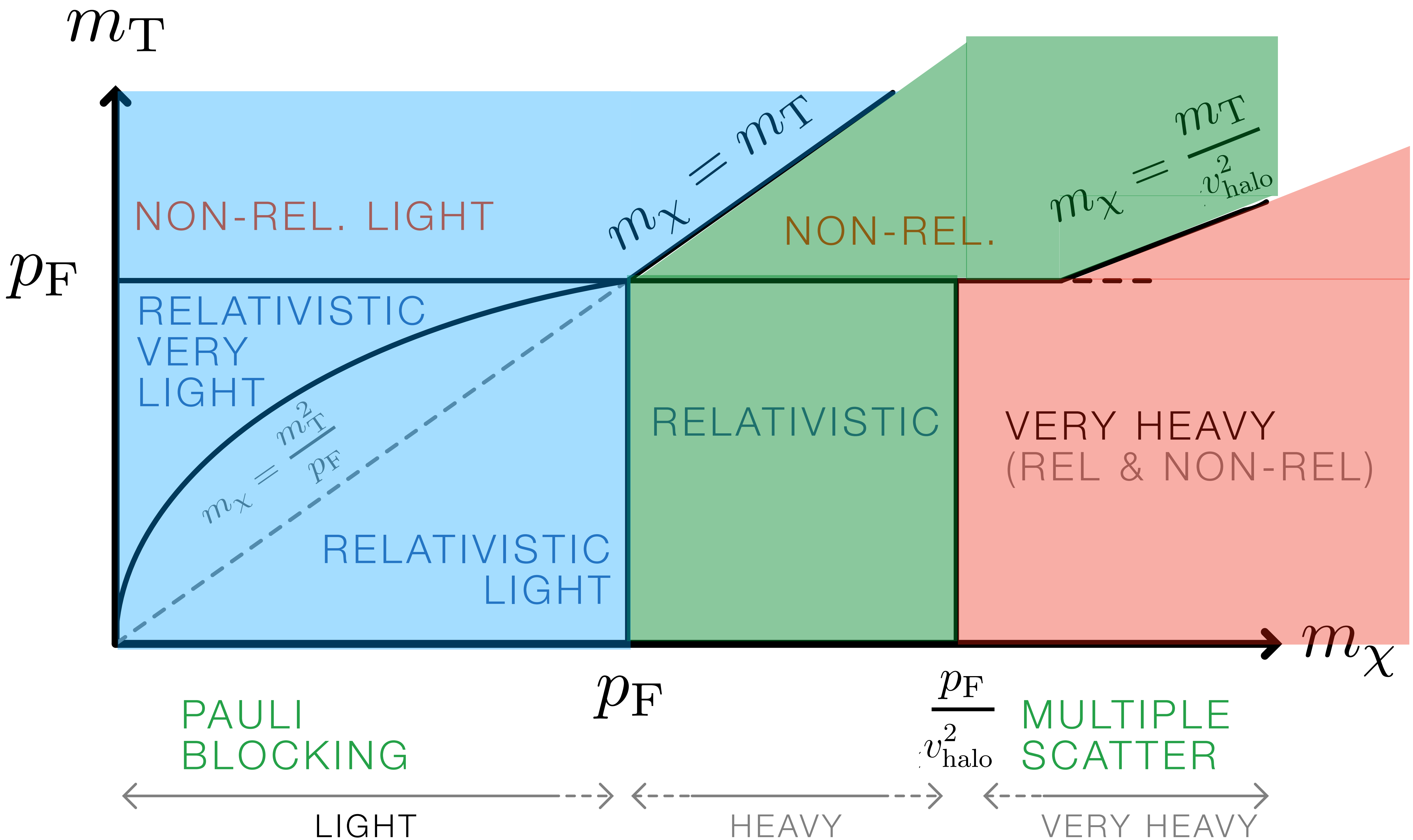
# Why did we get this result?

## The "phase space" of scattering




apples to oranges comparison

# Phase space of neutron star scattering



# What it all boils down to

## Understanding behavior of relativistic results

1. How does the differential cross section scale with $m_\chi$?
2. Is the phase space suppressed with $m_\chi$?
3. Does capture require multiple scatters?

capture efficiency

$$f \sim \frac{1}{N_{\text{hit}}} \int_{\cos\psi_{\text{max}}}^{1} d\cos\psi \int_{p_{\text{min}}}^{p_{\text{F}}} \frac{p^2 dp}{p_{\text{F}}^3} \frac{|\mathcal{M}|^2}{s}$$

For each operator in each regime, check scaling with dark matter mass.

