



Clustering with Light (but Massive) Relics

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LiMRs

- **Light (but Massive) Relics** are a general class of new particles which are light enough to be **relativistic after BBN** ...
- ... but are **non-relativistic today**
- so today they contribute to the **matter density** ...
- ... and may even have done so at recombination
- but they are not necessarily **cold dark matter**
 - even when non-relativistic, they may be fast enough to **free-stream** out of gravitational potentials
 - could lead to **deviations** from standard Λ CDM cosmology
- similar to neutrinos, warm/hot dark matter



eV-scale LiMRs and N_{eff}

- eV-scale LiMRs are interesting because they can reconcile energy injection in the early Universe with CMB constraints on ΔN_{eff}
- what's the issue?
 - consider a process which injects dark radiation in the early Universe (near or after BBN)
 - decay of dark sector matter, first-order phase transition, etc.
 - there are tight constraints on the amount of dark radiation at the time of recombination from CMB observations
 - for dark sector FOPT, constraints on latent heat also limit the amplitude of GW waves, making them hard to detect
- can avoid ΔN_{eff} constraints if dark radiation starts to redshift like matter before recombination



why eV-scale?

- for this purpose, we're most interested in LiMRs near **eV-scale** ... why?
- for $m_\chi \ll \text{eV}$, likely relativistic at recomb. \rightarrow **constrained by ΔN_{eff}**
- for $m_\chi \gg \text{eV}$, likely redshifting as matter well before matter-radiation equality ($T_{\text{MRE}} \sim 1 \text{ eV}$), in radiation-dominated epoch
 - density fraction then grows rapidly until around MRE
 - would end up with **too much matter**, unless injected radiation was very small
 - defeats the purpose
- **eV-scale LiMRs** are at the **sweet spot**
 - if eV-scale LiMRs are an **$O(1)$ fraction of radiation** at the time on injection, remain so until about MRE
 - at MRE, they start to redshift like matter, and can be an **$O(1)$ fraction of matter**
 - avoid **constraints from ΔN_{eff}**



but what's the catch?

- the catch is that even if eV-scale LiMRs are non-relativistic at recombination, they **still may not be cold dark matter**
- if LiMRs are fast enough, will **free-stream** out of small gravitational potentials
- this essentially **erases matter structures**
- this effect is constrained by **weak lensing of the CMB**, for example
- one of the ways we know that all dark matter can't be warm/hot
- so what can we add to this story?



goal

- this strategy has been considered before ... (Xu, Munoz Dvorkin 2107.09664, etc.)
- ... but under specific **assumptions** about the LiMR **cosmological history**
 - assumptions about LiMR temperature, coupling to SM, clustering
 - often lead to very **tight constraints** on LiMR mass, abundance
- we will **loosen assumptions** on LiMR cosmological history
- will find that **LiMRs can be an $O(1)$ fraction of the DM without erasing structure** in conflict with observation
- at some level, this is obvious \rightarrow if you make the LiMR cold enough, it can't free-stream
- but we will quantify this, using some **approximations**



what we do and don't do

- we **don't perform a global analysis** of any model
- only focusing on constraints arising from the **growth of structure**, parameterized by the **mass** and **effective temperature** of the LiMRs
- for any particular model, there are a variety of **other constraints** which depend on the properties and initial conditions of all other sectors
 - precise values of Ω_m , Ω_{cdm} , m_ν , etc.
- we instead perform an **analytic analysis** of how LiMRs affect the structure growth, illustrating **degeneracy** between LiMRs and other **unknowns**



overview of how matter clusters

- assume LiMR is a spin-0 particle
 - mass = m_x
 - effective temperature $T_x = r T_\gamma$
- decoupled while relativistic
- T_x determines occupancy of momentum modes
- redshifts like matter today
- c_i = propagation speed of i^{th} species $\sim [\delta\rho_i/\bar{\rho}_i]^{1/2}$
- δ_i = Fourier trans. of matter density contrast ($\delta\rho_i/\bar{\rho}_i$)

