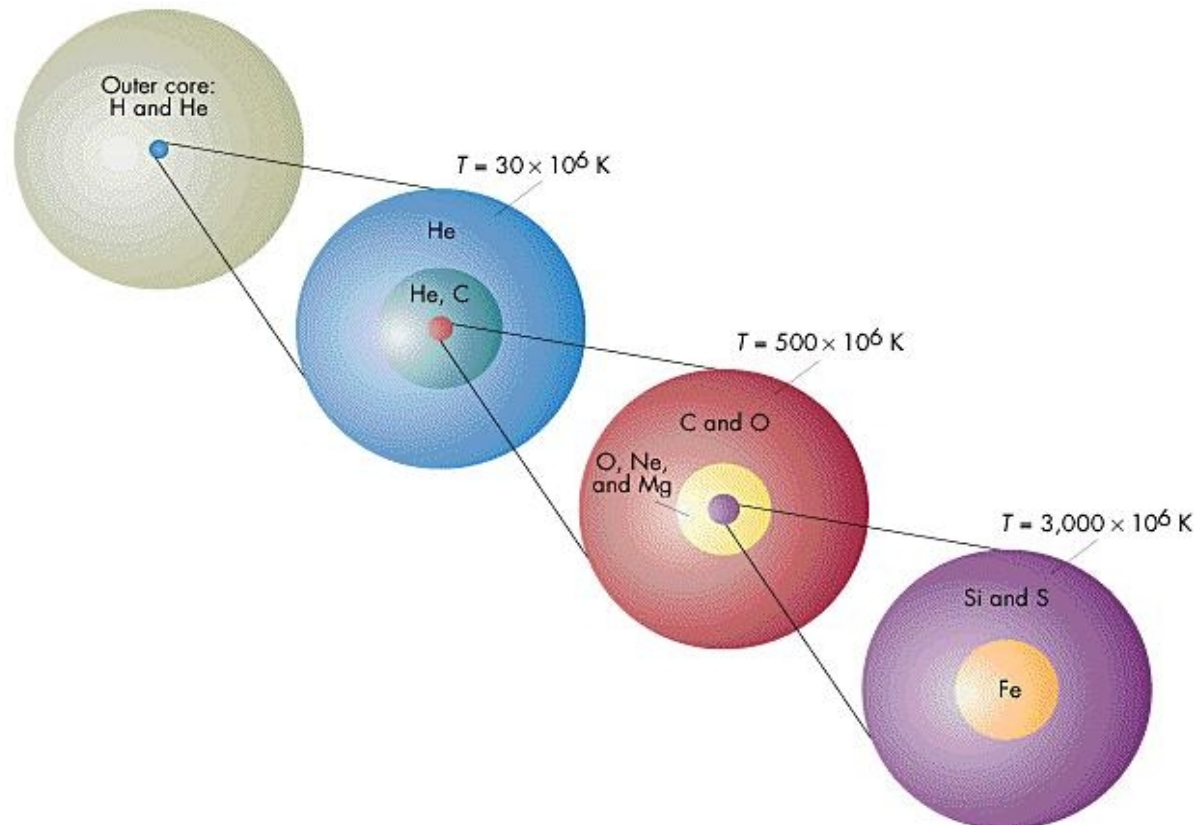


Supernova Theory: Models

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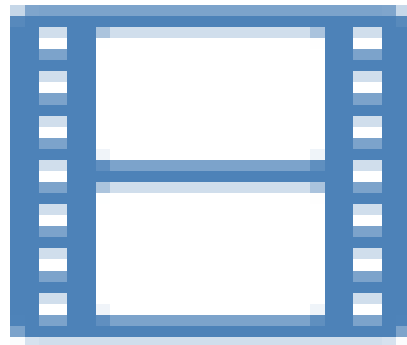


- Towards the end of their lives, stars with initial masses greater than $\sim 8 M_{\odot}$ will have a layered (onion-like) internal structure.
- Each layer is a mix of elements: from the inside out: iron, silicon, sulfur, neon, oxygen, carbon, helium, hydrogen.
- The star can be as large as 10^7 km across: the iron core has a radius of ~ 1000 km.