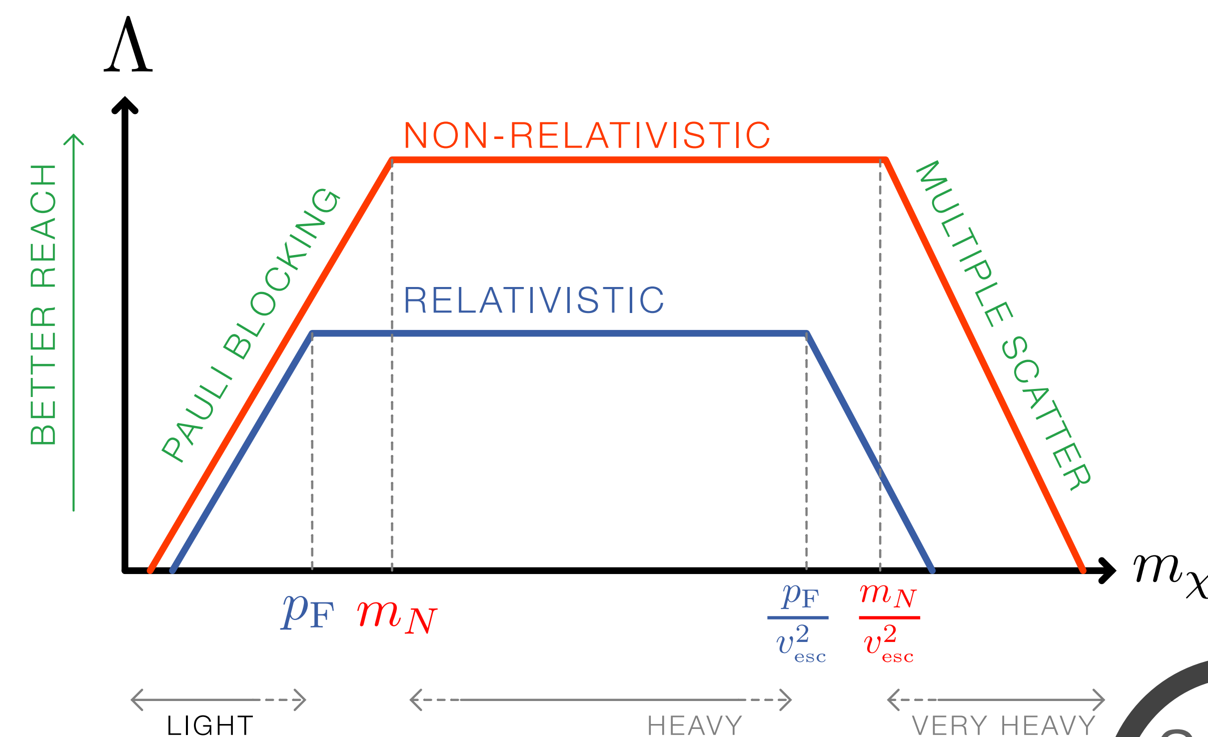
# Into the weeds…

$$\frac{|\mathcal{M}|^2}{s} \approx \frac{m_\chi^2 E_p^2}{s\Lambda^4} \approx \frac{m_\chi^2 m_\text{T}^2}{s\Lambda^4}\left(1+\frac{E_\text{F}^2}{m_\text{T}^2}\right)$$

| TARGET | DARK MATTER | $1-\cos\psi$ | $\Delta p/p_\text{F}$ | $s^{-1}$ | $\lvert\mathcal{M}\rvert^2$ | $N_\text{hit}^{-1}$ | $f$ |
|---|---|---|---|---|---|---|---|
| | VERY HEAVY | | | $m_\chi^{-2}$ | $m_\chi^2$ | $m_\chi^{-1}$ | $m_\chi^{-1}$ |
| NON-REL | HEAVY | | | $m_\chi^{-2}$ | $m_\chi^2$ | | $1$ |
| REL | HEAVY | | | $m_\chi^{-2}$ | $m_\chi^2$ | | $1$ |
| NON-REL | LIGHT | | $m_\chi$ | | $m_\chi^2$ | | $m_\chi^3$ |
| REL | VERY LIGHT | | $m_\chi$ | | $m_\chi^2$ | | $m_\chi^3$ |
| REL | LIGHT | $m_\chi$ | $m_\chi$ | $m_\chi^{-1}$ | $m_\chi^2$ | | $m_\chi^3$ |