$$f(p) = \frac{1}{\exp[p/rT_\gamma] - 1}$$

$$f_x \equiv \frac{\rho_x}{\rho_M} = 10^{-2} \frac{m_x}{0.075 \text{ eV}} r^3$$

$$c_x \sim \frac{\langle p_x \rangle}{m_x} = 3T_\gamma \frac{r}{m_x}$$



growth of structure

- starting point is **continuity** and **Euler eqns.**
- perturbations **grow** due to **grav. pot.** from all matter overdensities
- perturbations **damped** by **Hubble expansion** and **pressure**
- assume **matter-dominated** era
 - $a \propto t^{2/3}$, $H = 2/3t$
- define $\alpha_i = (3/2)(kc_i t/a)^2 = (k/k_{fs})^2$
- k_{fs} = comoving **free-streaming wavenumber**
- determines k for which **pressure matters**

$$\frac{d^2\delta_i}{dt^2} + 2H\frac{d\delta_i}{dt} = -\frac{k^2}{a^2}c_i^2\delta_i + \frac{3}{2}H^2\sum_j f_j\delta_j$$

Hubble pressure gravity

$$\frac{d^2\delta_i}{dt^2} + \frac{4}{3t}\frac{d\delta_i}{dt} = -\frac{2}{3t^2}\alpha_i\delta_i + \frac{2}{3t^2}\sum_j f_j\delta_j$$



erasing structure

- say $\alpha \gg 1$
 - pressure dominates over grav.
 - approximate α as constant
- density contrast has damped oscillation
- intuition \rightarrow particles stream out of potentials
 - suppresses structure
- α is time-dep., but won't need the details of how δ damps out

$$\frac{d^2\delta_i}{dt^2} + \frac{4}{3t} \frac{d\delta_i}{dt} \approx -\frac{2}{3t^2} \alpha_i \delta_i$$

$$\delta_i \propto t^\gamma$$

$$\gamma = -\frac{1}{6} \pm \frac{i}{6} \sqrt{24\alpha - 1}$$



clustering, and its suppression

- consider species for which $\alpha \ll 1$
- collectively, contrast is δ_{cl}
- f_{fs} = free-streaming matter frac.
 - doesn't source grav. pot.

- if $f_{fs} = 0$ (all matter clusters), density grows as $\delta \propto t^{2/3} \propto a$
- free-streaming has **two effects**
 - reduces **fraction of matter which clusters**
 - suppresses the **grav. pot** which causes structures to grow

$$\frac{d^2\delta_{cl}}{dt^2} + \frac{4}{3t} \frac{d\delta_{cl}}{dt} \approx \frac{2}{3t^2} (1 - f_{fs}) \delta_{cl}$$

$$\delta_{cl} \propto t^\gamma$$

$$\gamma_+ = \frac{2}{3} - \frac{2}{5} f_{fs} + O(f_{fs}^2)$$



weak lensing power spectrum

- growth of matter structures can be probed by **lensing of the CMB**
- we are interested in the CMB lensing convergence power spectrum C_{ℓ}^{KK}
- **integrated** effect over **all conformal times** from recomb. (η_*) to now (η_0)
 - $d\eta/dt = 1/a \rightarrow ds^2 = a^2 [d\eta^2 - dx^2]$
- so at fixed angular scale (ℓ), we are interested in **all comoving scales** which could subtend the angular scale at some conformal time
- at **large ℓ** which obs. probe, dominated by transverse wave numbers (**Limber approximation**)
 - $k = \ell / (\eta_0 - \eta)$ = comoving wavenumber
- $P(k; \eta)$ = matter power spectrum = $\langle \delta\delta \rangle \propto k^{-3} t^{4/3}$ (if no free-streaming)

$$C_{\ell}^{KK} \cong 2\pi^2 \ell \int_{\eta_*}^{\eta_0} d\eta \eta \left[\frac{9\Omega_m^2(\eta)(aH)^4}{8\pi^2} \frac{P(\ell / (\eta_0 - \eta); \eta)}{\ell / (\eta_0 - \eta)} \right] \left(\frac{\eta_* - \eta}{(\eta_0 - \eta_*)(\eta_0 - \eta)} \right)^2$$



analytic result

- if matter free-streams, power spectrum suppressed \rightarrow suppresses C_{ℓ}^{KK}
- two effects
 - only a fraction of matter clusters, ...
 - and structures grow more slowly
- but clustering only suppressed while X redshifts as matter
- and only suppressed if $k > k_{fs}$, which depends on η
- we will approximate break as sharp \rightarrow either $\alpha \ll 1$ or $\alpha \gg 1$
 - at any η and k , either X clusters or doesn't
- how is lensing affected by LiMRs which don't always free-stream?