- When the iron core grows too massive and hot, it begins to collapse due to gravity.
- The collapse is halted when supernuclear densities are reached. The core rebounds which will create a shockwave that propagates outwards into the star.
- The energy released by the collapse is approximately 3×10^{53} ergs – more energy than the Sun emits in its entire lifetime.
- The nuclei in the infalling matter fragment into neutrons and protons. Electron capture reactions turn most of the protons into neutrons producing lots of neutrinos.
- The hot ball of neutrons is a proto-neutron star (PNS).
- The high temperatures in the PNS also produces copious amounts of neutrinos.
- Neutrinos are the principle transporter of energy, entropy, and lepton number.

- We have been simulating core-collapse supernovae for ~60 years often with the world's biggest / best computers.
 - The first simulations assumed spherical symmetry for the explosion i.e. they were in 1D.
 - In the 80s the symmetry was relaxed to cylindrical i.e. 2D
 - We can now do simulations in 3D (no imposed symmetry) and with ever improving microphysics and spatial fidelity.
- 3D simulations regularly explode and indicate that the explosion is due to a combination of neutrino heating and turbulence.



courtesy of David Calvert (NCSU)

- Core-collapse simulations in 3D are computationally expensive and still require some mixture of approximations to make them feasible.
 - The approximations are typically in the neutrino transport but simplifications appear elsewhere too e.g. the nucleosynthesis.
- One important piece of missing physics is the flavor transformation of neutrinos.
 - but see [Stapleford, Fröhlich & Kneller, PRD, 102, 081301 \(2020\)](#)

Supernova neutrino signals

- In addition to their importance in the explosion mechanism, neutrinos provide a way of observing the core of a supernova.
- Neutrinos are also the indicator that a supernova is occurring in the Milky Way (or nearby).
 - The neutrinos will get to Earth hours to days before the surface of the star explodes.
- The SNEWS 2.0 collaboration is working to improve the alert and the observational follow-up campaign.
- A core-collapse supernova at the center of the Milky Way will produce ~2500 - 6000 events in DUNE.
- We can do a lot with this signal.

- A lot of different physics determines the ability of neutrinos to transport energy, entropy and lepton number in a supernova, and the signal we receive from a supernova.

Neutrinos

- Neutrino mass ordering
- Number of ν flavors
- Self-interaction effects,
- MSW effects,
- Turbulence effects
- Non-standard interactions,
- Non-standard self interactions
- Magnetic moments,
- SUSY,
-

Nuclear Physics / Astrophysics

- Progenitor structure,
- Neutrino opacities,
- Equation of State,
- Stalled shock duration,
- Shock position / velocity,
- Standing Accretion Shock Instability,
- Lepton Emission Self-sustained Asymmetry (LESA)
- Nucleosynthesis conditions,
-

- Supernovae make great neutrino laboratories.

- To compare with theory we need to turn a simulation into an expected detector signal.
- Most of the simulations cannot be used in their raw form because they don't account for the flavor transformation:
- Flavor transformation occurs at three different places:
 - flavor transformation within the SN,
 - the decoherence outside the SN on the trip to Earth,
 - Earth matter effects
- To study supernova neutrino flavor transformation we can post-process a simulation. This only works if the flavor transformation is not important for the explosion.
 - these calculations often make a number of approximations of their own to make them feasible.
- Computing the flavor transformation well by post-processing a single simulation can take $10^4 - 10^6$ core hours.

SNEWPY

- SNEWPY is a software pipeline in four parts to avoid the bottleneck between simulations and detector signals.

Baxter et al, ApJ, **925**, 107 (2022)

- SNEWPY has four parts:
 - **generate**: takes simulation neutrino data and turns it into a time series of neutrino fluxes and/or a total fluence at Earth by convolving with a prescription for the flavor transformation.
 - **simulate**: sends the time series through the SNOwGLoBES software for all the neutrino detectors SNOwGLoBES can model.
 - **SNOwGLoBES**: takes a neutrino spectral fluence and computes the number of events in various channels for neutrino detectors.
 - **collate**: collates the output from SNOwGLoBES into observable channels.

- **SNEWPY** comes with data for ~300 supernova simulations and 15 prescriptions for the flavor transformation.