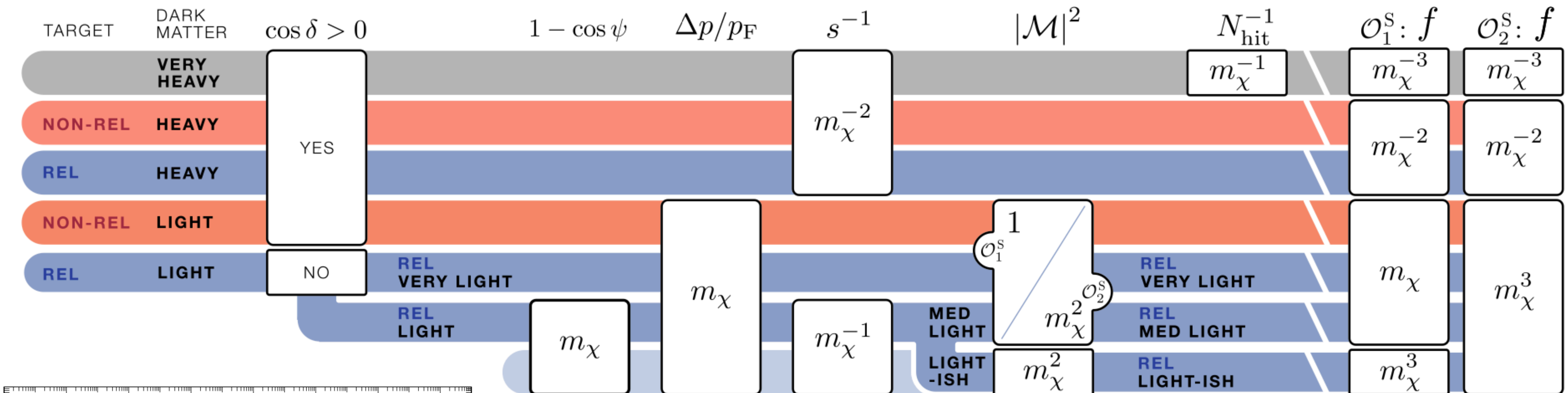
$$f \sim \frac{1}{N_\text{hit}}\int_{\cos\psi_\text{max}}^{1} d\cos\psi \int_{p_\text{min}}^{p_\text{F}} \frac{p^2\,dp}{p_\text{F}^3}\,\frac{|\mathcal{M}|^2}{s}$$

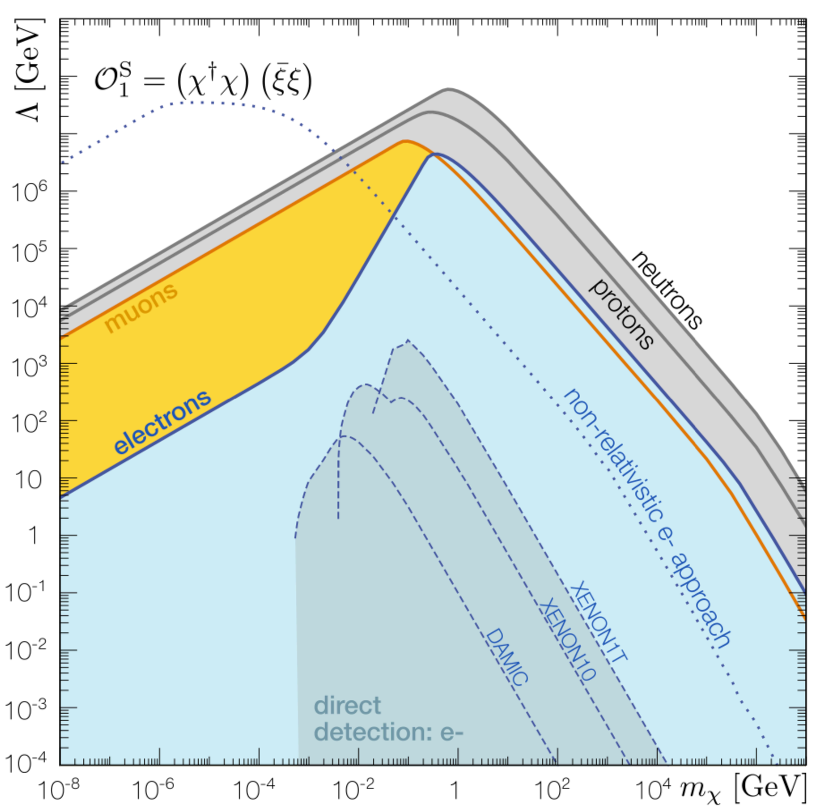
$$s \approx \begin{cases} m_\text{T}^2 & m_\chi \ll m_\text{T}^2/p_\text{F} \\ m_\chi E_p & m_\text{T}^2/E_\text{F} \ll m_\chi \ll p_\text{F} \\ m_\chi^2 & p_\text{F} \ll m_\chi \end{cases}$$



# … some cases are more complex

## But you we wrote up heuristic flow charts




In the this operator there is no plateau.