suppression of lensing

- dimensionless variables
 - $\tilde{\eta} = \eta / (3t_0)$, $\tilde{k} = k (3t_0)$
 - X redshifts as matter after $\tilde{\eta}_x$
- F encodes **suppression of clustering** ($O(f_x)$)
 - if all matter clusters, $F=1$
- X **free-streams at sufficiently late times**, but may not at early times

$$C_\ell^{\text{KK}} \propto \int_{\tilde{\eta}_*}^{\tilde{\eta}_0} d\tilde{\eta} \tilde{\eta} (\tilde{\eta}_* - \tilde{\eta})^2 (\tilde{\eta}_0 - \tilde{\eta})^2 \times F(\tilde{\eta}; \ell)$$

$$\tilde{\eta}_x = \left[71.4 \left(\frac{m_x / r}{0.05 \text{eV}} \right) \right]^{-1/2}$$

$$\tilde{k}_{\text{fs}} = 174 \frac{m_x / r}{0.05 \text{eV}} \tilde{\eta}$$

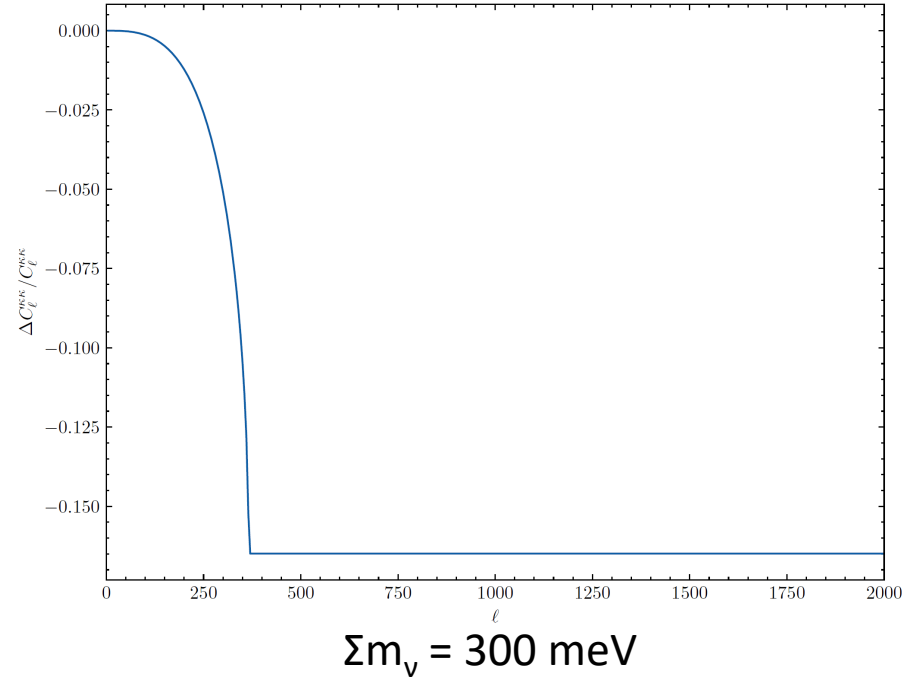
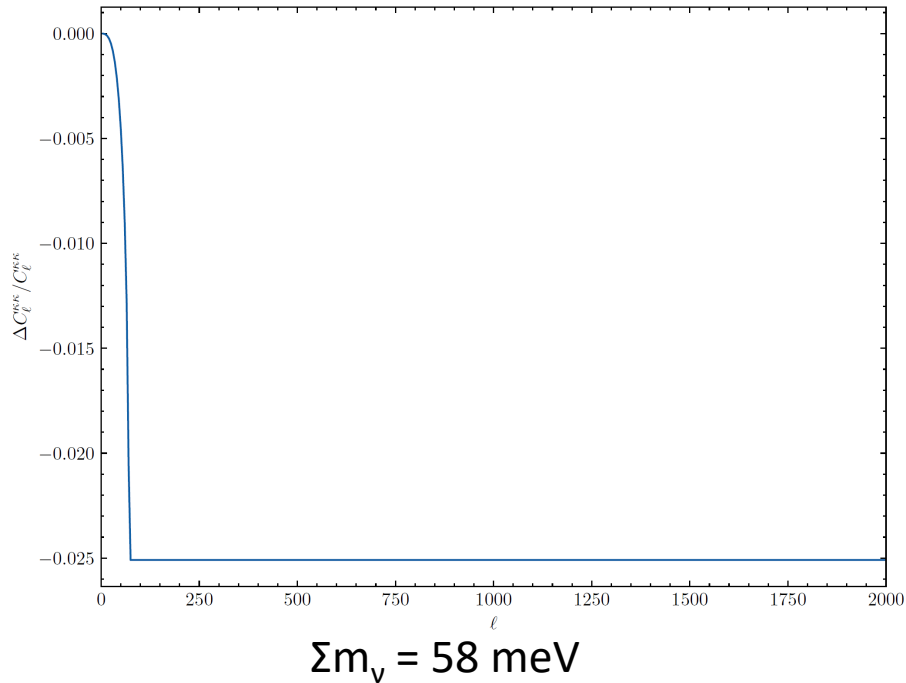
$$F = \exp \left[\ln \left(1 - 2f_x - \frac{12}{5} f_x \ln \frac{\tilde{\eta}}{\max[\tilde{\eta}_x, \tilde{\eta}_*]} \right) \times \theta(\tilde{\eta} - \tilde{\eta}_x) \times \theta \left(\frac{\tilde{\ell}}{\tilde{\eta}_0 - \tilde{\eta}} - \tilde{k}_{\text{fs}} \right) \right]$$