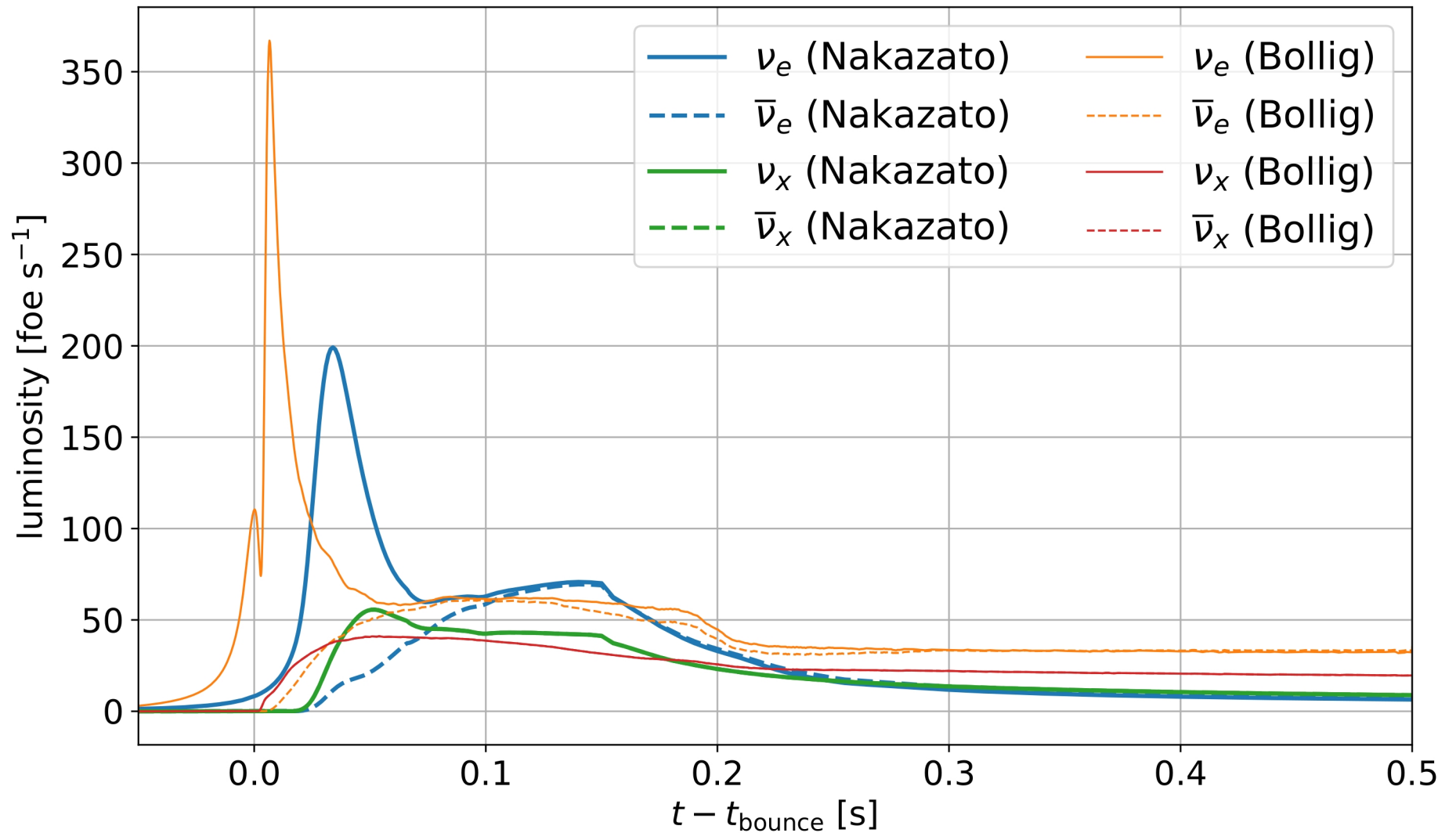
For three flavors:

- No Oscillations and Complete Exchange
- Adiabatic MSW in both mass orderings
- NonAdiabatic MSW H resonance in both mass orderings
- Two Flavor Decoherence at the H resonance in both mass orderings.
- Three Flavor Decoherence
- Neutrino Decay of the heaviest mass state to the lightest with variable lifetime and neutrino mass, in both mass orderings,

For four flavors:

- Adiabatic MSW of four flavors in both mass orderings
- MSW where the 'outer' es resonance is non adiabatic, for both mass orderings.