The electrons are relativistic and have an additional regime where dark matter is "light-ish" compared to the Fermi momentum.

Verified in Monte Carlo calculation (left)

2004.09539

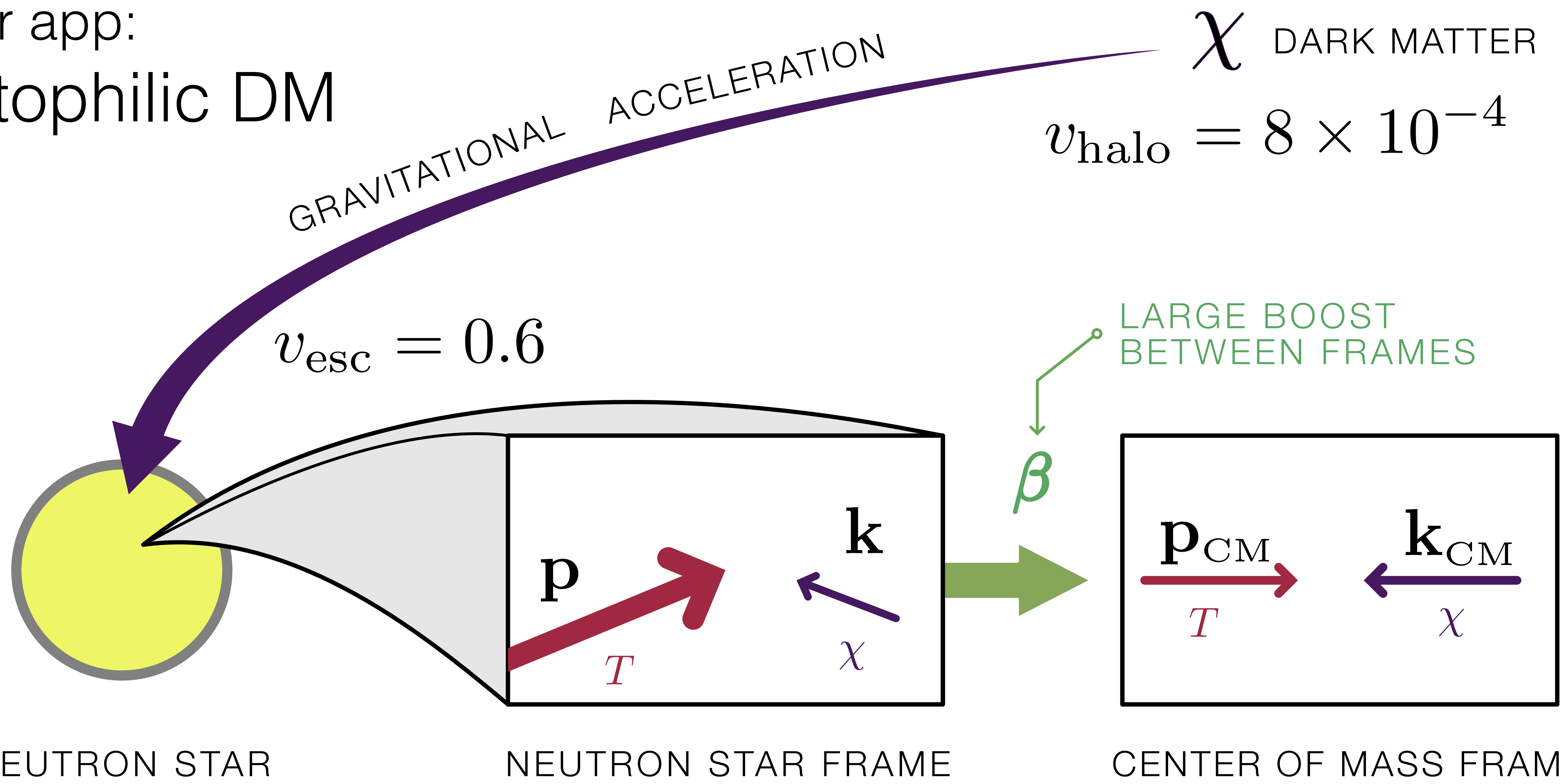
# Thanks!