fraction which clusters

growth suppression



let's apply to neutrinos

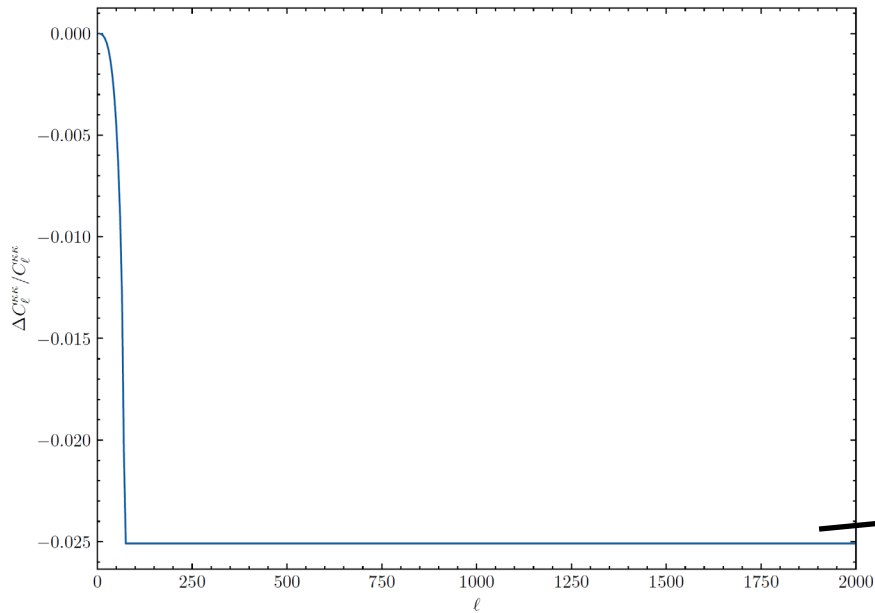


plotting $[C_{\ell}^{\text{KK}} - C_{\ell}^{\text{KK}}(F=1)] / C_{\ell}^{\text{KK}}(F=1)$ as a function of ℓ ; observ. probe $\ell \sim 2000$