- SNEWPY has a lot of flexibility.
- Its custom interfaces with the simulation data standardizes the extraction of the data.



Experiment	Type	Mass [kt]	Location	11.2 M _⊙	27.0 M _⊙	40.0 M _⊙
Super-K	H ₂ O/ $\bar{\nu}_e$	32	Japan	4000/4100	7800/7600	7600/4900
Hyper-K	H ₂ O/ $\bar{\nu}_e$	220	Japan	28K/28K	53K/52K	52K/34K
IceCube	String/ $\bar{\nu}_e$	2500*	South Pole	320K/330K	660K/660K	820K/630K
KM3NeT	String/ $\bar{\nu}_e$	150*	Italy/France	17K/18K	37K/38K	47K/38K
LVD	C _n H _{2n} / $\bar{\nu}_e$	1	Italy	190/190	360/350	340/240
KamLAND	C _n H _{2n} / $\bar{\nu}_e$	1	Japan	190/190	360/350	340/240
Borexino	C _n H _{2n} / $\bar{\nu}_e$	0.278	Italy	52/52	100/97	96/65
JUNO	C _n H _{2n} / $\bar{\nu}_e$	20	China	3800/3800	7200/7000	6900/4700
SNO+	C _n H _{2n} / $\bar{\nu}_e$	0.78	Canada	150/150	280/270	270/180
NOνA	C _n H _{2n} / $\bar{\nu}_e$	14	USA	1900/2000	3700/3600	3600/2500
Baksan	C _n H _{2n} / $\bar{\nu}_e$	0.24	Russia	45/45	86/84	82/56
HALO	Lead/ ν_e	0.079	Canada	4/3	9/8	9/9
HALO-1kT	Lead/ ν_e	1	Italy	53/47	120/100	120/120
DUNE	Ar/ ν_e	40	USA	2700/2500	5500/5200	5800/6000
MicroBooNe	Ar/ ν_e	0.09	USA	6/5	12/11	13/13
SBND	Ar/ ν_e	0.12	USA	8/7	16/15	17/18
DarkSide-20k	Ar/any ν	0.0386	Italy	-	250	-
XENONnT	Xe/any ν	0.006	Italy	56	106	-
LZ	Xe/any ν	0.007	USA	65	123	-
PandaX-4T	Xe/any ν	0.004	China	37	70	-

Table from SNEWS 2.0 whitepaper

Al Kharusi et al, New Journal of Physics, **23**, 031201 (2021)

Quantum Supernova Simulations

- Many recent studies of neutrinos in core-collapse supernovae indicate flavor transformation might occur close to the proto-neutron star and therefore affect the dynamics.

Sawyer, PRD **72** 045003 (2005)

Izaguirre, Raffelt & Tamborra, PRL **118** 021101 (2017)

Abbar et al, PRD **100** 043004 (2019)

Johns et al, PRD **101** 043009 (2020)

and many many more

- We need to include flavor transformation into simulations.

How hard can it be?

- The spatial resolution of the simulations will have to increase considerably.
 - The best current CCSN simulations have grid zones ~100m - 1km
 - The oscillation lengthscale around the PNS is ~10 microns
- As the spatial grid zones become smaller, the time steps shrink.
- Including quantum neutrino transformation in simulations will increase the runtime of even a 1D simulation by a lot.
 - a back-of-the-envelope estimate is that a 1D simulation with quantum transport would be $\sim 10^{14}$ more expensive than a classical sim.
 - it takes a good supernova code such as 1D Agile-Boltztrann ~100 to 1000 core hours to run to 1 s postbounce.
- How can we reduce the computational expense of a quantum supernova without losing too much of the physics?

Neutrino oscillations with moments

Myers et al, arXiv:2111.13722

- Many classical supernova simulation codes calculate the neutrino transport using angular moments.
- It is possible to generalize a classical moment to a quantum moment, and to do neutrino transformations with them.