Opportunities for *direct detection* with neutron stars

New formalism for relativistic, degenerate targets

Killer app:

leptophilic DM




**arXiv:1911.13293 & arXiv: 2004.09539**

# Additional slides

## … someone asked a clever question

# Möller velocity review

$$v_{\text{rel}} = \left.\frac{k}{E_k}\right|_{\text{T}} = \left.\frac{\sqrt{E_k^2 - m_\chi^2}}{E_k}\right|_{\text{T}} = \frac{\sqrt{(p\cdot k)^2 - m_{\text{T}}^2 m_\chi^2}}{p\cdot k} \qquad v_{\text{Møl}} = \frac{\sqrt{(p\cdot k)^2 - m_{\text{T}}^2 m_\chi^2}}{E_p E_k}$$

---

$$\mathcal{R} = \frac{d\nu}{\Delta V \Delta t} = (d\sigma\, v_{\text{rel}}\, dn_{\text{T}}\, dn_\chi)_{\text{T}} \quad \longrightarrow \quad A = \frac{p\cdot k}{E_T E_\chi}(d\sigma\, v_{\text{rel}})_{\text{T}}$$

$$\mathcal{R} = (A\, dn_{\text{T}} dn_\chi)_F = \left(A \frac{E_T E_\chi}{m_{\text{T}} m_\chi}\right)_F d\hat{n}_{\text{T}} d\hat{n}_\chi$$

---

$$\mathcal{R} = d\sigma_{\text{CM}} \left(\frac{p\cdot k}{E_T E_\chi} v_{\text{rel}}\right) dn_{\text{T}} dn_\chi = d\sigma_{\text{CM}}\, v_{\text{Møl}}\, dn_{\text{T}} dn_\chi$$

see, e.g. Cannoni 1605.00569 for a review

# Möller velocity: in a nutshell

## From a boost of the "fixed-target" lab frame

In the rest frame of one particle 1 we already said that $v_{\rm rel} = |\boldsymbol{v}_2|$. The 4-momenta are $p_1 = (m_1, 0)$, $p_2 = (E_2, \boldsymbol{p}_2)$ with scalar product $p_1 \cdot p_2 = m_1 E_2$. It follows that $v_{\rm rel} = |\boldsymbol{p}_2|/E_2 = \sqrt{E_2^2 - m_2^2}/E_2$ can be written as[6]

$$v_{\rm rel} = \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{p_1 \cdot p_2}. \qquad (18)$$

Using (12) and $p_1 \cdot p_2 / E_1 E_2 = 1 - \boldsymbol{v}_1 \cdot \boldsymbol{v}_2$, in terms of velocities Eq. (18) becomes

1605.00569

$$v_{\rm rel} = \frac{\sqrt{(\boldsymbol{v}_1 - \boldsymbol{v}_2)^2 - (\boldsymbol{v}_1 \times \boldsymbol{v}_2)^2}}{1 - \boldsymbol{v}_1 \cdot \boldsymbol{v}_2}. \qquad (19)$$

Cannoni, 1605.00569

# Möller velocity: in a nutshell

## From the flux

The Möller velocity cancels the transformation of densities in the expression for the flux.

From Eq. (17) and (18) the invariant flux is 1605.00569

$$F = n_1 n_2 \frac{\cancel{p_1 \cdot p_2}}{E_1 E_2} \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{\cancel{p_1 \cdot p_2}},$$

which gives Eq. (8), or, in terms of velocities,

$$F = n_1 n_2 \cancel{(1 - \boldsymbol{v}_1 \cdot \boldsymbol{v}_2)} \frac{\sqrt{(\boldsymbol{v}_1 - \boldsymbol{v}_2)^2 - (\boldsymbol{v}_1 \times \boldsymbol{v}_2)^2}}{\cancel{1 - \boldsymbol{v}_1 \cdot \boldsymbol{v}_2}},$$