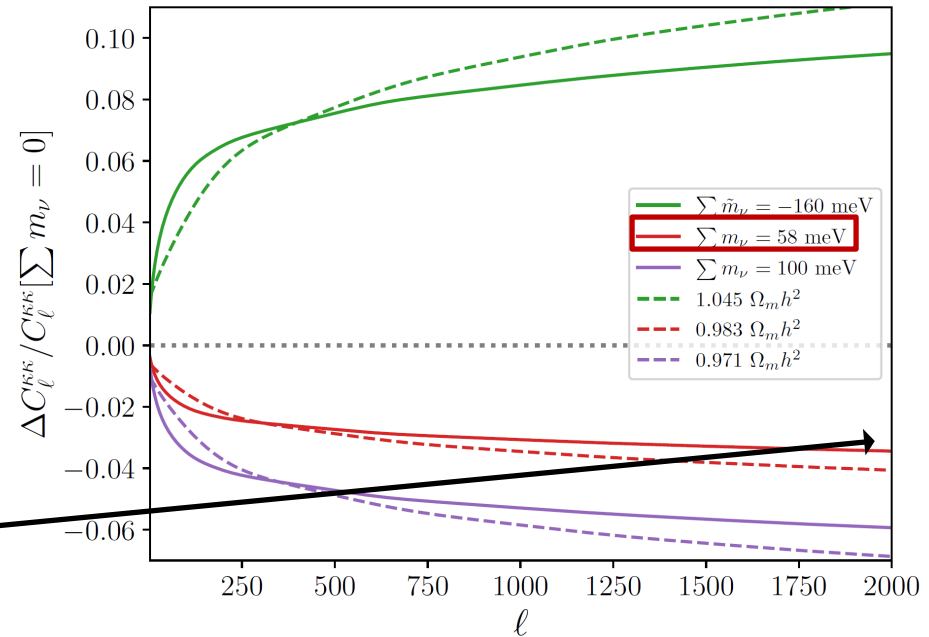
curve flattens at large ℓ , since ν free-streams at all relevant $k \rightarrow$ never clusters



comparison to full numeric result



$\Sigma m_\nu = 58 \text{ meV}$



Craig, Green, Meyers, Ragendran
2405.00836



caveats

- **shape** of lensing power spectrum does not match full numerical calculation of Boltzmann equations, but **asymptotic value does**
- we essentially took $\Omega_M = 1$, which **fails at late times**
 - doesn't matter much at large ℓ , but for smaller ℓ , late times more relevant
- for a **complete analysis**, need to make many **model/cosmology choices**
 - Ω_{cdm} , Ω_b , etc.
 - but we want to estimate the effect of X free-streaming on clustering, without fixing other choices
- in fact, both X and neutrinos will free-stream, so we should include both
 - we are computing to linear order in f , so **contributions add**
- we are computing **one piece of several**



our approach

- in some sense, that is one lesson from the DESI preference for “**negative neutrino mass**” in Λ CDM (DESI 2404.03002)
- really a preference for **additional CMB lensing** beyond expectation
 - interpreted as a preference for negative m_ν in a cosmology in which that contribution is least constrained by cosmology
 - but the lensing power spectrum is a sum of **several contributions**, and one of them has to give
- that’s the **approach we will take** here
- if the contribution of X to CMB lensing lies in the range of neutrino models which are largely allowed by current constraints ...
- ... then this contribution can be **hidden within neutrino slop** which we don’t know, so is likely **allowed by lensing**