Strack and Burrows, PRD **71** 093004 (2005)

Zhang and Burrows, PRD **88** 105009 (2013)

- A quantum angular moment is defined as

$$M_n(q) = \int q \cos^n \theta F d\Omega$$

- where q is the energy of the neutrino, θ the angle relative to the radial direction, F is the neutrino distribution matrix
- The first few moments have well-known names
 - $n = 0$ is the (differential) energy density E_q
 - $n = 1$ is the (differential) radial component of the energy flux F_q
 - $n = 2$ is the 'rr' component of the (differential) pressure tensor P_q

- The moments evolve according to

$$\frac{\partial E_q}{\partial t} + \frac{\partial F_q}{\partial r} + \frac{2F_q}{r} = -i[H_V + H_M + H_E, E_q] + i[H_F, F_q]$$

$$\frac{\partial F_q}{\partial t} + \frac{\partial P_q}{\partial r} + \frac{3P_q - E_q}{r} = -i[H_V + H_M + H_E, F_q] + i[H_F, P_q]$$

⋮

- the absorption / emission / collisions have been omitted, H_V is the vacuum Hamiltonian, H_M the matter Hamiltonian, H_E and H_F are the two contributions to the self-interaction,
- The infinite tower of equations can be truncated at what ever level one desires.
 - Typically one uses a one-moment (M0) or a two-moment (M1) truncation.
- We need an additional relationship between the moments in order to solve the equations.
- This relationship is called 'The Closure'

Are moment-based approaches any good?

- We want to compare moment-based approaches to quantum neutrino transport against less-approximate methods.
- One comparison can be made with ‘multi-angle calculations’ which are based on the neutrino Bulb Model.

Duan et al PRL **97** 241101 (2006)

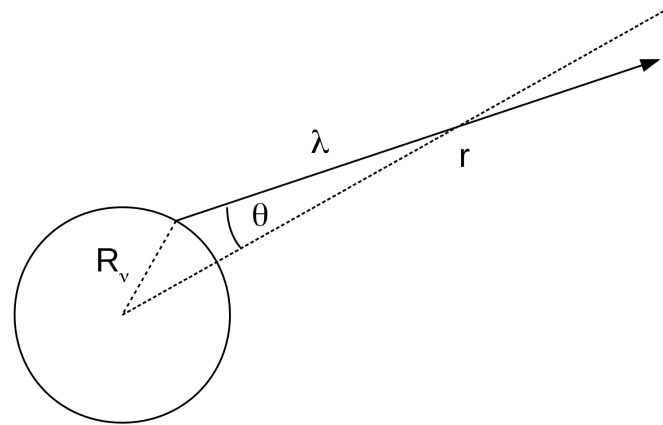
- The neutrinosphere is a hard surface with spherically symmetric neutrino emission.
- No collisions or absorption / emission beyond the neutrinosphere.
- The neutrino field is in steady state.
- The neutrino field has axial symmetry around the radial direction.

- We used a set of neutrino spectral parameters which produce a flavor instability close to the neutrinosphere.

	L [ergs/s]	$\langle E \rangle$ [MeV]	T [MeV]	η
ν_e	2.05×10^{49}	9.4	2.1	3.9
$\bar{\nu}_e$	2.55×10^{49}	13	3.5	2.3
ν_x	1.975×10^{48}	15.8	4.4	2.1
$\bar{\nu}_x$	1.975×10^{48}	15.8	4.4	2.1