Cannoni, 1605.00569

# Energy transfer kinematics

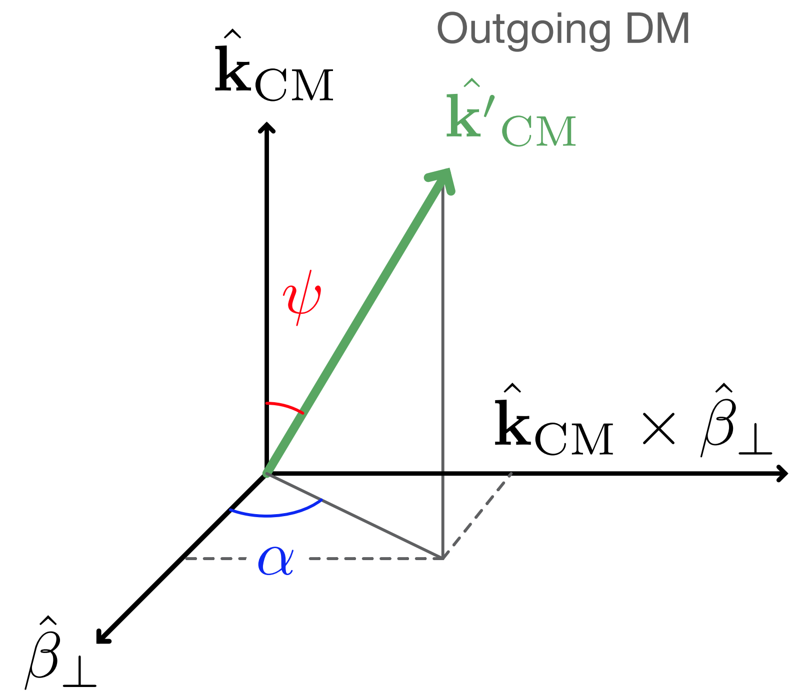
DM momentum

$$k^\mu_{\text{CM}} = \begin{pmatrix} \gamma & -\gamma\boldsymbol{\beta} \\ -\gamma\boldsymbol{\beta} & \gamma \end{pmatrix} \begin{pmatrix} E_k \\ \mathbf{k} \end{pmatrix}$$

Target momentum

$$p^\mu_{\text{CM}} = \begin{pmatrix} \gamma & -\gamma\boldsymbol{\beta} \\ -\gamma\boldsymbol{\beta} & \gamma \end{pmatrix} \begin{pmatrix} m_{\text{T}} \\ \mathbf{0} \end{pmatrix}$$

$$\mathbf{p}_{\text{CM}} + \mathbf{k}_{\text{CM}} = 0 \qquad \boldsymbol{\beta} = \frac{\mathbf{k}}{E_{\text{esc}} + m_{\text{T}}}$$



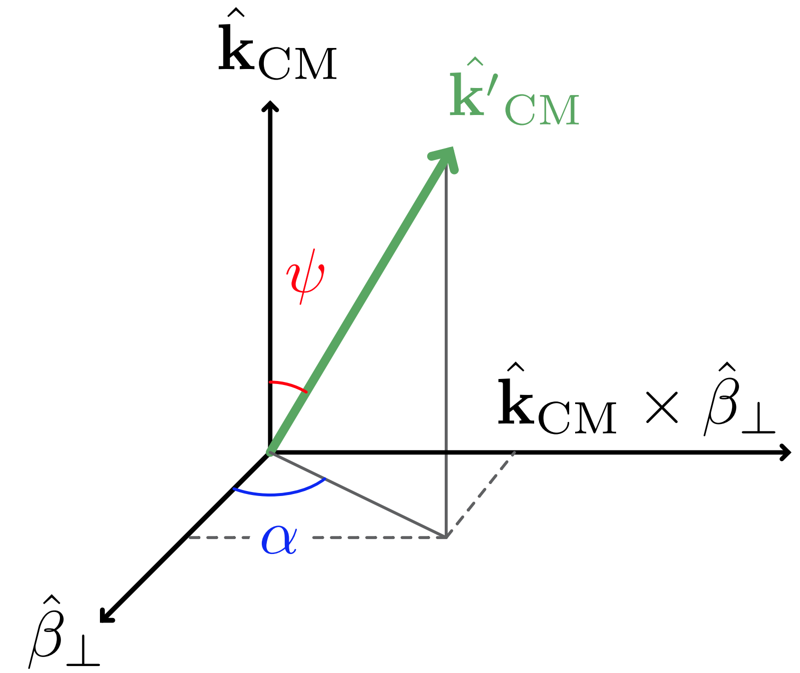

$$q^\mu_{\text{CM}} = k^\mu_{\text{CM}} - k'^\mu_{\text{CM}} = (0, \mathbf{q}_{\text{CM}})^T$$

$$\Delta E = q^0 = \gamma\boldsymbol{\beta}\cdot\mathbf{q}_{\text{CM}} = \frac{\gamma\mathbf{k}\cdot\mathbf{q}_{\text{CM}}}{E_k + m_{\text{T}}} = \frac{\gamma^2 m_{\text{T}}\mathbf{k}^2(1-\cos\psi)}{(E_k+m_{\text{T}})^2}$$

$$\Delta E = \frac{m_{\text{T}} m_\chi^2}{m_\chi^2 + m_{\text{T}}^2 + 2\gamma_{\text{esc}} m_\chi m_{\text{T}}} \frac{v_{\text{esc}}^2}{1 - v_{\text{esc}}^2}(1-\cos\psi) \ ,$$