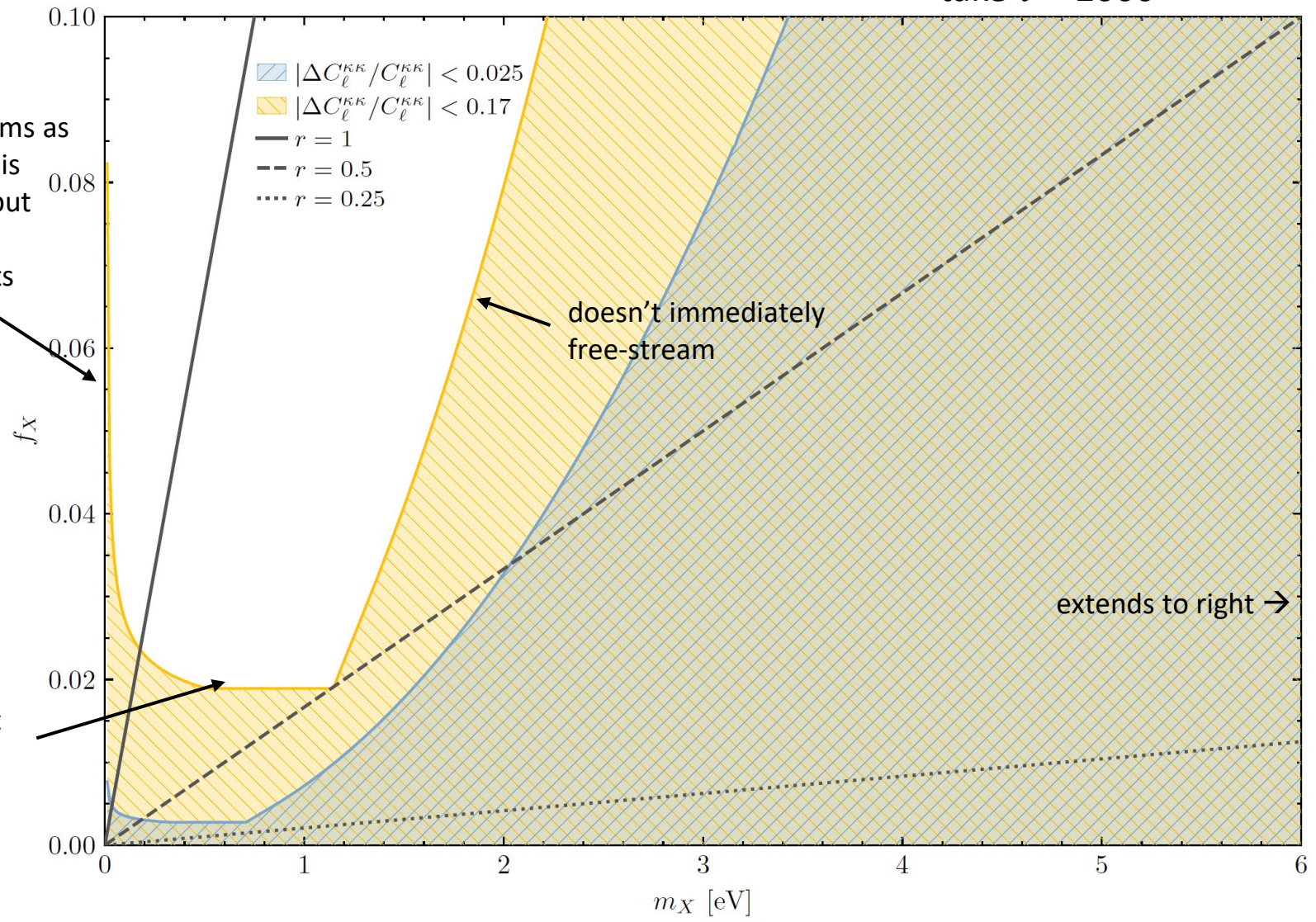


result

take $\ell = 2000$

free-streams as soon as it is non-rel., but has ΔN_{eff} constraints

non-rel. at recomb.





upshot

- eV-scale LiMRs can be **consistent with lensing constraints**
 - intuitively sensible ... no matter how light, **if LiMR is slow enough, it will cluster**
- early deposition of relativistic particles **may not be constrained by ΔN_{eff}**
- if they are **LiMRs**, then main constraint will be from **clustering**, which **can be satisfied**
- but a **detailed** model and cosmology are needed to **test all CMB constraints**
 - need to specify neutrino sector, Ω_{cdm} , etc.
 - with a complete model, can go beyond linear approx. to a **full numerical calculation**, but main result should be **robust**

conclusion

- a non-negligible fraction of dark matter could be **eV-scale LiMRs**, consistent with constraints on clustering
- if cold enough, don't free-stream until very late
- can avoid tight constraints on **dark radiation** by hiding it as LiMRs

- prospects for **direct and indirect detection** of eV-scale DM?



Backup Slides



clustering matter

$$f_{cl} = 1 - f_{fs} = \sum_{i \in cl} f_i$$

$$f_{cl} \delta_{cl} = \sum_{i \in cl} f_i \delta_i$$

$$\frac{d^2 \delta_i}{dt^2} + 2H \frac{d\delta_i}{dt} \approx \frac{3}{2} H^2 \sum_{j \in cl} f_j \delta_j$$

$$\frac{d^2 \left(\sum_{i \in cl} f_i \delta_i \right)}{dt^2} + 2H \frac{d \left(\sum_{i \in cl} f_i \delta_i \right)}{dt} \approx \frac{3}{2} H^2 \left(\sum_{i \in cl} f_i \right) \sum_{j \in cl} f_j \delta_j$$

$$f_{cl} \frac{d^2 \delta_{cl}}{dt^2} + 2H f_{cl} \frac{d\delta_{cl}}{dt} \approx \frac{3}{2} H^2 f_{cl} (f_{cl} \delta_{cl})$$

$$\frac{d^2 \delta_{cl}}{dt^2} + 2H \frac{d\delta_{cl}}{dt} \approx \frac{3}{2} H^2 (1 - f_{fs}) \delta_{cl}$$