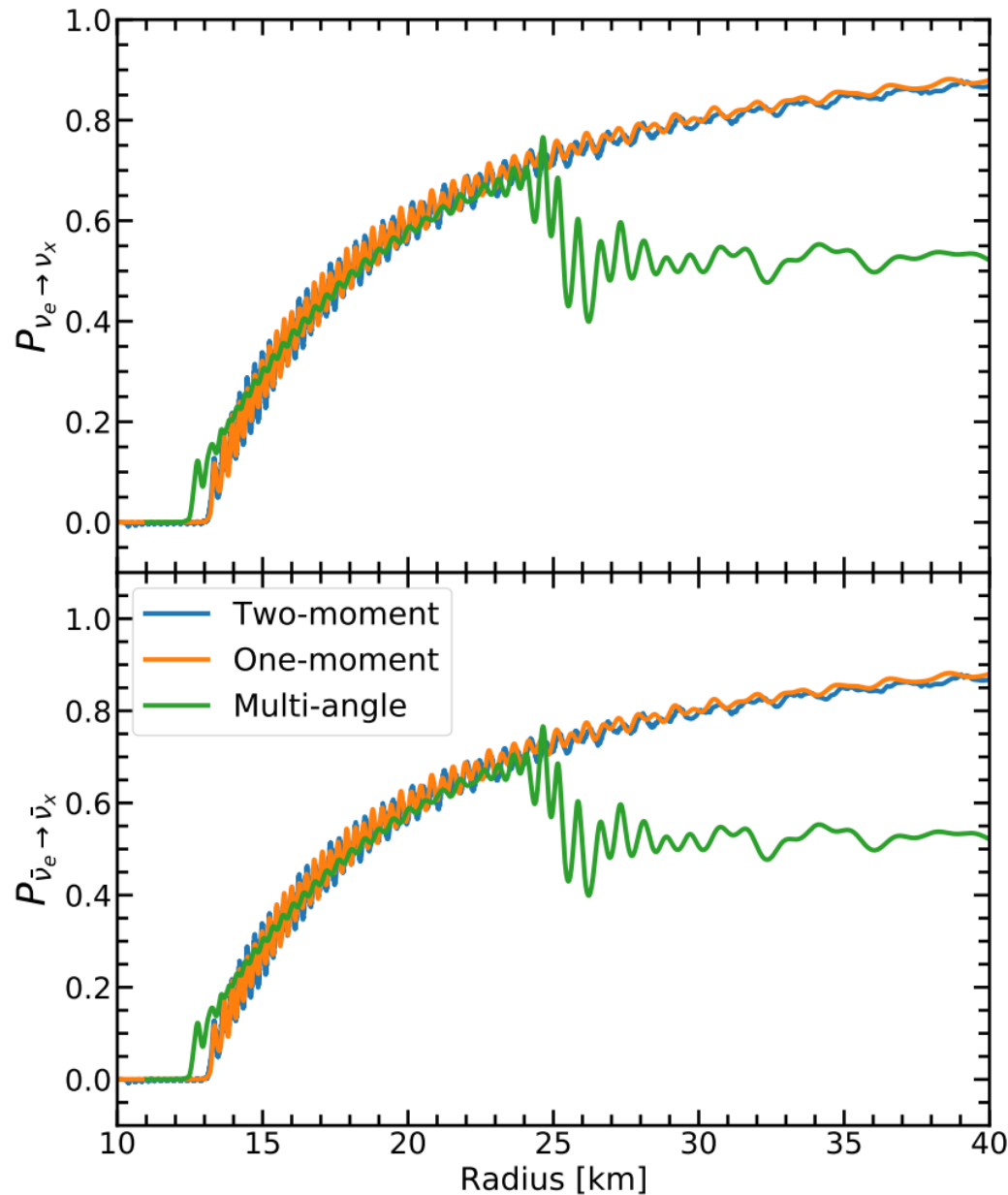
- For the **M0** moment calculation, we use for the closure the exact relation between the flux and energy density in the limit of no oscillations

$$F_q = \frac{(1 + \cos \theta_{max})}{2} E_q$$



- where θ_{max} is the largest angle between the neutrino velocity vectors at some radius r , and the radial direction.
- For the **M1** calculation the closure is the exact relation between the energy density and the pressure in the limit of no oscillations.

$$P_q = \frac{(1 - \cos^3 \theta_{max})}{3(1 - \cos \theta_{max})} E_q$$



- The different approaches are in agreement about where the instability occurs.
- The multi-angle separates from the moments at ~ 23 km.
- The M0 and M1 calculations are almost identical.

The moment code is ~ 100 times faster than a multi-angle code.

Summary

- 3D simulations of supernovae explode and we really need a nearby supernova to test our theories.
- **SNEWPY** bridges the gulf between supernova simulations and detector signals.
- **Version 1.2 is publicly available.**
 - <https://github.com/SNEWS2/snewpy/releases/tag/v1.2>
- Please send me your suggestions for new features. My email is jim_kneller@ncsu.edu
- **Despite some issues that need to be addressed, moments are promising, less computationally expensive, approach to including quantum neutrino transport in simulations.**