# Energy transfer: relativistic

$$\Delta E = E_k - E_{k'} = \gamma\left[(E_k)_{\text{CM}} + (E_k)_{\text{CM}}\right] + \gamma\boldsymbol{\beta}\cdot(\mathbf{k}_{\text{CM}} - \mathbf{k}'_{\text{CM}}) = \gamma\boldsymbol{\beta}\cdot\mathbf{q}_{\text{CM}}$$




$$\Delta E = \gamma\boldsymbol{\beta}\cdot\left[\mathbf{k}_{\text{CM}}\,(1-\cos\psi) - k_{\text{CM}}\sin\psi\,\cos\alpha\,\hat{\boldsymbol{\beta}}_\perp\right]$$

$$= \gamma(\boldsymbol{\beta}\cdot\mathbf{k}_{\text{CM}})\,(1-\cos\psi) - \gamma\sqrt{\boldsymbol{\beta}^2\,\mathbf{k}^2_{\text{CM}} - (\boldsymbol{\beta}\cdot\mathbf{k}_{\text{CM}})^2}\,\sin\psi\,\cos\alpha\,.$$

$$\boldsymbol{\beta}\cdot\mathbf{k}_{\text{CM}} \equiv \beta k_{\text{CM}}\cos\delta = \frac{E_p k^2 - E_k p^2 + (E_p - E_k)\mathbf{p}\cdot\mathbf{k}}{E E_{\text{CM}}}$$

# Maximum energy transfer

$$\frac{\Delta E}{\gamma\beta k_{\text{CM}}} = \cos\delta\ (1-\cos\psi) - |\sin\delta|\cos\alpha\ \sin\psi$$

We may succinctly write the conditions for the maximum energy transfer as

$$\cos\alpha = -1 \qquad \cos\psi = -\cos\delta \qquad \sin\psi = |\sin\delta| = \sqrt{1-\cos^2\delta}$$

$$\frac{\Delta E_{\text{max}}}{\gamma\beta k_{\text{CM}}} = \cos\delta(1+\cos\delta) + \sin^2\delta = \cos\delta + 1$$

$$\cos\delta = \frac{E_p k^2 - E_k p^2 + (E_p - E_k)\mathbf{p}\cdot\mathbf{k}}{E\beta E_{\text{CM}} k_{\text{CM}}}$$

One may then evaluate this in various limits.

# Heuristics for phase space scaling

## … hand-wavy, but tested

**Rule of Thumb 1** (Independent Integration Assumption). *We assume that the phase space integrals are independent of one another. For simplicity, we ignore the dependence on phase space integrals in the differential cross section, $d\sigma/d\Omega_{\text{CM}}$. This is sufficient to understand the scaling behavior with respect to the dark matter mass.*

**Rule of Thumb 2** (Weak Condition). *First $\Delta E > 0$. This is a sufficient, but not necessary condition.*

**Corollary of Thumb 1** (Weaker condition). $\cos\delta > 0$ *is a sufficient condition that $\Delta E > 0$ for a unsuppressed part of phase space. This is a sufficient, but not necessary condition.*

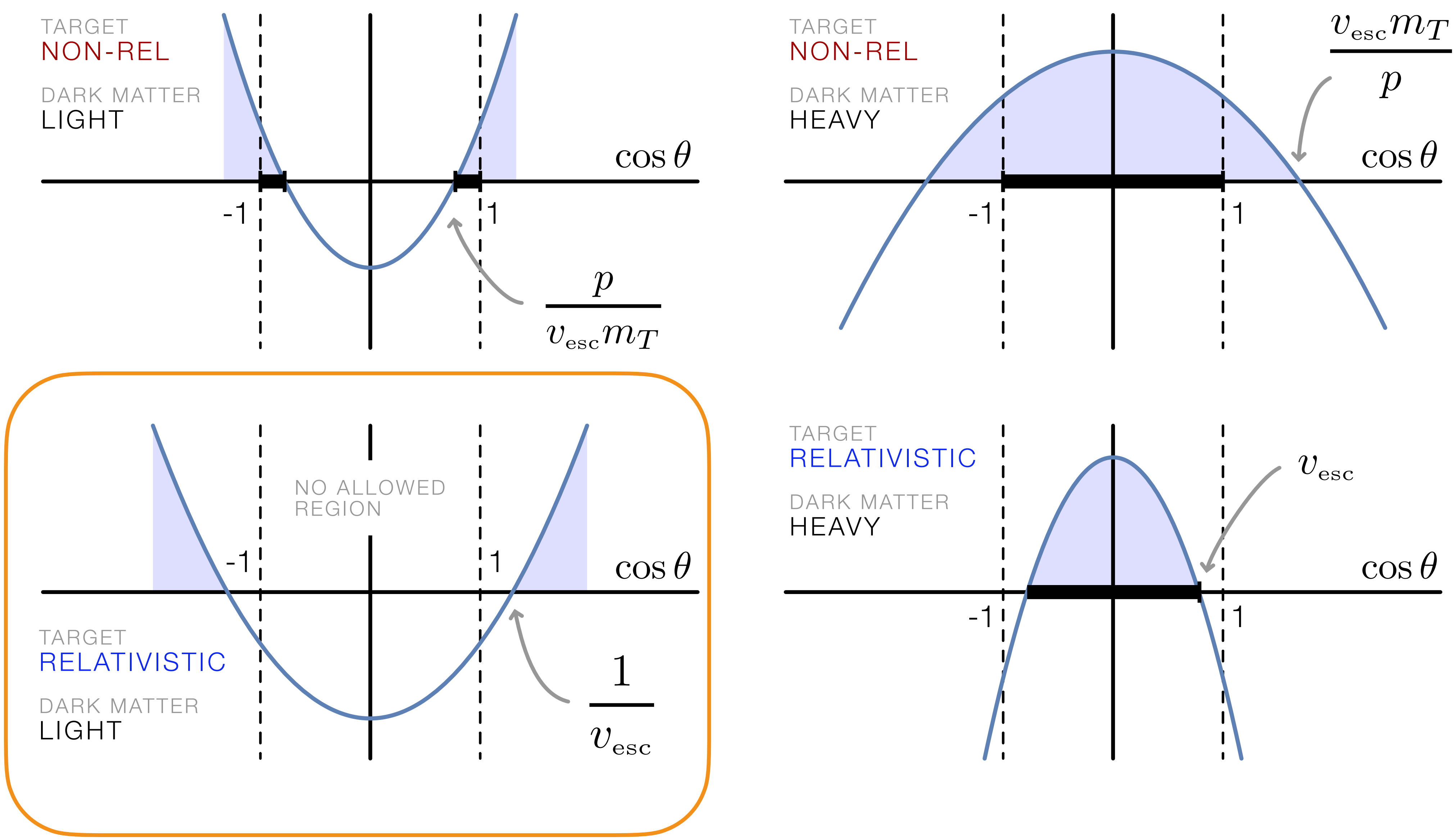
*Proof.* This comes from positivity of the right-hand side and the range $0 \leq \psi \leq \pi$, since $\psi$ is a polar angle. □

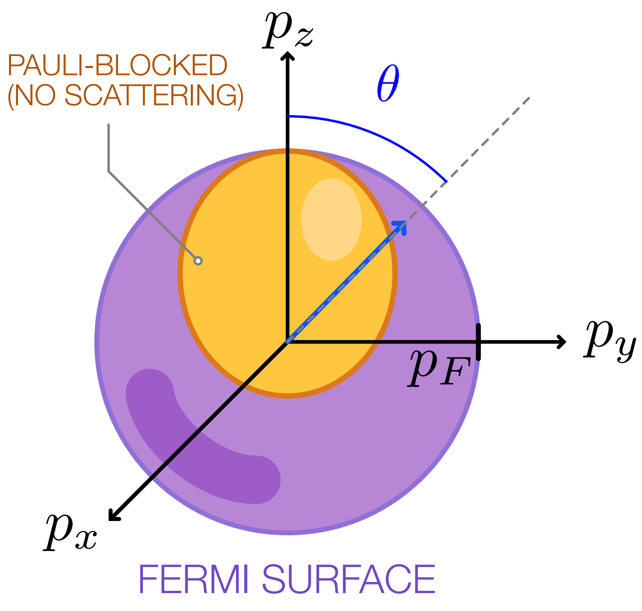
**Rule of Thumb 3** (Strong Condition). *The phase space for the initial target momentum must be large enough that the outgoing target after scattering has momentum larger than the Fermi momentum. For this diagnostic, we check relative to the maximum kinematically allowed energy transfer, $\Delta E_{max}$:*

$$p + \Delta E_{max} > p_F \ . \tag{F.4}$$

2004.09539

# Initial state suppression




**Corollary of Thumb 1** (Weaker condition). $\cos\delta > 0$ *is a sufficient condition that* $\Delta E > 0$ *for a unsuppressed part of phase space. This is a sufficient, but not necessary condition.*

$$\left(m_{\text{T}}^2 + p^2\right)\left(\gamma_{\text{esc}}^2 v_{\text{esc}}^2 m_\chi^2 + p\gamma_{\text{esc}} v_{\text{esc}} m_\chi \cos\theta\right)^2 > \gamma_{\text{esc}}^2 m_\chi^2 \left(p^2 + p\gamma_{\text{esc}} v_{\text{esc}} m_\chi \cos\theta\right)$$